Stress In Thin Films

4.0 Introduction: The Theoretical Strength

Thin films are seldom used in electronic devices as structural parts to carry mechanical loads. Nevertheless, stress or strain does commonly exist in thin films as a result of constraints imposed by their substrates. A thin film and its substrate generally have different thermal expansion coefficients, so stress is produced during temperature changes occurring in deposition and annealing. Stress in thin films is known to cause yield and reliability problems in microelectronic devices. In some optoelectronic devices the stress can affect the actual device properties. In this chapter, we shall discuss the nature of stress in thin films, the chemical potential in a stressed solid, and the time-dependent response of a solid to applied stresses.

A piece of solid is under stress when its atoms are displaced from their equilibrium positions by a force. The displacement is governed by the interatomic potential. It is well known that the potential \( \phi \) and the internal force \( F = -\frac{\partial \phi}{\partial r} \) between two atoms as a function of interatomic distance generally obey the schematic relations shown in Fig. 2.1a and Fig. 2.1b. When we consider instead an applied external force, we define

\[
F_{ex} = +\frac{\partial \phi}{\partial r}
\]  

(4.1)

where we have changed the sign to be positive compared to the internal force between two atoms. An external tensile force tends to lengthen the solid and in turn to increase the interatomic distance. On the basis of the sign convention given in Chapter 2, Section 2.2, a force which increases the interatomic distance is positive, and hence the tensile force (or stress) is positive. An external compressive force (or stress) which tends to shorten the solid is negative. The interatomic potential, the external force, and the sign of the force are shown schematically in Fig. 4.1a to 4.1c, respectively.

Clearly, Fig. 4.1b is an inverted diagram of Fig. 2.1b. We define the point \( F_{\text{max}} \) to be the maximum force which corresponds to the dissociative distance \( r_D \). \( F_{\text{max}} \) is the maximum tensile force needed to pull the solid apart, because the force needed to increase the interatomic distance beyond \( r_D \) is less than \( F_{\text{max}} \). We can regard \( F_{\text{max}} \) as the theoretical strength of the solid. To calculate \( F_{\text{max}} \), we require that
\[
\frac{\partial^2 \phi}{\partial r^2} = 0 \quad \text{at} \quad r = r_D. \quad \text{If we assume that the solid obeys the Lennard–Jones potential and that the function } \phi \text{ is given by Eq. (2.3), we obtain its second derivative with respect to } r,
\]

\[
\frac{\partial^2 \phi}{\partial r^2} = \varepsilon_6 \left( \frac{a_0}{r} \right)^6 \left( 13 \left( \frac{a_0}{r} \right)^6 - 7 \right) = 0 \quad \text{at} \quad r = r_D \quad (4.2)
\]

where \(a_0\) is the lattice spacing.

Theoretically, it follows that:

Furthermore, the potential energy is not true at all points in the potential or in the system. We consider the energy stored in the system, which is given by the force applied to the atoms. The force on an atom under stress is described by Hooke's Law in the elastic region, and the elastic limit of the solid such that the nature of the deformation is:

4.1 Elasticity

Consider a point in the material. If we apply a force, the material will deform.

where \(\sigma\) is the stress and is less than the yield stress.

\[
F = \frac{1}{r} \left( \frac{a_0}{r} \right)^6
\]

FIGURE 4.1 (a) Interatomic potential function plotted against interatomic distance. The dotted curve shows the anharmonicity of atomic vibration. (b) Applied force plotted against atomic displacement. (c) The direction and sign of applied force by convention.
where \( a_0 \) is the equilibrium interatomic distance. The solution of Eq. (4.2) shows that:

\[
r_{0} = 1.11a_0
\]

Theoretically the solid can be stretched (strained) by about 11% before it breaks! Furthermore, if stretched just below that strain, it would return to the original condition when the external force is removed. Experimentally, these observations are not true at all. Most polycrystalline metals, whether they obey the Lennard–Jones potential or not, have an elastic limit of only 0.2%; beyond that, plastic deformation sets in. We consider elastic behavior in 4.1.

