MOSFET Small-Signal:

\[ g_{mb} = \frac{V_{gs} \cdot 2}{Z \cdot V_{sd} + Z \cdot f} = \frac{1}{Z} \]

**KCL:**

\[ g_m \cdot V_{gs} + g_{mb} \cdot V_{ds} + g_{ds} \cdot V_{ds} + I_{test} = 0 \]

\[ V_{gs} = V_{g} - V_{s} = 0 - V_{test} = -V_{test}; \quad V_{ds} = 0 - V_{test} = -V_{test} \]

\[ V_{g} = 0 - V_{test} = -V_{test} \]

\[ \Rightarrow g_{s} = \frac{I_{test}}{V_{test}} = \frac{g_m + g_{mb} + g_{ds}}{V_{test}} \]

**What is effective s.s. transconductance with emitter degeneration?**

\[ V_i = T_{id} R_{e} \Rightarrow \]

**KCL at D:**

\[ g_m (V_i - V_s) + g_{mb} (0 - V_s) + g_{ds} (0 - V_s) = i_d \]

\[ g_m V_i - V_s (g_m + g_{mb} + g_{ds}) = i_d \]

But, at \( S \), \( i_d = g_m V_s \)

\[ \Rightarrow g_m V_i = i_d + (g_m + g_{mb} + g_{ds}) R_{e} i_d \]

\[ G_m = \frac{i_d}{V_i} = \frac{g_m}{1 + (g_m + g_{mb} + g_{ds}) R_{e}} \]

**Inspection analysis with body effect:** Replace \( g_m \rightarrow g_{mb} + g_{ds} \) in denominators.
Example:

\[
\frac{U_i}{U_{DD}} \quad \Rightarrow \quad \frac{U_i}{U_{0}}
\]

\[
\begin{align*}
U_i & \quad \Rightarrow \quad V_i \\
\end{align*}
\]

\[
\begin{align*}
\text{Inspection} \Rightarrow \quad \frac{U_0(s)}{U_i} & = \frac{g_m}{g_{out}} \left( \frac{1 + sC_{gs}/g_m}{1 + sC_{out}/g_{out}} \right) \\
\end{align*}
\]

\[
\begin{align*}
\therefore \quad W_2 & = -\frac{g_m}{C_{gs}} \quad (\text{Left-half-plane zero}) \\
\end{align*}
\]

\[
\begin{align*}
C_{out} & = C_L + C_{sb} + C_{gs} \quad (C_{gb} \text{ driven by ideal voltage source; } C_{sb} \text{ shorted to } \text{ground}) \\
\end{align*}
\]

\[
\begin{align*}
\therefore \quad W_p & = -\left( \frac{g_m + g_{mb} + g_{ds} + g_E}{C_{gs} + C_{sb} + C_L} \right) \quad (\text{Left-half-plane pole}) \\
\end{align*}
\]

**Frequency Response Analysis:**

Consider the four different singularities:
(A.) Left-half-plane (LHP) pole:

\[ G(s) = \frac{1}{1+\frac{s}{\omega_p}} \]

Recall: Set \( s = j\omega \) to evaluate frequency response:

\[ A(j\omega) = \frac{1}{1 + j\omega/\omega_p} \]

\[ |A(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_p)^2}} \]

\[ \angle A(j\omega) = -\tan^{-1}(\omega/\omega_p) \]

For Bode plot:

\[ \text{dB} = 20 \log_{10} |A(j\omega)| \]

\[ = 20 \log_{10} \left( \frac{1}{\sqrt{1 + (\omega/\omega_p)^2}} \right) = -10 \log_{10} \left( 1 + (\omega/\omega_p)^2 \right) \]

i) For \( \omega \ll \omega_p \), \( \text{dB} = -10 \log_{10} 1 = 0 \text{ dB} \)

ii) For \( \omega = \omega_p \), \( \text{dB} = -10 \log_{10} 2 = -3 \text{ dB} \)

iii) For \( \omega \gg \omega_p \), \( \text{dB} = -10 \log_{10} \left( \frac{\omega}{\omega_p} \right)^2 = -20 \log_{10} \left( \frac{\omega}{\omega_p} \right) \) (\(-20 \text{ dB/decade}\))

