Lecture 12: Op Amps & Emitter Coupled Pair

- Announcements:
  - Pre-Lecture materials online
  - HW#1A online for 240A folks
  - Lab#2 starts tomorrow
  - Lab#1 due tomorrow at the beginning of your lab section; turn it in to your TA
  - HW#5 due tomorrow
  - Midterm will be on the date specified in your syllabus: Thursday, March 21, 3:30-5 p.m. in 390 Hearst Mining Building

- Lecture Topics:
  - Op Amp Review
  - Emitter Coupled Pair (ECP)
  - Half Circuits

- Last Time:

Current Source Matching Considerations

\[
\Delta I_0 = \frac{\Delta W/L}{W/L} \Delta V_{GS} \quad (V_{GS}/2)
\]

Aside: When I draw current sources, they're just biased transistors

There are circuits that set the current values, e.g.,

This is a voltage generator that takes the current from generator \( V_{BAS1} \) and \( V_{BAS2} \)
and so forth.
Now, start on Op Amps & ECP using the Pre-Lecture handout

- Half Circuits:

\[ V_{CC} \]
\[ Q_1 = Q_2 \]
\[ R_{C1} = R_{C2} = R_C \]

\[ V_{i1} \]
\[ V_{o1}, V_{o2} \]
\[ V_{i2} \]

\[ I_{FE}, I_{EE} \]

\[ V_{EE} \]

...a whatever is needed to generate \( I_{inf} \) the needed supply \& temperature independence
**Ideal Op Amps**

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**Ideal Voltage Amplifier**
- Ideal when \( \frac{N_2}{N_1} \cdot A_{in} \); i.e., when source and load \( R_l \) do not influence the gain of the amplifier.

For this to occur, the voltage division at the input and output must be eliminated.

This happens when:
- \( R_s = \infty \)
- \( R_o = 0 \)

These resistance values define an ideal voltage amplifier.

We'll look at other amplifier types later.

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**Ideal Operational Amplifier**

- The workhorse of analog electronics.
- Combination of op-amp and feedback components allows the implementation of analog computer, sampled-data systems, analog filters, A/D converters, DAC's, instrumentation amplifiers.

In general, have a minimum of 5 terminals:

\[
\begin{align*}
\text{Input} & \quad \begin{cases} 
\text{Inverting} & \quad V^+ \to A_{in} \to V^- \\
\text{Non-Inverting} & \quad V^+ \to V^- \to A_{in} 
\end{cases} \\
\text{Output} & \quad V_o \to A_{out} \to V_o
\end{align*}
\]

Perhaps the best way to define an op amp is thru its equivalent circuit:

**Equivalent Circuit of an Ideal Op Amp**

\[
N_o = A(N_s - N_i) = A(N_s - N_i) 
\]

Single-ended input

Differentiated output

Voltage-Controlled Voltage Source (VCVS)
### Ideal Op Amps

**Properties of Ideal Op Amps:**

1. $R_\text{in} = \infty$ leads to $i_+ = 0$.
2. $R_\text{o} = 0$.
3. $A = \infty$ leads to $v_+ = v_-\,$, assuming $v_\text{o} = \text{finite}$.
4. $v_\text{o} = A(v_+ - v_-) = \text{finite}$.
5. $v_+ - v_- = 0 \Rightarrow v_+ = v_- = \text{virtual short curr.}$
6. $\frac{v_+}{v_-} = \text{virtual ground}$.

**Big assumption! ($v_+ = \text{finite}$)**

How can we assume this? Only when there is an appropriate negative feedback path!

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### Negative Feedback

**Even Signal:**

- $\Delta S_0$ is amplified by $\frac{v_0}{S_0}$.
- $\Delta S_+ = \text{virtual ground}$.
- $\Delta S_- = A \Delta S_0$.