At the equilibrium position \( a_0 \), the external force is zero, and the potential corresponds to the minimum potential energy \( e_0 \) between the atoms. At a small displacement in either direction from \( a_0 \), the force is linearly proportional to the displacement. This is the origin of the elastic behavior in a solid aggregate of atoms under stress. The elastic behavior observed is described by Hooke’s law. Within the elastic region, the displacement disappears when the force is removed. Beyond the elastic limit, permanent deformation occurs. In permanent damage, a structural ductile solid such as steel deforms by dislocation motion, but a brittle solid such as glass will deform by fracture via crack propagation. The major difference is due to the nature of chemical bonds and crystal structure in these solids.

### 4.1 Elastic Stress–Strain Relationship

Consider a piece of thin solid film of dimensions \( l \times W \times t \) as shown in Fig. 4.2. If we apply a force \( F \) to the area \( A = Wt \) to stretch the film length \( l \) by \( \Delta l \), we have

\[
\frac{F}{A} = \frac{Y \Delta l}{l} \quad \text{or} \quad \sigma = Ye
\]

(4.3)

where \( \sigma = F/A \) and \( \varepsilon = \Delta l/l \) are stress and strain, respectively, and \( Y \) is Young’s modulus. This is Hooke’s law. In addition

\[
\frac{\Delta l}{l} = \frac{\Delta W}{W} = -\nu \frac{\Delta l}{l}
\]

(4.4)

where \( \nu \) is Poisson’s ratio. This ratio is a positive number for almost all materials and is less than one-half. Notice that there is a negative sign before \( \nu \) which means

![Figure 4.2](image)

**Figure 4.2** A piece of thin film of dimensions \( l \times W \times t \) under tension.
that while we stretch $l$, both $W$ and $t$ shrink. An easy way to measure Poisson’s ratio is to observe the change in lattice parameter by x-ray diffraction in the direction normal to the tensile stress. For example, take a single crystal film of cubic crystal structure and stretch it by bending as shown in Fig. 4.3. The cubic unit cell deforms into a tetragonal cell (dashed rectangle). The interplanar spacing normal to the substrate surface decreases. This decrease is measured by the shift in the x-ray diffraction angle.

Different materials have different values of $Y$ and $v$. The elastic behavior of a polycrystalline material is characterized by just these two parameters. Sometimes other parameters such as shear modulus and bulk modulus are given, but they are interrelated. An example is given in the following discussion.

We consider shear strain and illustrate it with the example shown in Fig. 4.4a. It is a schematic diagram of the cross-sectional view of a Si chip joined by two solder joints to a ceramic pad in the flip-chip packaging scheme. During operation, the device will experience a temperature rise of $\sim100^\circ$C. Since Si expands more than the ceramic, the solder joints experience a shear strain. The strain is actually cyclic because the device is being turned on and off frequently. The cyclic strain has been found to cause reliability failure of the solder joints. Furthermore, we can imagine that if we increase the chip size, the solder joints at the edges of the chip will feel a greater shear strain. Therefore, we cannot increase the chip size arbitrarily. Shear strain is a critical factor limiting the yield of the device and also the size of the Si chip.

By shear strain, we mean (Fig. 4.4b) that the square block is deformed by a pair of forces in such a way that the bottom side is held down to prevent rotation. At equilibrium the net force and torque are zero. The shear strain $\theta$ is defined by

$$\theta = \frac{\delta}{l}$$

(4.5)

where $\delta$ is the shear displacement of material of length $l$ (Fig. 4.4b). To relate the strain to the shear stress, we translate the shear stress to a combination of tensile and compressive stresses on the rectangular block as shown in Fig. 4.4c. The tensile force is equal to $\sqrt{2}S$, yet the length and hence the area is also increased by a factor of $\sqrt{2}$. So, the tensile stress is $S/A$, which is the same magnitude as the shear stress. The compressive stress is the same except for the sign. Consider the stress-strain relation also...
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(4.5)
elate the nside and e tensile a factor ar stress. ss—strain

\[ \frac{\Delta B}{B} = \frac{1}{Y} \frac{S}{A} + \frac{-v}{Y} \left( \frac{-S}{A} \right) = \frac{1 + v}{Y} \frac{S}{A} \]