\[ \angle A(j\omega) = -\tan^{-1}(\omega/\omega_p) \]

i) For \( \omega \ll \omega_p \), \( \angle A(j\omega) = -\tan^{-1} 0 = 0^\circ \)

ii) For \( \omega = \omega_p \), \( \angle A(j\omega) = -\tan^{-1} 1 = -45^\circ \)

iii) For \( \omega \gg \omega_p \), \( \angle A(j\omega) = -\tan^{-1} 0 \approx -90^\circ \)

* For completeness at \( \omega_p/10 \) and 10 \( \omega_p \) used for asymptotic analysis:

i) \( \omega = \omega_p/10 \); \( \angle A(j\omega) = -\tan^{-1} \omega/\omega_p \)

\[ = -\tan^{-1} 1/10 \approx -5.71^\circ \]

ii) \( \omega = 10 \omega_p \); \( \angle A(j\omega) = \tan^{-1}(10) = -84.29^\circ \)
The four singularities:

**LHP pole**

\[ \frac{w}{w_p} \]

\[ -20 \text{ dB/dec.} \]

\[ 0 \]

\[ -45^\circ \text{ dec.} \]

\[ \frac{w_{10}}{w} \]

\[ \frac{w_{10}}{w_{100}} \]

- **S-plane**

**RHP pole**

\[ \frac{jw}{w_p} \]

\[ -20 \text{ dB/dec.} \]

\[ 0 \]

\[ -45^\circ \text{ dec.} \]

\[ \frac{w_{10}}{w} \]

\[ \frac{w_{10}}{w_{100}} \]

- **S-plane**

**LHP zero**

\[ \frac{1}{jw} \]

\[ +20 \text{ dB/dec.} \]

\[ 45^\circ \]

\[ \frac{w_{10}}{w} \]

\[ \frac{w_{10}}{w_{100}} \]

- **S-plane**

**RHP zero**

\[ \frac{1}{jw} \]

\[ +20 \text{ dB/dec.} \]

\[ 45^\circ \]

\[ \frac{w_{10}}{w} \]

\[ \frac{w_{10}}{w_{100}} \]

- **S-plane**

*Store these in your memory for inspection analysis*

**Example Bode plot:** (Assume poles and zeros are widely spaced, e.g., more than 100x)

\[ A(s) = \frac{A_0 (1 + s/w_2)}{(1 + s/w_1)(1 + s/w_2)} \]

\[ |A(jw)| = 20 \log \frac{A_0 |1 + jw/w_2|}{|1 + jw/w_1||1 + jw/w_2|} \]

So, just add logs of individual terms to get overall Bode magnitude plot.
Note: phase bump in frequency response

Add phase plots to get overall phase
Dominant Pole Estimation:

Motivation: An opamp may have many internal poles. But, we want it to be stable in (worst-case) closed-loop application (unity-gain configuration). Note: -180° phase shift corresponds to a sign change; i.e., negative F.B. becomes positive F.B. and circuit can oscillate. Goal: Limit total phase shift to < 180°. So, to a first-order, design for single-pole response = -90 phase shift.

General Transfer Function:

\[ A(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_m s^m}{1 + b_1 s + b_2 s^2 + \cdots + b_n s^n} \quad (m \leq n) \]

Many times only poles and unimportant or no zeros

\[ A(s) = \frac{K}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2}) \cdots (1 - \frac{s}{p_n})} \quad \text{after factoring with constant} \]

\[ = \frac{K}{1 + s\left(\frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_n}\right) + s^2\left(\frac{1}{p_1 p_2 p_3} + \cdots \right)} \]

\[ b_1 = \sum_{i=1}^{n} \frac{1}{p_i} \quad \text{from above} \]

\[ \text{Dominant pole } \Rightarrow |p_1| < 1 \text{ or } |p_2|, |p_3|, \text{ etc.} \]

\[ x \quad \text{or} \quad \frac{1}{p_i} \]

\[ \sigma \quad (\text{note: change in terminology: } p_i \text{ is negative for LP pole}) \]
From above: \[ |A(j\omega)| = \frac{K}{\sqrt{1 + (\omega/p_1)^2} \cdot \sqrt{1 + (\omega/p_2)^2} \cdots \sqrt{1 + (\omega/p_n)^2}} \]

\[ = \frac{K}{\sqrt{1 + (\omega/p_1)^2}} \text{ for dominant pole} \]

