Lecture 6: Frequency Response Inspection Analysis

• Announcements:
  • This is our make-up lecture
  • We will have lecture tomorrow (Tuesday), as well, at our regular time and place

• Lecture Topics:
  • Amplifier Bode plot
  • Open Circuit Time Constant (OCTC) Analysis
  • Frequency Response Inspection Analysis
  • Frequency Response Examples

• Last Time:

Mos Inspection Formulas w/ Substrate Grounded

- \( B \rightarrow \infty \)
- \( r_m = \frac{g_m}{\beta} \rightarrow \infty \)
- \( g_m \rightarrow g_m + g_{mb} \)

- Only difference from substrate tied to source is that \( g_m \) is replaced by \( g_m + g_{mb} \) in some of the formulas

- Particularly over where the source is involved!

\[
\begin{align*}
R_g &= 0 \\
R_s &= \frac{1}{g_m + g_{mb}} \\
R_d &= R_b \left[ 1 + (g_m + g_{mb}) R_b \right]
\end{align*}
\]

Remind: When the substrate is tied to the source, \( g_{mb} = 0 \).

Effect of \( g_{mb} \) (one more example)

\[
\begin{align*}
\frac{V_d}{V_s} &= -G_m R_{d} \quad \frac{G_m}{1 + (g_m + g_{mb}) R_f} \\
\frac{V_d}{V_s} &= -G_m R_{d} \quad \frac{G_m R_f}{1 + (g_m + g_{mb}) R_f} \\
\frac{V_d}{V_s} &= \frac{g_m R_f}{1 + (g_m + g_{mb}) R_f}
\end{align*}
\]

\[
\begin{align*}
R_s &= \frac{1}{g_m + g_{mb} + g_d} \\
R_s &\approx \frac{1}{g_m + g_{mb}}
\end{align*}
\]
Recall that the transfer function of a general amplifier can be expressed as a function of frequency via:

\[ A(s) = \frac{A_m F_L(s) F_H(s)}{\omega_p c_{ou}} \]

where:
- \( A_m \) is the midband gain.
- \( F_L(s) \) is the high frequency shaping.
- \( F_H(s) \) is the low frequency shaping.

We've already found this using small signal analysis.

High Freq. Response Determination Using Open Clkt. Time

In general:

\[ F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \ldots + a_n s^n}{1 + b_1 s + b_2 s^2 + \ldots + b_m s^m} \]

From which:

\[ b_1 = \frac{1}{\omega_{p1} + \omega_{p2} + \ldots + \frac{1}{\omega_{p1} \omega_{p2} \ldots \omega_{pN}}} = \sum_{k=1}^{N} \frac{1}{\omega_{pk}} \]

\[ c_0 = \frac{1}{\omega_{p1} + \omega_{p2} + \ldots + \frac{1}{\omega_{p1} \omega_{p2} \ldots \omega_{pN}}} = \sum_{k=1}^{N} \frac{1}{\omega_{pk}} \]
Through network theory, one can prove that: (see Gray & Meyer, Chpt. 7)
\[ \sum_{j=1}^{n_p} \frac{1}{Z_{pi}} = \sum_{j=1}^{n_p} \frac{1}{Z_{ji}} = \sum_{j=1}^{n_p} \frac{1}{Z_{ji}} \]

where \( C_j \) are capacitors in the H.F. ckt., i.e., small ones
\( R_{jo} \) driving pt. resistance seen between the terminals of \( C_j \) determined with

1. all small (<1nF) capacitors open-circuited
2. all independent sources eliminated (i.e., short voltage source, open current source)
3. short all large (coupling/bypass) capacitors (i.e., >1μF or >1nF)

In calculating the H.F. response, we use the dominant pole approximation:

1. \( \omega_H \ll \omega_{p1}, \ldots, \omega_{p_n} \)
2. \( \Gamma(s) \approx \frac{1}{1 + j \omega_H} \)
3. \( b_i \approx \frac{1}{\omega_{pi}} \approx \omega_H \approx \frac{1}{b_i} = \frac{1}{\omega_H} = \frac{1}{j C_j R_j} \)

When there is no dominant pole, an approximate expression for \( \omega_H \) is:

\[ \omega_H \approx \sqrt{\frac{1}{\omega_{p1}^2 + \omega_{p2}^2 + \ldots + \frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \ldots}} \] (just FYI)
Find \( \tau_0 \):
\[
\tau_0 = (R_0 || R_4)C_{C1}
\]

\[
\omega_H = \frac{1}{\tau_0 + \frac{1}{\omega_0} + \omega_0}
\]

Now, we Miller’s Theorem:

\[
C_m(1 - \omega) = C_m(1 + \omega_m R_C)
\]

Gain across \( \omega \)
(from base to collector)

\[
\tau_0 = (R || R_2)(1 + \omega_m C_m)
\]

\[
\tau_2 = (R_4 || R_6)(C_m + C_m(1 + \omega_m R_C))
\]

\[
\omega_H = \frac{1}{\tau_0 + \frac{1}{\omega_0} + \omega_0}
\]

Multi-Stage Ex.

\[
\tau_{in} + (\frac{R_1}{C_{m1}}) = \tau_{in} + \tau_{EE} = \frac{C_m}{g_m}
\]

\[
\tau_{m1} = C_m \left( \frac{R_{in}}{R_{in} + g_m} \right)
\]

\[
\tau_{m2} = \frac{1 + \frac{1}{\tau_{EE}} \left( \frac{1}{g_m + \frac{1}{R_{in}}} \right)}{g_m}
\]

\[
\omega_H = \frac{1}{\tau_0 + \tau_{m1} + \tau_2 + \tau_0}
\]