(4.6)

On the other hand, we have defined

\[ \theta = \frac{\delta}{l} = \frac{\sqrt{2} \Delta B}{l} = \frac{2 \Delta B}{B} \]

Then

\[ \theta = 2 \left( \frac{1 + v}{Y} \right) s \]

(4.7)
where \( \sigma = \frac{S}{A} \) is the shear stress. If we define the shear modulus as

\[
\mu = \frac{s}{\theta}
\]

then

\[
\mu = \frac{Y}{2(1 + \nu)} \tag{4.8}
\]

Shear stress can be regarded as a pair of normal stresses, and vice versa.

The relationship between the elastic constants for single crystals is given in Appendix B.

### 4.2 Strain Energy

It is interesting to estimate the magnitude of the energy involved in elastic strain. Consider the case at the elastic limit. The elastic energy is given by

\[
E_{\text{elastic}} = \int \sigma \, d\varepsilon = \frac{1}{2} Ye^2 \tag{4.9}
\]

To estimate the elastic energy, we take values of Young's Modulus from Table 2.4 in Chapter 2, or choose one of the stiffest materials, steel, with \( Y = 2.0 \times 10^{12} \) dyne/cm\(^2\) and \( 8.4 \times 10^{22} \) atoms in 1 cm\(^3\). We take \( \varepsilon = 0.2\% \), then

\[
E_{\text{elastic}} = \frac{1}{2} Ye^2 = 4 \times 10^6 \text{ dyne/cm}^2 \approx 3 \times 10^{-5} \text{ eV/atom}
\]

The value of elastic energy obtained is three to four orders of magnitude smaller than the typical chemical energy, say the formation energy of silicide, which is about 0.5 eV/atom. Therefore, in chemical reactions such as compound formation, the effect of elastic strain energy or stress effect is negligible. This is the reason that in measuring interdiffusion coefficients during silicide formation, the part of the driving force due to stress is ignored. We can also conclude that elastic energies are small from Fig. 4.1a. The potential energy corresponding to a strain of 0.2% is still very close to the binding energy.

Strain energy, although small, is important in cases where solids are near equilibrium. At equilibrium the forces are balanced, so any small additional force will be able to tilt the balance; it affects the equilibrium. Strain energy due to epitaxial misfit can stabilize metastable phases. Furthermore, in an epitaxial structure where the dislocation slip system is nonoperative or the nucleation of dislocations is difficult, the elastic limit can be greatly extended (for instance, up to a few percent), so that the strain energy can be two orders of magnitude greater. But when a solid is far away from equilibrium, the strain energy tends to be unimportant in most kinetic processes.
4.3 The Origin of Stress in Thin Films

There are intrinsic and extrinsic stresses in a thin film. Intrinsic stress comes from defects such as dislocations in the film. The origin of extrinsic stress in a thin film comes mainly from adhesion to its substrate. Stress can be introduced in a thin film due to differential thermal expansion between the film and its substrate, due to lattice misfit with its substrate, or due to chemical reaction with its substrate when the intermetallic compound formed is coherent to the film but has a slight lattice misfit. It has also been suggested that in thin film grain growth, the removal of grain boundaries—and hence the reduction of the excess volume in the grain boundaries—will induce stress in the film when it is constrained by the substrate.

To illustrate the stress induced by differential thermal expansion, we shall consider a Pb thin film on a Si substrate. The coefficients of thermal expansion of Pb and Si are $29.5 \times 10^{-6}/\degree C$ and $2.6 \times 10^{-5}/\degree C$, respectively. We deposit Pb on Si at room temperature and then cool them to 4.2 K where the Pb becomes superconducting. The sample experiences a temperature drop of about 300 K, and the net change in linear dimension is 0.86% for Pb. While the Pb tries to shrink, the Si substrate restricts it from doing so; hence in cooling, the Pb is under tension. The tensile stress will be relaxed to some extent because of the yielding of the Pb film. Upon heating the sample back to room temperature, the Pb tends to expand and again it is restricted by the Si substrate. The Pb is under compression upon heating. Since room temperature is about half the melting point of Pb, atomic diffusion is substantial. The Pb film will release its compressive stress partly by atomic diffusion, hence hillock formation occurs. A picture of a Pb hillock is shown in Fig. 15.4 in Chapter 15. This is a well-known phenomenon in Josephson junction devices where Pb has been used as electrodes and has experienced the temperature cycling between room temperature and 4.2 K. Hillock formation causes rupture of the ultrathin oxide layer used for junction tunneling and the device fails.

