Recap: DFT (Cosine Transform) and OMP in Imaging Context

DFT - Motivated by wireless (search for a basis that would be an eigen-basis for "channels with echoes") → modeled using circulant matrices.

Important Properties of DFTs:
1. The DFT basis is an eigenbasis for all circulant matrices of DFT.
2. DFT basis is orthonormal.
3. The $p^{th}$ eigenvalue of $C_{\vec{h}}$ is just $\sqrt{n} \langle \vec{u}_p, \vec{h} \rangle = \sqrt{n} (\vec{u}_p^* \vec{h})$, where $C_{\vec{h}}$ is a circulant matrix in the time domain with $\vec{h}$ as its first vector.

The DFT complex basis can be transformed using Euler’s formula ($e^{i\theta} = \cos(\theta) + i \sin(\theta)$) into a sine/cosine basis which is real.

$V$ is basis:

$$
\begin{bmatrix}
\cos 0 \\
\cos 1 \\
\sin 1 \\
\cos 2 \\
\sin 2
\end{bmatrix}
$$

$C_{\vec{h}}$ is circulant and real:

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Exercise: Look at the $2 \times 2$ blocks on the diagonals. Show these are scaled “rotation” matrices.

Fact: We can view real vectors as sums of phase-shifted cosines.