**Odd Signal:**

- $\Delta S_0$ is subtracted from input.
- $S_0 = a(S_2 - 60) + \beta S_0$.
- $S_0(1 + \beta) = a S_2$.
- $\frac{S_0}{S_2} = \frac{a}{1 + \beta}$.
- $\frac{a}{\beta} > \frac{1}{\beta} \Rightarrow \text{finite!}$
- $S_0 = a \rightarrow S_2 = \text{finite}$

**In Summary:**

1. Neg. FB can insure $S_0$ finite even with $a = \infty$.
2. Gain dependent (or overall T.F.) depends only on external components (e.g., $\beta$).
3. Overall (closed-loop) gain $\frac{S_0}{S_2}$ is independent of amplifier gain $a$.

Very important (as you'll see, when designing amplifiers using transistors, it's very hard to get large gain, but it's hard to get an exact gain).

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**Notes:**

- $70,000 \approx 50,000$ instead.
**EE 140: Analog Integrated Circuits**  
**Lecture 12m: Op Amps & Differential Circuits**  

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### Op Amp Circuits

- **Positive Feedback**: 
  - $S_i \uparrow$  
  - $S_{out}$  
  - $S_{in}$  
  - $S_{out} \rightarrow$ output blow up!  
  - (for $a > 1$)  
  - $N$  
  - will be the case for $a \rightarrow \infty$  
  - $\Rightarrow$ usually get oscillation!

Thus, for a bounded, controllable function, avoid negative FB around an op amp.

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- **Op Amp Properties**

  1. Verify that there is negative FB.
  2. $N_+ \rightarrow N_-$ implies $\frac{N_+}{N_-} \rightarrow$ node at terminal $G$ is virtual ground.
  3. $i_o = i_2$  
  - $\frac{N_+}{R_1} = \frac{N_+}{R_2}$  
  - $N_o = -\frac{N_+}{R_2}$

Note: Gain dependent on $R_1$ and $R_2$ (external components), not on $a$ or op amp gain.

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- **Example 1**: 
  - $N_{out}$  
  - $N_{in}$  
  - $R_2$  
  - $\Rightarrow$ $N_{out}$ depends on $R_1$ and $R_2$.

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- **Example 2**: 
  - $N_{in}$  
  - $N_{out}$  
  - $R_2$  
  - $\Rightarrow$ $N_{out}$ depends on $R_1$ and $R_2$.

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How does one make an op amp? (It turns out, you already know.)

1. Gain (voltage gain).
2. Two inputs (+ and -).
3. One output equal to the difference of the inputs multiplied by some gain.

In fact, the differential pair you studied in EE105 can by itself serve as an op amp!

(need only decide which of the inputs is the (+) and (-) terminals)

Why have 2 inputs?

1. To get a virtual short for op amp nets.
2. To suppress common-mode noise:

Input line

Nearby noisy line

Can avoid this w/ a differential input:

Input 1

Nearby noisy line

Input 2

killed common-mode noise
Differential Pair (Bipolar)

Differential Gain: $A_d = \frac{V_{1d}-V_{2d}}{V_{id}}$ (want this to be large for high differential amplification)

Common-Mode Gain: $A_{cm} = \frac{V_{1c}-V_{2c}}{V_{cm}}$ (want this to be small so that the amp rejects common-mode signals)

Common-Mode Rejection Ratio: $CMRR = \frac{A_{cm}}{A_{cm}}$ (should be very high to form the differential and reject the common-mode)

Note: No need for bypass capacitors (bypasses to the inputs or outputs) → can just use direct coupling!

Biasing/Long Signal Common-Mode Behavior

Case: $R_{ee} = \infty$ → ideal current source biasing → $I_{ee} = I_{ee} = \frac{I_{ee}}{2}$ → $V_{1c} \sim V_{2c} \to V_{dd}$

If $V_{cm}$ up → $V_{ee}$ up, but cannot draw from $I_{ee}$ source (cannot source $I_{ee}$ to $V_{cm}$)

$\frac{1}{2} I_{ee}$

Case: $R_{ee}$ finite → $V_{ee} = \frac{V_{cm}}{2}$

If $V_{cm}$ up → $V_{ee}$ up → $V_{ee} \sim V_{cm}$ (cannot source $I_{ee}$ to $V_{cm}$)

In general, $R_{ee}$ will be large, so this component would be large, and the bias point would be much