The difference in thermal expansion coefficient between the film and the substrate causes thermal stress. The thermal expansion coefficient is an intrinsic property of a pure element. Indeed large differences in thermal expansion coefficients between different kinds of materials (metals, semiconductors, and insulators) may be one of the limiting factors in growing high-quality epitaxial structures which combine these kinds of materials.

If the interatomic potential well of an atom is parabolic, the atom's displacement will be linearly proportional to the driving force. The atom undergoes a harmonic oscillation, so that its average position does not change and there is no thermal expansion upon heating. This is the same as a simple pendulum whose mean position remains unchanged in oscillation. But in reality, the interatomic potential well is not parabolic; the Lennard–Jones potential shows that resistance to compression is stronger than tension. Thus, thermal vibration tends to drive atoms apart. The stronger the vibration, the greater the separation. This leads to thermal expansion by the anharmonicity of the atomic potential. The dotted line in Fig. 4.1a depicts the increase of $a_0$ with energy or temperature. Since we can also change $a_0$ by stress...
and the change is described by Young’s modulus, the thermal expansion coefficient and this modulus are related. We recall that in Section 2.7 the relation between Young’s modulus and interatomic potential was discussed. The equation of state of solids which relates changes of pressure, temperature, and volume is given by Grüneisen’s equation (see Mott and Jones).

### 4.4 Biaxial Stress in Thin Films

On a planar substrate, the stress due to differential thermal expansion experienced by the thin film is biaxial. As shown in Fig. 4.5a, the stresses act along the two principal axes in the plane of the film, but there is no stress in the direction normal to the film free-surface, and yet there is strain in the normal direction. Another example of biaxial stress is the stresses on the surface of a balloon. The in-plane stress results in strain normal to the balloon surface and it becomes thinner as it expands. To express the biaxial stress, we start with a three-dimensional isotropic system in which the stress $\sigma$ and strain $\varepsilon$ are related by the following basic equations

$$
\varepsilon_x = \frac{1}{Y}[\sigma_x - \nu(\sigma_y + \sigma_z)]
$$

$$
\varepsilon_y = \frac{1}{Y}[\sigma_y - \nu(\sigma_x + \sigma_z)]
$$

$$
\varepsilon_z = \frac{1}{Y}[\sigma_z - \nu(\sigma_x + \sigma_y)]
$$

(4.10)

![Diagram of biaxial stress in thin films](image)

**FIGURE 4.5** (a) Biaxial stress in a thin film deposited on a rigid substrate. (b) Cross-sectional view of a thin film under compression on a bent substrate. (c) A schematic diagram showing the stress distribution in film and substrate and the corresponding forces and bending moments.
In thin films, there is stress within the plane of the film (x and y) but no stress in the z direction ($\sigma_z = 0$). Therefore

$$\varepsilon_x = \frac{1}{Y} (\sigma_x - \nu \sigma_y)$$

$$\varepsilon_y = \frac{1}{Y} (\sigma_y - \nu \sigma_y)$$

$$\varepsilon_z = \frac{-\nu}{Y} (\sigma_x + \sigma_y)$$
From these equations, we have
\[
\varepsilon_x + \varepsilon_y = \frac{1 - \nu}{\gamma} (\sigma_x + \sigma_y)
\] (4.11)
and
\[
\varepsilon_z = \frac{-\nu}{1 - \nu} (\sigma_x + \sigma_y)
\]
In two-dimensional isotropic systems where \(\varepsilon_x = \varepsilon_y\),
\[
\varepsilon_z = \frac{-2\nu}{1 - \nu} \varepsilon_x
\]
We shall apply these relations in Chapter 7 to calculate the tetragonal distortion (lattice constant change) in heteroepitaxial growth of thin films.

