**PROBLEM SET #8**

*Issued: Tuesday, Nov. 4, 2014*

*Due: Wednesday, Nov. 12, 2014, 8:00 a.m. in the EE 143 homework box near 140 Cory*

- **Ion Implantation/Diffusion**

1. Consider the following cross-section that is to be doped with As using ion implantation to form the source/drain regions. Assume the Si substrate is initially doped with B with a uniform concentration of $10^{16}$ cm$^{-3}$.

   ![Cross-section diagram](https://via.placeholder.com/150)

   (a) Assume that the SiO$_2$ and polysilicon layers have the same ion stopping power as Si, and that SiO$_2$ thickness is 60 nm. What are the ion implantation dose and energy required to achieve a peak concentration of $10^{19}$ cm$^{-3}$ of As at the SiO$_2$ and Si interface in the source/drain regions (i.e., $y = 60$ nm)?

   Projected range, $R_p = 60$ nm $\rightarrow$ Required energy = 105 keV and straggle, $\Delta R_p = 23$ nm [From graph]

   Thus, the total dose $Q = \sqrt{2\pi\Delta R_p}N_p = 5.76 \times 10^{13} cm^{-2}$

   (b) Continuing from (a), calculate the junction depth of the source/drain regions.

   Background doping concentration, $N_B = 10^{16} cm^{-3}$

   Junction depth, $x_j = R_p + \Delta R_p \sqrt{2\ln\left(\frac{N_p}{N_B}\right)} = 145.49 nm$

   So the source/drain junctions are 85.49nm deep in the silicon substrate.

   (c) What is the minimal thickness of the gate polysilicon for the polysilicon and SiO$_2$ stack to serve as an effective implantation mask that decreases the As concentration in the channel region below 1/10th the background concentration?

   Assuming the gate thickness is $T_G$.

   Dopant concentration in the channel is $N(T_G + T_{ox}) = N_p \exp\left(-\frac{(T_G + T_{ox} - R_p)^2}{2(\Delta R_p)^2}\right)$

   For $N(T_G + T_{ox}) = N_B/10,$
\[ T_G = R_p + \Delta R_p \sqrt{2\ln\left(\frac{10N_p}{N_B}\right)} - T_{ox} = 98.71\text{nm} \]

So the gate thickness has to be \( > 98.71\text{nm} \).

(d) Continuing from (a), a following drive-in step at 1100°C yields a final junction depth of 2 \( \mu \text{m} \) (counted from the SiO\(_2\) and Si interface). Estimate the final sheet resistance in the S/D regions.

Total dose in silicon, 
\[ Q' = \frac{Q}{2} = \frac{5.76 \times 10^{13}}{2} \text{cm}^{-3} = 2.88 \times 10^{13}\text{cm}^{-2}. \]

Assuming a Gaussian distribution,
\[ Q' = N_0\sqrt{\pi Dt} = 2.88 \times 10^{13}\text{cm}^{-2} \quad (i) \]

While the junction depth is found to be
\[ x_j = 2\sqrt{Dt \ln \left(\frac{N_0}{N_B}\right)} = 2\mu\text{m} = 2 \times 10^{-4}\text{cm} \quad (ii) \]

Solving equations (i) and (ii) for \( Dt \) we obtain,
\[ Dt = 2.94 \times 10^{-9}\text{cm}^2 \]

which is indeed very larger than \( Dt/\Delta R_p^2 \).

Hence \( N_0 = \frac{Q'}{\sqrt{\pi Dt}} = 3 \times 10^{17}\text{cm}^{-3} \)

According to Irvin’s curves, for background concentration, \( N_B = 10^{16}\text{cm}^{-3} \) and surface concentration, \( N_0 = 3 \times 10^{17}\text{cm}^{-3} \), sheet resistance-junction depth product \( = R_s x_j = 9 \times 10^2 = 900\text{ohm} - \mu\text{m} \)

Since \( x_j = 2\mu\text{m} \), \( R_s = 450\text{ohms/square} \)

(e) Continuing from (d), estimate the required drive-in time.

