Lecture 6: Frequency Response Inspection Analysis

Announcements:
- HW#2 due tomorrow at 8 a.m.
- Lab#1 is online; HW#3 online soon

Lecture Topics:
- Amplifier Bode plot
- Open Circuit Time Constant (OCTC) Analysis
- Frequency Response Inspection Analysis

Last Time:

Effects of $g_{m_b}$ on Input impedance

Only difference from "substrate tied to source case" is that $g_m$ is replaced by $g_m+g_{m_b}$ in some of the formulas. Particularly over where the source is involved!

$R_s = \frac{1}{g_{m} + g_{m_b}}$

$R_d = V_b \left[ 1 + (g_m + g_{m_b})R_s \right]$

$\frac{N_d}{V_b} = \frac{g_m R_s}{1 + (g_m + g_{m_b}) R_s}$

$\frac{N_d}{V_s} = -g_m R_d$

$G_m = \frac{g_m}{1 + (g_m + g_{m_b}) R_s}$

$\frac{N_d}{V_x} = -g_m R_x$

Remark: When the substrate is tied to the source, $g_{m_b} = 0$. 

If continue to analyze all quantities, $g_m \rightarrow g_m + g_{m_b}$ (for all denominator) (but not some numerator).
Recall that the transfer function of a general amplifier can be expressed as a function of frequency via:

\[ A(s) = A_m F_L(s) F_H(s) \]

\[ s = j\omega \]

Midband gain

High frequency shaping

Low freq. shaping

We've already found this using small-signal analysis.

High Freq. Response Determination Using Open Clkt. Time Constant (OCTC) Analysis

In general:

\[ F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_{n_H} s^{n_H}}{1 + b_1 s + b_2 s^2 + \cdots + b_{n_P} s^{n_P}} \]

where \( n_P > n_H \)

\[ F_L(s) = \frac{1}{1 + \sum_{j=1}^{n_H} \frac{s}{\omega_j}} \]

From which:

\[ b_1 = \frac{1}{\omega_{P_1}} + \frac{1}{\omega_{P_2}} + \cdots + \frac{1}{\omega_{P_{n_P}}} = \sum_{k=1}^{n_p} \frac{1}{\omega_{P_k}} \]

Coef. of the 1st order term
Through network theory, one can prove that: (see Gray & Meyer, Chpt.7)

$$\mathbf{\Sigma} C_{j} R_{j0} = \mathbf{\Sigma} \tau^{j}$$

where $C_{j}$ are capacitors in the H.F. ckt, i.e., small ones

$R_{j0} = \text{driving pt. resistance seen between the terminals of } C_{j} \text{ determined with}$

1. all small (<1nF) capacitors open-circuited
2. all independent sources eliminated (i.e., short voltage source, open current source)
3. short all large (coupling/bypass) capacitors (i.e., >1µF or >1nF)

In calculating the H.F. response, we use the dominant pole approximation:

a) $\omega_{1} = \omega_{p1}, \ldots, \omega_{pnp}$

$$S(\omega) \frac{F_{p}(s)}{H(\omega)} = \frac{1}{1 + \frac{s}{\omega_{1}}}$$

b) $\frac{b_{i}}{\omega_{p1}} \rightarrow \omega_{1} = \omega_{p1} \approx \frac{1}{b_{i}} = \frac{1}{\omega_{1}} = \frac{1}{j C_{j} R_{j0}}$

When there is no dominant pole, an approximate expression for $\omega_{1}$ is:

$$\omega_{1} \approx \sqrt{\frac{1}{\omega_{p1}^{2} + \omega_{p2}^{2} + \cdots + \frac{1}{\omega_{e1}^{2}} - \frac{1}{\omega_{e2}^{2}} - \cdots}}$$

(just FYI)

- Now, go to inspection formula sheet and go over how to use the frequency response parts


Find \( T_\sigma \): 

\[ T_\sigma = C_{CS}(\frac{1}{R_0} + R_4) \times C_{CS}R_4 \]

\[ \omega_H = \frac{1}{T_\sigma + T_\epsilon} \]

Using Miller's Theorem:

\[ T_\sigma = (R_5(R_0 + R_1 + R_2) + C_{CS} + C_{SR})(1 + \frac{1}{\omega_H C_{SR}}) \]

\[ T_\sigma = R_5(C_{CS} + C_{SR}) \]

\[ T_\sigma = R_\epsilon(C_{CS} + C_{SR}) \]

Ex. Multi-Transistor Ctl:

\[ T_\sigma = \frac{R_\epsilon}{C_{CS} + C_{SR} + \frac{R_\epsilon}{R_\epsilon'}} \]

\[ \omega_H = \frac{1}{T_\sigma + T_\epsilon + T_\pi} \]
Ex. 1905 Two-Stage Amplifier

\[ V_0 = \left[ C_{gd1} \left( 1 + g_m R_D \right) + C_{gd2} \right] R_S \]

\[ V_O = \left[ C_{db1} + C_{gd1} + C_{gd2} \right] \left( C_{gs1} R_D \right) \]

\[ V_C = C_{sb2} \left( R_{s2} \parallel \frac{1}{g_{m2} + g_{m2}} \right) \]

\[ \omega_n = \frac{1}{\frac{1}{V_0} + \frac{1}{V_O} + \frac{1}{V_C} + \frac{1}{V_S}} \]