With thin films on a circular substrate, it is convenient to use cylindrical rather than Cartesian coordinates, and we have
\[
\sigma_r = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + 2\tau_{xy} \sin \theta \cos \theta
\]
\[
\sigma_\theta = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta - 2\tau_{xy} \sin \theta \cos \theta
\] (4.12)
where \(\tau_{xy}\) is the shear stress. If there is no curl in the stress field (\(\sigma_\theta = 0\)), we obtain
\[
\sigma_r = \sigma_x + \sigma_y
\]
Similarly
\[
\varepsilon_r = \varepsilon_x + \varepsilon_y
\]
Then
\[
\varepsilon_r = \frac{1 - \nu}{\gamma} \sigma_r \quad \text{and} \quad \varepsilon_z = \frac{-\nu}{\gamma} \sigma_z
\] (4.13)
We shall use these relationships, for example, when we consider the growth of a circular hillock in thin films under a compressive stress in Chapter 15.

The biaxial stress discussed above assumes that the substrate is rigid. We shall now consider the bending of the substrate under the biaxial stress. For example, if we deposit an Al film on a Si wafer at liquid nitrogen temperature and bring them up to room temperature, the Si wafer bends because Al has a larger thermal expansion coefficient than Si. Their cross section with a concave curvature is sketched in Fig. 4.5b. The Al film is constrained by the substrate, assuming a very good adhesion, and is under compression. The compressive stress in the Al film can be determined by measuring the curvature of the Si wafer and using the following analysis.

We begin the analysis by assuming that the film thickness \(t_f\) is much less than that of the substrate \(t_s\), so the neutral plane where there is no stress can be taken to be at the middle of the substrate. In Fig. 4.5c, we enlarge one end of the substrate to show the neutral plane, the stress distribution in the film and in the substrate, and the corresponding forces and moments. At equilibrium, the moment produced by the stress in the film must equal to that produced by the stress in the substrate. Since we have as film thickn
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we have assumed that the film thickness is thin, the stress $\sigma_f$ is uniform across the film thickness. The moment $M_j$ (force times perpendicular distance) due to the force in the film with respect to the neutral plane is

$$M_j = \sigma_f W t_f \frac{r^2}{2} \quad (4.14)$$

where $W$ is the width of film normal to $t_f$. To calculate the moment of the substrate, we first obtain the geometrical relation,

$$\frac{d}{r} = \frac{\Delta d}{t_f/2} \quad (4.15)$$

and so

$$\frac{1}{r} = \frac{\Delta d}{t_f/2} \frac{t_f}{t}$$(4.16)

where $r$ is the radius of curvature of the substrate measured from the neutral plane, $d$ is an arbitrary length of the substrate measured at the neutral plane, and $\Delta d/d = \varepsilon_{max}$ is the strain measured at the outer surfaces of the substrate. In the substrate, the elastic strain is zero at the neutral plane, yet it increases linearly with distance $z$ measured from the neutral plane (i.e., it obeys Hooke’s law and increases linearly with stress), so that

$$\varepsilon(z) = \frac{\varepsilon_{max}}{2} \frac{z}{r} \quad (4.17)$$

where $\varepsilon(z)$ is the strain in a plane which is parallel to and at a distance of $z$ from the neutral plane. Then, by assuming a state of biaxial stress in the substrate, we have from Eq. (4.11) that

$$\sigma(z) = \left(\frac{Y}{1-\nu}\right) \varepsilon(z) = \left(\frac{Y}{1-\nu}\right) \frac{z}{r} \quad (4.18)$$

Therefore the moment produced by the stress in the substrate is

$$M_s = W \int_{-t_f/2}^{t_f/2} \sigma(z) dz = W \int_{-t_f/2}^{t_f/2} \left(\frac{Y}{1-\nu}\right) \frac{z^2}{r} dz$$

$$= \left(\frac{Y}{1-\nu}\right) \frac{W t_f^2}{12r} \quad (4.19)$$

By equating $M_s$ to $M_j$, we have Stoney’s equation,

$$\sigma_f = \left(\frac{Y}{1-\nu}\right) \frac{r^2}{6rt_f} \quad (4.20)$$
where the subscripts $f$ and $s$ refer to film and substrate, respectively. Eq. (4.20) shows that by measuring the curvature and the thicknesses of the film and the substrate, and by knowing Young’s modulus and Poisson’s ratio of the substrate, we can determine the biaxial stress in the film. The curvature can be measured by laser interference or by stylus profiling.