The diffusion coefficient of arsenic at 1100°C is found to be
\[ D = 0.32\exp\left(-\frac{3.56 \times 1.6 \times 10^{-19}}{138 \times 10^{-25} \times 1373}\right) = 2.73 \times 10^{-14}\text{cm}^2/\text{sec} \]

So the diffusion time is found to be
\[ t = \frac{2.94 \times 10^{-9}}{2.73 \times 10^{-14}} = 107,355.4\text{sec} = 29.82\text{hr} \]

2. Problem 4.3, 4.4, 4.6, and 4.19 in the textbook.
(Note: Problem 4.4(b): Fig. 4.21 \( \rightarrow \) Fig. 4.12; Problem 4.6 (c): Fig. 4.20(e) \( \rightarrow \) Fig. 4.11)
Problem 4.3 in the textbook.
A boron diffusion into a 1-ohm-cm n-type wafer results in a Gaussian profile with a surface concentration of $5 \times 10^{18} \text{ cm}^{-3}$ and a junction depth of $4 \mu\text{m}$.

(a) How long did the diffusion take if the diffusion temperature was 1100°C.

Ans:
The background concentration of a 1-ohm-cm wafer is $N_B = 4.5 \times 10^{15} \text{ cm}^{-3}$ [From textbook figure 4.8]

For a Gaussian profile, $x_j = 2 \sqrt{Dt \ln \left( \frac{N_0}{N_B} \right)} = 4 \mu\text{m} = 4 \times 10^{-4} \text{ cm}$

Diffusion coefficient at 1100°C is $D = 10.5 \exp \left( -\frac{3.69 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-25} \times 1373} \right) = 2.996 \times 10^{-13} \text{ cm}^2/\text{sec}$.

Time required for diffusion, $t = \frac{1}{D} \frac{x_j^2}{4 \ln \left( \frac{N_0}{N_B} \right)} = 19037 \text{ s} = 5.29 \text{ hr}$

(b) What was the sheet resistance of the layer?

Ans:

![Figure 1: Surface impurity concentration versus sheet-resistance-junction depth product](image)

Using Fig. 1, for background concentration, $N_B = 4.5 \times 10^{15} \text{ cm}^{-3}$ and surface concentration, $N_0 = 5 \times 10^{18} \text{ cm}^{-3}$, sheet resistance-junction depth product $R_s x_j = 3.3 \times 10^2 = 330 \text{ ohm - } \mu\text{m}$

Since $x_j = 4 \mu\text{m}$, $R_s = 82.5 \text{ ohms/square}$

(c) What is the dose in the layer?

Ans: Dose in the layer, $Q = N_0 \sqrt{D t \pi} = 6.69 \times 10^{14} \text{ cm}^{-2}$.
(d) The boron dose was deposited by a solid-solubility-limited diffusion. Design a diffusion schedule (temperature and time) for this predeposition step.

**Ans:** For a solid-solubility limited diffusion, \( Q = 2N_0 \sqrt{Dt/\pi} = 6.69 \times 10^{14} \text{cm}^{-2} \)

Assuming the diffusion temperature in the predeposition step to be 950°C, the boron surface concentration is found to be \( N_0 = 1.4 \times 10^{20} \text{cm}^{-3} \) [From Figure 2].

It leads to

\[
Dt = \pi \frac{Q^2}{4N_0^2} = 1.79 \times 10^{-11} \text{cm}^2
\]

A temperature of 950°C yields a diffusion coefficient of \( D = 10.5 \exp \left( -\frac{3.69 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-25} \times 1223} \right) = 6.54 \times 10^{-15} \text{cm}^2/\text{sec} \).

So required time, \( t = 2742.26 \text{ sec} = 45.7 \text{ min} \)

**Problem 4.4 in the textbook.**

The boron diffusion in Problem 4.3 is followed by a solid-solubility-limited phosphorus diffusion for 30 mins at 950°C. Assume that the boron profile does not change during the phosphorus diffusion.

(a) Find the junction depth of the new phosphorus layer.

Assume an erfc profile.

