Lecture 19: Compensation

- Announcements:
  - HW#9 online
  - Lab#2 due this Friday
    - Grade depends heavily on the report
    - Make sure you spend enough effort on the report per your TA's requirements
  - Lab#3 update online (update for 240A folks)
  - No lecture next Tuesday; this lecture is 2 hours to compensate
  - Two lectures after next Tuesday also 2 hours

- Lecture Topics:
  - Compensation

- Last Time:
  
  Stability + Compensation

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When does instability come from? Practical:
- Anytime FB becomes unstable under certain conditions → must compensate to suppress instability!

Ex: Non-Inverting Amplifier

\[ V_o \approx a(s) V_i \]
\[ V_e = V_i \cdot V_f \]
\[ V_f \cdot f V_o \]

\[ A(s) = \frac{V_o}{V_i}(s) = \frac{a(s)}{1 + a(s) f} = \frac{a(s)}{1 + T(s)} \]

Closed loop gain

Loop transmission: \[ T(s) \approx a(s) f \]

Instability occurs when \[ A(s) \rightarrow \infty \]

⇒ \[ A(s) \frac{GCD}{1 + a(s) f} \rightarrow A \rightarrow \frac{a(s)}{1 - 1} \rightarrow \infty \]

Will also go unstable if denominator is (s)
In general:

If \(|a(s)f| > 1\) when \(\alpha(s)f = -180^\circ\), \(\Rightarrow\) Instability.

This is a simplified form of the Nyquist Criterion.

Stability of a FB Ckt. Using a Single-Pole OpAmp:

For a single pole OpAmp: \(a(s) = \frac{a_0}{1 - \frac{s}{p}}\),

Thus: closed loop T.F.

\[ A(s) = \frac{a(s)}{1+a(s)f} = \frac{a_0}{1 + \frac{s}{p(1+a(s)f)}} \]

\(A_0\): closed loop deg of \(\times\) (1+a(s)f) smaller than \(a_0\). 

To: \(a(s)f\) - loop gain (defined @ dc)

\(T(s)\), \(a(s)f\) - loop transmission (defined for general frequencies).

Bode Plot: use to determine:
- \(\alpha(s)f\) when \(\alpha(s)f = 1\)
- then can determine stability.

\[ A(s) = \frac{V_a(s)}{V_o(s)} = \frac{a_0}{1 + a(s)f} \]

\(20\log(A(s)) = 20\log(a(s)f)\)

Original open loop op amp T.F.

\[ 20\log(A(s)) \approx 20\log(a(s)f) \]

Closed loop T.F.

\[ 20\log(a(s)f) \approx 20\log(a(s)f) \]

This is stable!
Remarks:

1. For the case of a single-pole op amp, FB cannot ever reach $\angle a(j\omega) = -180^\circ$. (90° is the limit.)

2. Thus, a single-pole op amp in FB with $f = \text{const.}$, i.e., $f$ is a function of $\omega$, is always stable!

But in reality, op amp also has non-dominant poles! Therefore can get $\angle a(j\omega) 
\neq -180^\circ$

Con mitigate instability.

Use again a Bode plot to investigate.

Stability of a FB Ckt. Using a mul:

Acme: dominant pole: $p_1$, non-dominant pole: $p_2$

$|P_1 P_2| > |P_2|$
For the more general case where \( A(s) \) has multiple poles:

- \( A(s) \) has the same additional poles (i.e., \( \text{pfreq} \))

  \[ A(s) \approx \frac{A_0}{\left(1 - \frac{s}{\text{pfreq}}\right)\left(1 - \frac{s}{\text{p1}}\right)\left(1 - \frac{s}{\text{p2}}\right)} \]

  - But can also get peaking

**Definitions:**

- \( \text{Phase Margin} = 180^\circ + (\angle A(j\omega))_{\text{pfreq}, \text{omega}} (\left| A(j\omega) \right| = 1) \)
  - \( 180^\circ - 160^\circ = 20^\circ \)
  - \( 180^\circ - 100^\circ = 80^\circ \)

  - \( \text{Phase margin must be} > 0^\circ \) for stability
    - For stability: \( \text{Phase Margin} > 0^\circ \)

  - In design safety, design for \( \text{Phase Margin} \geq 45^\circ \)

  - Even safer (for settling time): \( \text{PM} \geq 60^\circ \)

Can see this peaking on a root-locus analysis.