Equation (4.20) has been applied to measure surface stress during epitaxial growth of a film on a substrate. The pseudomorphic (commensurate) growth as shown in Fig. 1.4b induces a stress between the film and the substrate. When the substrate is sufficiently thin, the misfit stress can bend the substrate as discussed. In the extreme case of one monolayer pseudomorphic growth of Ge on a 0.1 mm thick (001) Si strip, the bending is large enough to be detected by laser reflection (Schell-Sorokn and Tromp, 1990). In essence, the force on the cross section $W_f$ of a film is $F_f = \sigma_f W_f$, or $F_f/W = \sigma_f$. Rearranging Eq. (4.20), we have

$$r = \left( \frac{Y}{1 - v} \right) \frac{6}{W} \left( \frac{F_f}{W} \right)$$  \hspace{1cm} (4.20a)

This shows that by measuring $r$ (or determining it as a function of $f_f$), we determine $F_f/W$.

The dimension of $F_f/W$ is force per unit width of the film (i.e., it is a measure of surface stress). Recall the discussion of surface energy in Section 2.3 where we have shown that surface energy and surface tension of a liquid have the same magnitude. This is not so for solids. Liquids cannot take shear stresses and the surface of a liquid cannot sustain a compressive stress along the surface, yet solids can. On the other hand, neither solid nor liquid surfaces can have a normal stress.

### 4.5 Chemical Potential in a Stressed Solid

If the stress is maintained (either under constant stress or constant load) and even if it is within the elastic limit, the film or the material will respond by a slow deformation (i.e., relaxation by diffusional creep). A typical example is the sagging of lead pipes by their own weight in some very old houses. Room temperature is a relatively high temperature for lead, which melts at 327°C, therefore atomic diffusion is sufficient for creep to occur. A modern application of creep is the use of pure and well-annealed copper rings as pressure seals in ultrahigh vacuum systems. In general, creep is a high-temperature phenomenon except where grain-boundary diffusion becomes dominant, and then only a moderate temperature is required. We have discussed in Section 4.2 that elastic strain energy is small as compared to chemical energy, so we shall deal with creep in pure elements only, in absence of chemical reactions.

We consider the chemical potential in a stressed solid. In Appendix B, we have the change of Helmholtz free energy $F$ as

$$dF = -SdT - p\,dV$$
Atomistic Film Growth and Resulting Film Properties:

Residual Film Stress

By Donald M. Mattox, SVC Technical Director

The residual stress found in PVD films is due to differences in the coefficient of expansion of the film and substrate giving stress on cooldown from the deposition temperature, and the growth of the film not allowing adatoms to condense in their least energetic association with one another ("growth" or "quenched-in" stresses). In the case of tensile stresses, the atoms are farther apart than they would be in the annealed state, and in the case of compressive stresses, the atoms are closer together than they would be in the annealed state. These residual stresses are stored energy that can be released in subsequent processing or with time in storage or in service. Total film stress is a function of film thickness, elastic modulus, and the morphology and density of the film. The film stress can easily reach the level where plastic deformation of the material can occur. For high-modulus materials such as chromium, tungsten, or metal oxides, very high stresses can be attained with a great deal of stored energy. For low-modulus materials such as gold or silver, plastic deformation will relieve the stresses before high stress levels are attained. The stresses may not be uniform through the film thickness (i.e., a stress gradient in the deposit) if the growth mode and film density change with thickness and/or if some region has been annealed more than others during the deposition process. If the stress is not uniform through the film thickness, the film will curl when separated from the substrate—uniform, the separated film will lie flat.