$V_{cc}$
Differential Mode Analysis

Small-Signal Analysis of Diff. Pair

\[ N_{d1} = -\frac{1}{2} g_m R_C \]
\[ N_{d2} = +\frac{1}{2} g_m R_C \]
\[ V_{O1} = V_{O2} = 0 \]

Differential Mode Analysis

Assume a ch, we only diff. input:

Total current thru \( I_{EE} \) cont.

\[ V_{EE} \text{ cont. as input changes} \]
\[ \Theta \text{ acts as an unwound gain} \rightarrow V_{O} = 0 \text{V (always):} \]

1. We can ground \( \Theta \), and then have a different half ch.

Note: Can usually only use this for a purely symmetrical ch.
Common-Mode Analysis

Assume a pure CM input → the inputs together

\[ V_{cm} = \frac{V_{R1} - V_{R2}}{2} \]

By symmetry, \( V_{R1} = V_{R2} \) →

\[ V_{cm} = 0 \]

Thus, totally how to equalize CM

Can split the circuit into CM half-circuits!

S.S. CM half-circuit.

Common-Mode Rejection Ratio (CMRR): \( CMRR = \frac{A_{cm}}{A_{cm} + 2g_m R_E} \)

\[ A_{cm} = \frac{-g_m R_E}{1 + g_m (2R_E)} \]

\[ R_{cm} = \frac{-g_m}{1 + g_m (2R_E)} \]

What could be large CMRR: want

\[ R_{cm} \rightarrow \infty \]

\[ R_E \rightarrow 0 \]

want as large as possible

Read a current source

Having looked at S.S. parameters, we now turn to large-signal performance. Here, we will be particularly interested in the linear range of the ECF.
Large Signal ECP Performance

Find $I_{C}$:

$\frac{I_{C}}{I_{E}} = \exp\left(\frac{v_{BE}}{V_{T}}\right)$

$\Rightarrow I_{C} = I_{E}\exp\left(\frac{v_{BE}}{V_{T}}\right)$

Combine (1) and (2) to get:

$I_{C1} = \frac{\alpha}{1 + \exp\left(-\frac{v_{BE}}{V_{T}}\right)}$

$I_{C2} = \frac{\alpha I_{E}}{1 + \exp\left(\frac{v_{BE}}{V_{T}}\right)}$

Find $V_{CE}$:

$V_{CE} = V_{BE} - I_{C}R_{C}$

$V_{BE} = \frac{I_{C1}R_{C}}{I_{E} + I_{C1}}$

Using (3):

$V_{CE} = \frac{I_{C1}R_{C}}{I_{C1} + I_{C2}}$

$x = \exp\left(\frac{v_{BE}}{V_{T}}\right)$

$\Rightarrow \frac{1}{x} - \frac{1}{x + 1} = \frac{1}{x + 1}$

$\Rightarrow \frac{1}{x - 1} = \frac{1}{x}$

$\Rightarrow x = \cosh\left(\frac{v_{BE}}{V_{T}}\right)$

$\Rightarrow u = \frac{V_{CE}}{2V_{T}}$

$\Rightarrow \begin{bmatrix} \sinh u = \frac{1}{2} (e^{u} - e^{-u}) \\ \cosh u = \frac{1}{2} (e^{u} + e^{-u}) \end{bmatrix}$

To find $V_{CE}$, use the Taylor series for sinh x:

$\sinh x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

This is fairly linear for small $V_{CE}$, but gets nonlinear abruptly when $V_{CE}$ approaches a threshold value.
In the above curve, the $V_u/V_j$ X-fm function is nearly linear for $V_u < V_f$ but beyond $V_f$, start to enter the non-linear realm of curve → causes signal distortion. e.g., phase locking up, television static.

To linearize, add emitter degeneration (same trick as used before to single X-fm amplifier).

If $I_{JE}$ is large, then:
- $I_{EC} = I_{JE} + I_{EC}$
- This can still be $I_{JE} < V_f$ if this absorbs some of the input voltage.

Alternative Emitter Technique: H Head Logam DC Currents:

- Some JFETs perform via the need to drop a DC voltage across $R_E$ → gain
- Can use linear $V_{CE}$ or $V_{BE}$.