derivation of beam bending equation

new segment length $ds$
$$ds = \frac{dx}{\cos \theta}$$

slope of $w(x)$
$$\frac{dw}{dx} = \tan \theta$$

radius of curvature $\rho$
$$ds = \rho d\theta$$

small angle approximation
$$ds \approx dx$$

$w(x)$ – neutral axis as a function of position along the original beam $x$
derivation of beam bending equation

\[ \frac{d^2 w}{dx^2} = -\frac{M}{EI} \]
\[ \frac{d^3 w}{dx^3} = -\frac{V}{EI} \]
\[ \frac{d^4 w}{dx^4} = \frac{q}{EI} \]

internal bending moment \( M \)
point shear force \( V \)
distributed load \( q \)

\[ \theta \approx \frac{dw}{dx} \]
\[ \frac{d\theta}{dx} \approx \frac{1}{\rho} \]

substitute
\[ \frac{1}{\rho} = \frac{d^2 w}{dx^2} \]
\[ \frac{d^4 w}{dx^4} = \frac{q}{EI} \quad \text{how to solve?} \]

- Laplace or Fourier transforms
- eigenfunction expansion
- **guess and check**

\[ w = A + Bx + Cx^2 + Dx^3 + Ex^4 \quad \text{trial solution} \]

\[ \frac{dw}{dx} = B + 2Cx + 3Dx^2 + 4Ex^3 \]

\[ \frac{d^2 w}{dx^2} = 2C + 6Dx + 12Ex^2 \quad \text{boundary conditions} \]

\[ \frac{d^3 w}{dx^3} = 6D + 24Ex \]

\[ \frac{d^4 w}{dx^4} = 24E \]
\[ \rho = \frac{EI}{W} \]

\[ I = \frac{\rho W}{E} \]