Films under compression will try to expand, and if the substrate is thin, the film will bow the substrate with the film being on the convex side. If the film has a tensile stress, the film will try to contract bowing the substrate so the film is on the concave side. Tensile stress will relieve itself by microcracking of the film and the peeling of the cracked surface from the

---

**Figure 1.** Deformation and fracture in stressed films.
Substrate. Compressive stress will relieve itself by buckling. The stress distribution in a film may be anisotropic and will give cracking or buckling patterns that depend on the stress distribution. Figure 1 shows some of the failure patterns that can be observed with stressed films.

The most common technique for measuring the total film stress is by measuring the deflection (bowing) of a thin substrate (beam or disk) on which the film has been deposited. By knowing the mechanical properties of the substrate and film material, the film thickness, and the deflection, the film stress can be calculated. Beam deflection can be measured by optical interferometry, laser beam reflection (optical lever arm), or the change of electrical capacitance between two surfaces. The stress due to atom displacement (strain) in a lattice can also be measured by X-ray techniques, but this technique does not take into account film morphology effects nor the effect of a high concentration of lattice defects and grain boundaries.

The most simple case is the deflection of a long, narrow, and thin substrate beam. The narrow width minimizes the “angle-iron stiffening” effect and the thin substrate allows the deflection to be easily determined. In some cases it may be necessary to determine the modulus of the film material independently. This can be done by loading the coated beam with a known stress and measuring the deflection.

One equation for calculating the film stress distribution, and the counterring substrate stress distribution, is given by:

**Film Stress (\(\sigma_f\))**:

\[
\sigma_f = \frac{6p}{l^2} \left( \frac{Y_f}{E_f} \right) \left[ \frac{1 + \nu_f}{1 + \nu_s} \frac{Y_s}{Y_f} \right] \quad \text{where} \quad \frac{1}{2} \leq \frac{Y_s}{Y_f} \leq \frac{1}{2}
\]

**Substrate Stress (\(\sigma_s\))**:

\[
\sigma_s = \frac{6p}{l^2} \left( \frac{1 + \nu_s}{1 + \nu_f} \frac{Y_f}{Y_s} \right) \left[ \frac{1 + \nu_s}{1 + \nu_f} \frac{Y_s}{Y_f} \right] \quad \text{where} \quad \frac{1}{2} \leq \frac{Y_s}{Y_f} \leq \frac{1}{2}
\]

- \(Y_f\) = Film Thickness
- \(Y_s\) = Substrate Thickness
- \(p\) = Radius of Curvature
- \(Y_p\) = Distance from neutral plane to a point in the substrate
- \(Y_f\) = Distance from neutral plane to a point in the film
- \(E_s\) = Substrate Young’s Modulus
- \(E_f\) = Film Young’s Modulus

Figure 2 shows a sample calculation of the film stress that appears in the film, at the interface (both sides) and in the substrate. In this case a molybdenum film (6 microns thick, 47 x 10^9 psi modulus) was deposited on a thin glass beam (Corning microsheet 0211, 2 mils thick, 10.7 x 10^9 psi modulus) and there was a deflection of the beam equivalent to a radius of curvature equal to 6.31 inches.

High total residual film stresses are often the cause of adhesion failure (deadhesion), particularly when the film is a high-modulus material and the film thickness is large. For example, if one uses several thousand angstroms of chromium on a glass substrate, the film stress should be controlled, since a high total stress can cause adhesion failure at the interface or in the glass material. If the film thickness is less than 500 angstroms, the total film stress will be too low to cause failure in the glass. High film stress can also cause voids to form in the film (tensile stresses) or the film to form “bulges” (compressive stress) when the film has good adherence to the substrate.

Film stresses can be controlled by changing the deposition parameters or by altering the growth mode of the film. For example, the sputtering pressure in sputter deposition (i.e., concurrent bombardment by reflected high-energy neutrals) and the amount of energy deposited by concurrent bombardment in ion plating have important effects on the resulting film stress.

**References**


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![Stress distribution in film and substrate.](image)

**Figure 2.** Stress distribution in film and substrate.