1 HMMs

Consider the following Hidden Markov Model.

Suppose that we observe $O_1 = a$ and $O_2 = b$.
Using the forward algorithm, compute the probability distribution $P(W_2|O_1 = a, O_2 = b)$ one step at a time.

(a) Compute $P(W_1, O_1 = a)$.

(b) Using the previous calculation, compute $P(W_2, O_1 = a)$.

(c) Using the previous calculation, compute $P(W_2, O_1 = a, O_2 = b)$.

(d) Finally, compute $P(W_2|O_1 = a, O_2 = b)$. 
2 Particle Filtering

Let’s use Particle Filtering to estimate the distribution of \( P(W_2|O_1 = a, O_2 = b) \). Here’s the HMM again.

We start with two particles representing our distribution for \( W_1 \).

\( P_1 : W_1 = 0 \)

\( P_2 : W_1 = 1 \)

Use the following random numbers to run particle filtering:

\[
[0.22, 0.05, 0.33, 0.20, 0.84, 0.54, 0.79, 0.66, 0.14, 0.96]
\]

(a) **Observe**: Compute the weight of the two particles after evidence \( O_1 = a \).

(b) **Resample**: Using the random numbers, resample \( P_1 \) and \( P_2 \) based on the weights.

(c) **Predict**: Sample \( P_1 \) and \( P_2 \) from applying the time update.

(d) **Update**: Compute the weight of the two particles after evidence \( O_2 = b \).

(e) **Resample**: Using the random numbers, resample \( P_1 \) and \( P_2 \) based on the weights.

(f) What is our estimated distribution for \( P(W_2|O_1 = a, O_2 = b) \)?