2. \( \omega_{-3dB} \approx \frac{1}{b_1} \)

**Millar Effect:**

\[ V_{dd} \quad R_D \quad V_0 \]

\[ ^{V_{dd}} \]

\[ R_D \quad ^{V_0} \]

(Case: assume B
 grounded
 unless indicated
 otherwise)

**Millar Effect Values:**

\[ C_m = (1 - A_vo) \cdot C_{gd} = (1 + \frac{g_m}{g_D + g_ds}) \cdot C_{gd} \]

- Can be much larger than \( C_{gd} \); i.e., "Millar Multiplied Capacitance"

\[ C_x = (1 - \frac{1}{A_vo}) \cdot C_{gd} = (1 - \frac{g_D + g_ds}{g_m}) \cdot C_{gd} \approx C_{gd} \text{ for large gain values (>5vdb)}\]

**Millar Effect greatly simplifies dominant pole frequency calculations**

\[ Z_v = \text{Zero Value Time Constant Method} \]

\[ \Rightarrow |b_1 = \frac{Z_v}{T_o}| \]

Read 3.3 in S.M.
\[ Z_{UTC} : \ b_1 = \sum T_i = \sum C_i R_i \]

- \( C_i \) = important caps for high freq. response; e.g., transistor caps and \( C_L \) treat large bypass and coupling capacitors as short circuits.
- \( R_i \) = Driving point resistance for each \( C_i \).

**Example:**

Imagine where important caps are (as shown):
- \( C_{pi}, C_{mu}, C_{es}, C_L \)

**Strategy:**
1) First, consider caps) connected between gain nodes (A to B) and apply Miller Effect to simplify:

\[ AV_0 = -\frac{g_m}{g_c + g_0} \]

\[ C_m = C_{mu} \left( 1 + \frac{g_m}{g_c + g_0} \right) \]

\[ C_X = C_{mu} \text{ for } AV_0 > 5 \]
(2) Consider parallel capacitors together as one capacitance:

\[
\begin{align*}
& \text{by inspection:} \\
& R_{D0} = R_s / (s \pi) \\
& g_{D0} = g_s + s \pi
\end{align*}
\]

(3) Replace capacitors one at a time with \( V_{\text{test}} \) source to get driving point resistance:

i) \((C_T + C_m)\):

\[
R_{D0} = g_{m} \left( \frac{1 + \frac{g_m}{g_c + g_0}}{s \pi} \right)
\]

By inspection:

\[
\begin{align*}
R_{D0} &= R_s \parallel \pi \\
g_{D0} &= g_s + g_{\pi}
\end{align*}
\]

ii) \((C_L + C_x)\):

\[
R_{D0} = \frac{1}{g_c + g_0}
\]

Again, by inspection:

\[
T_{D0} = R_{D0} (C_x + C_L)
\]

\[
= \frac{1}{g_c + g_0} \left[ C_x + C_L \right]
\]
So, \( b_i = \frac{1}{2T_i} \approx \frac{1}{W_{-3dB}} \) (dominant pole)

\[
W_{-3dB} = \frac{1}{2T_i} = \frac{1}{\frac{1}{g_s + g_n} \left[ \frac{C_m + C_m (1 + g_m)}{g_x + g_o} \right] + \left( \text{Cut off} \right)} \frac{1}{g_x + g_o}
\]

How does this compare?

Suppose \( R_s = 0 \) (\( g_s = \infty \)).

\[
W_{-3dB} = \frac{g_x + g_o}{g_m + g_L} = \frac{g_{out}}{2g_{out}}
\]

\( \text{Same as we saw for ideal (no R_s) single-stage CE amp!} \)