**Ans:**

Since the diffusion temperature in the predeposition step to be 950°C, the phosphorus surface concentration is found to be \( N_{01} = 7 \times 10^{20} \text{cm}^{-3} \) [From Figure 2].

Diffusion time, \( t_1 = 30 \text{ min} = 1800 \text{ sec} \). A temperature of 950°C yields a diffusion coefficient of \( D_1 = 10.5 \exp \left( -\frac{3.69 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-25} \times 1223} \right) = 6.54 \times 10^{-15} \text{cm}^2/\text{sec} \),

which leads to \( D_1 t_1 = 1.176 \times 10^{-11} \text{cm}^2 \).

Now the background doping includes uniform n-type doping of \( 4.5 \times 10^{15} \text{cm}^{-3} \) and Gaussian boron distribution.

\( Dt \) product for Gaussian boron diffusion \( Dt = 5.7 \times 10^{-9} \text{cm}^2 \gg D_1 t_1 (\text{erfc}) \)

So at the junction of the new phosphorus layer the boron concentration can be assumed same as the surface concentration.

Background concentration for phosphorus diffusion,

\[
N_{B1} = N_0 - N_B = 5 \times 10^{18} - 4.5 \times 10^{15} \approx 5 \times 10^{18} \text{cm}^{-3}
\]

For an erfc profile, junction depth \( x_{j1} = 2 \sqrt{D_1 t_1 \text{erfc}^{-1} \left( \frac{N_{B1}}{N_{01}} \right)} = 0.13 \mu m \)

Hence the junction depth for the phosphorus layer is 0.13 \( \mu m \).

At \( x = 0.13 \mu m \), boron concentration= \( N_0 \exp \left( -\frac{x^2}{4D_1t} \right) = 4.96 \times 10^{18} \text{cm}^{-3} \), which is almost same to \( N_0 \). So the assumption regarding the boron concentration is valid.
(b) Find the junction depth based on the concentration-dependent diffusion data presented in Fig. 4.12.

**Ans:** Using the concentration-dependent diffusion data in Figure 3, we obtain that at 950°C for 30 min diffusion of phosphorus,

\[ N(x) = N_{B1} = 5 \times 10^{18} \text{cm}^{-3} \]

when \( x = 0.75 \mu m \).

So the junction depth for the phosphorus layer is 0.75\( \mu m \). At \( x = 0.75 \mu m \), boron concentration

\[ N_0 \exp \left( -\frac{x^2}{4D_t} \right) = 3.9 \times 10^{18} \text{cm}^{-3}, \]

which is a little different than \( N_0 \). Still the assumption may be used for simplification.

(c) Calculate the total \( D_t \) product for Problem 4.3 and compare the result to the \( D_t \) product for this problem. Is the assumption in the problem statement justified?

**Ans:**

For problem 4.3,

\[ (D_t)_{predeposition} = 1.79 \times 10^{-11} \text{cm}^2 \]

\[ (D_t)_{drive-in} = (D_t)_{total} - (D_t)_{predeposition} = 5.7 \times 10^{-9} - 1.79 \times 10^{-11} \approx 5.7 \times 10^{-9} \text{cm}^2 \]

\[ (D_t)_{total} = 5.7 \times 10^{-9} \]

For problem 4.4,

\( D_t \) product of boron drive-in diffusion during the high-temperature phosphorus diffusion step is

\( (D_t)_{drive-in_2} = D_2 t_2 \)

Diffusion coefficient of boron at 950°C is

\[ D_2 = 10.5 \exp \left( -\frac{3.69 \times 16 \times 10^{-19}}{1.38 \times 10^{-28} \times 1223} \right) = 6.54 \times 10^{-15} \text{cm}^2/\text{sec} \]

\[ t_2 = 30 \text{min} = 1800 \text{sec} \]

Hence, \( (D_t)_{drive-in_2} = 1.176 \times 10^{-14} \text{cm}^2 \ll (D_t)_{total} \)

So the new \( D_t \) product for boron diffusion is

\[ (D_t)_{total(new)} = (D_t)_{total} + (D_t)_{drive-in_2} \approx 5.7 \times 10^{-9} \text{cm}^2 \]

which is almost same as the \( D_t \) product before phosphorus diffusion step.