As the PM continues to shrink, the peak grows.

- When the peak gets sufficiently large, the circuit will oscillate at its freq. \( \rightarrow \) Instability!
Definition:

Gain Margin = \[ 10 \log |a(j\omega)| \text{ in dB} \text{ @ freq. where } \arg(a(j\omega)) = -180^\circ \]

For stability: Gain Margin < 0 dB

Compensation of Op Amps:

To compensate, need distance between \( p_1 \) \( p_2 \) to be large enough to encompass the largest desired loop gain. \( \delta_{\text{max}} = \text{Tomax} \)

\[ |a(j\omega)| \text{ (dB)} \]

Two Ways to Compensate:

1. Narrowbanding
2. Pole Splitting

\[ 20 \log \left( \frac{p_2'}{p_1'} \right) = -20 \log \text{Tomax} \]

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\[ \frac{p_2'}{p_1'} \geq \text{Tomax} \]

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\[ \text{for } \phi = 45^\circ \]

\[ A_0 = \frac{R_2}{R_1} \]

\[ \text{smaller closed loop gain intended} \]
Narrowbanding

- Introduce a pole \( p_b \) so that there is sufficient separation between \( p_0 + p_i \), which becomes the dominant pole.

\[ |P_0| \quad \text{la}(\omega) [\text{dB}] \]

- New \( P_2 \)
- \( |P_{\text{new}}| \quad \text{la}(\omega) [\text{dB}] \)

- \( |P_{\text{new}}| \quad \text{log}(\omega) \)
- \( \angle \text{a}(\omega) \quad \text{log}(\omega) \)

- \( \angle \text{a}(\omega) = \pm 170^\circ \)
- \( \angle \text{a}(\omega) = \pm 45^\circ \)

Remember Narrowbanding:

1. Assumption: \( P_1, P_2, P_3 \) don't move when \( p_b \) is introduced (often not true, but that's not that bad)
2. Summarize: choose \( p_b \) such that \( |\text{la}(\omega)| = 0 \text{dB} \approx 1 \text{st} \), which becomes the new dominant pole.

\( \pm \text{this gives } \text{PM} = 45^\circ \) (for \( |P_{\text{new}}| > |P_1| \) \& \( |P_3| > |P_2| \))

3. Why do this? Wouldn't it be much better to just move the original \( P_1 \) (i.e., pole-splitting)?

5. Do it when you have no other choice, e.g., when you have a packaged op amp & have access only to a few terminals, not the optimum compensation node.

4. \[ |P_{\text{new}}| = \frac{|P_1|}{\text{Tomax}} \text{ maximum expected/needed loop gain} \]

Problem:

1. Often, \( |P_{\text{new}}| < |P_1| \); \( f_{3 \text{dB}} \) & phase will be very weak

2. \( \text{Wandering} = |P_1| \), which isn't that bad

Solution: Pole-Splitting

- Move \( P_1 \) down & \( P_2 \) up simultaneously.

\( \omega_{3 \text{dB}} = |P_1| \) \quad \omega_{\text{wandering}} = |P_{\text{new}}|
Pole-Splitting

\[ |a(j \omega)| \text{ [dB]} \]

- At some \( p_2 \) does not move
- \( p_1 \) moves down to the line

For any \( \omega \) in this range it's stable.

\( \Delta a(j \omega) \)

- \( C \) defines the maximum loop gain that is stable
- \( \Delta \omega \) is unstable
- \( \Delta \omega \) is unstable
- \( \Delta \omega \) is unstable

For pole-splitting, \( |p_1| = \frac{1}{p_{max}} \)

Unity Gain Stable Op Amps

- \( \text{Ex: } 741 \text{ op amp} \)
- Monolithically compensated w/ an internal \( C_c \)
- To be stable when \( A_0 = 1 \) (closed loop gain)

\[ |a(j \omega)| \text{ [dB]} \]

- Open-loop und \( C_c \)
- \( 20 \log(A_0) \)
- \( 20 \log(5) \)

Loss of BW for \( \text{gain} A_0 < 5 \)

What if this were the lowest gain you needed?
- \( \omega_{-3dB} \) would be this

Could have done this if compensated for the needed \( A_0 = 5 \rightarrow \omega_{-3dB} \)

Higher \( \omega \) if compensate for \( A_0 \), rather than for unity gain.