So the assumption in the problem that the boron profile does not change during phosphorus diffusion can be considered justified.
Problem 4.6 in the textbook.
(a) Calculate the $Dt$ product required to form a 0.2µm-deep source-drain diffusion for an NMOS transistor using a solid-solubility-limited arsenic deposition at 1000°C into a wafer with a background concentration of $3 \times 10^{16}$ cm$^{-3}$.

**Ans:** For a solid-solubility-limited arsenic deposition, assuming a erfc profile, the junction depth will be

Since the diffusion temperature is 1000°C, the arsenic surface concentration is found to be $N_0 = 1.05 \times 10^{21}$ cm$^{-3}$ [From figure 2]. The background concentration is $N_B = 3 \times 10^{16}$ cm$^{-3}$.

$$x_j = 2\sqrt{Dt \text{ erfc}^{-1}\left(\frac{N_B}{N_0}\right)} = 0.2\mu m = 2 \times 10^{-5} cm$$

We obtain

$$\text{erfc}^{-1}\left(\frac{N_B}{N_0}\right) = \text{erfc}^{-1}(2.86 \times 10^{-5}) = 2.96$$

So the $Dt$ product is found to be

$$Dt = \frac{x_j^2}{4\left(\text{erfc}^{-1}(\frac{N_B}{N_0})\right)^2} = 1.14 \times 10^{-11} cm^2$$

(b) What is the diffusion time? Does this time seem like a reasonable process?

**Ans:** The diffusion coefficient of arsenic at 1000°C is found to be

$$D = 0.32 \exp\left(-\frac{3.56 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-25} \times 1273}\right) = 2.58 \times 10^{-15} cm^2/sec$$

So the diffusion time is found to be

$$t = \frac{1.14 \times 10^{-11}}{2.58 \times 10^{-15}} = 4422.8 \text{ sec} = 1.23 \text{ hr}$$

which is a reasonable diffusion time.

(c) Recalculate $Dt$ based upon the model in Fig. 4.11 and Table 4.2.

**Ans:** Using concentration dependent model, arsenic diffusion can be modeled by first order dependence.

$$x_j = 2.29\sqrt{N_0Dt/n_i} = 0.2\mu m = 2 \times 10^{-5} cm$$

$n_i$ at 1000°C is found to be $n_i = 1.055 \times 10^{19}$ cm$^{-3}$.

So the $Dt$ product is found to be

$$Dt = \frac{x_j^2n_i}{(2.29)^2N_0} = 7.66 \times 10^{-13} cm^2$$

Problem 4.19 in the textbook.
Gold is diffused into a silicon wafer using a constant-source diffusion with a surface concentration of $10^{18}$ cm$^{-3}$. How long does it take the gold to diffuse completely through a silicon wafer 400µm thick with a background concentration of $10^{16}$ cm$^{-3}$ at a temperature of 1000°C?

**Ans:**
The diffusion coefficient of gold at 1000°C is found to be $D = 4 \times 10^{-7}$ cm$^2$/sec

Background concentration, $N_B = 10^{16}$ cm$^{-3}$.

Surface concentration, $N_0 = 10^{18}$ cm$^{-3}$.

Impurity concentration, $N(x) = N_0 \text{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$

For $N(x = 400\mu) = N_B$, 

\[ D_t = \frac{x^2}{4\left(\text{erfc}^{-1}\left(\frac{N_R}{N_0}\right)\right)^2} = 1.208 \times 10^{-4} \text{cm}^2 \]

where, \( \text{erfc}^{-1}\left(\frac{N_R}{N_0}\right) = \text{erfc}^{-1}(10^{-2}) = 1.82 \)

So the required time is found by
\[
t = \frac{D_t}{D} = \frac{1.208 \times 10^{-4}}{4 \times 10^{-7}} = 302 \text{ sec} = 5.03 \text{ min}
\]