Contrast and Positive Feedback

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Thus, for a bounded, controllable function, need negative FB around an op amp.

Op Amp Clfs.

Example. Inverting Amplifier

\[ \frac{V_o}{V_{in}} = -\frac{R_2}{R_1} \]

1. Verify that there is negative FB.
2. \( V_o = \text{finite} \rightarrow V_o \approx 0 \rightarrow \) node attached to (-) terminal is virtual ground
3. \( i_o = 0 \rightarrow i_1 = i_2 \)

\[ i_1 = \frac{V_{in}}{R_1} = \frac{V_o}{R_1} \]

\[ V_o = -\frac{(V_{in}/R_1) R_2}{R_1} = -\frac{R_2}{R_1} V_{in} \]

\[ \frac{V_o}{V_{in}} = -\frac{R_2}{R_1} \]

Note: Gain dependent only on \( R_1 + R_2 \) (external components), not on the op amp gain.

Analyze to find \( V_{in} \) input voltage.

Example.

\[ \frac{V_o}{V_{in}} = -\frac{R_2}{R_1} \]

1. Verify that there is neg. FB

\[ \Rightarrow \frac{V_o}{V_{in}} = \frac{R_2}{R_1} \]

\( \Rightarrow \) this op. will oscillate!

\[ \Rightarrow \text{usually get oscillation!} \]
How does one make an op amp? (It turns out, you already know!)

**Basic Needed Attributes:**

1. Gain (voltage gain).
2. Two inputs, (+) and (-).
3. One output equal to the difference of the inputs multiplied by some gain.

In fact, the differential pair you studied in EE 105 can by itself serve as an op amp!

(need only decide which of the inputs is the (+) and (-) terminal)

Why have 2 inputs?

1. To get a virtual short for op amp dc.
2. To suppress common-node noise:

Let's now look at the diff. pair in more detail! ... but first

Can avoid this w/o differential input:

Input 1

Input 2

Output line
Differential Pair (Bipolar)

**Differential Pair (Miller-Coupled Pair)**

**Purpose:** Amplify the difference between two signals regardless of their common-mode DC value (or their common-mode values in general).

**Definition:**
- \( V_{\text{diff}} = V_{i1} - V_{i2} \) (differential input)
- \( V_{\text{cm}} = \frac{V_{i1} + V_{i2}}{2} \) (common-mode input)

\[
\begin{align*}
V_{i1} &= V_{\text{cm}} + \frac{V_{\text{diff}}}{2} \\
V_{i2} &= V_{\text{cm}} - \frac{V_{\text{diff}}}{2}
\end{align*}
\]

**Differential Gain:** \( A_{\text{diff}} = \frac{V_{o1} - V_{o2}}{V_{i1} - V_{i2}} \) (want this to be large for this differential amplifier)

**Common-Mode Gain:** \( A_{\text{cm}} = \frac{V_{o1}}{V_{i1}} = \frac{V_{o2}}{V_{i2}} \) (want this to be small so that the amp rejects common-mode signals)

**Common-Mode Rejection Ratio:** \( \text{CMRR} = \frac{A_{\text{diff}}}{A_{\text{cm}}} \) (should be very high to force the differential mode and reject the common-mode)

- we also want a high Common-Mode Input Range to reject DC input offsets
- Note: No need for bypass capacitors (large) to the inputs or outputs — can just use direct coupling!

**Biasing/Setup Signal Common-Mode Behavior**

**Case:** \( R_{\text{ee}} = \infty \) -> ideal current source biasing, \( I_{\text{ee}} = \frac{I_{\text{rec}}}{2} \) -> \( V_{\text{o1}} = V_{\text{o2}} = V_{\text{ad}} = 0 \)

If \( V_{\text{cm}} \) \( \uparrow \) \( \Rightarrow \) \( V_{\text{th}} \) \( \uparrow \), but cannot draw from \( I_{\text{ee}} \); stays constant -> bias pt.

\[
V_{\text{cm}} \Rightarrow \frac{1}{2} I_{\text{ee}}
\]

**Case:** \( R_{\text{ee}} \) finite \( \Rightarrow \) \( V_{\text{th}} = V_{\text{ee}} - V_{\text{BE(on)}} \)

If \( V_{\text{cm}} \) \( \uparrow \) \( \Rightarrow \) \( V_{\text{th}} \) \( \uparrow \) \( \Rightarrow \) \( I_{\text{ee}} = I_{\text{ee}} + \frac{V_{\text{th}}}{R_{\text{ee}}} \) (cannot draw = \( I_{\text{ee}} \) + 0)

\( \Rightarrow \) in general, \( R_{\text{ee}} \) will be large, so this component will be large, and the bias pt. will be near

**Input common-mode range:**

- \( V_{\text{cm}} \) \text{ (max)} \( \uparrow \) \( \Rightarrow \) \( \frac{V_{\text{ee}} - V_{\text{BE}}}{2} \) \text{ (sat: KVL)}

**Output common-mode range:**

- \( V_{\text{cm}} \) \text{ (min)} \( \downarrow \) \( \Rightarrow \) \( \frac{V_{\text{ee}}}{2} \) \text{ (sat: KVL)}

- \( V_{\text{cm}} \) \text{ (max)} \( \downarrow \) \( \Rightarrow \) \( \frac{V_{\text{ee}}}{2} \) \text{ (sat: KVL)}

**Conclusion:**
- \( Q_{3} \) at edge of
- \( V_{\text{sat}} \) to \( V_{\text{cm}} \) \text{ (min)}
- \( V_{\text{cm}} \) \text{ (max)} = \( V_{\text{ee}} \) \text{ (sat)}
- \( V_{\text{BE}} \) at edge of
**Differential Mode Analysis**

**Small-Signal Analysis of Diff. Pair**

\[ N_{01} = -\frac{1}{2} g_m R_c N_{01} \]
\[ N_{02} = -\frac{1}{2} g_m R_c N_{02} \]
\[ + \frac{1}{2} g_m N_{01} R_c \]

\[ N_{01} - N_{02} = -2 g_m R_c (N_{01} - N_{02}) \]

\[ N_{01} - N_{02} = +\frac{1}{2} g_m R_c (N_{01} - N_{02}) \]

\[ N_{01} - N_{02} = -\frac{1}{2} g_m R_c (N_{01} - N_{02}) \]

\[ \frac{V_{01}}{V_{01}} = \frac{V_{02}}{V_{02}} \]

= Easiest to see this happening using the Spreadsheet (for those who must see the model only)

This time also get the \( \frac{N_{01} - N_{02}}{N_{01}} \) gain!

\[ N_{01} = -\frac{1}{2} g_m R_c N_{01} \]
\[ N_{02} = +\frac{1}{2} g_m R_c N_{02} \]

Ground to get only the contribution from \( N_{01} \)

\[ \frac{V_{01}}{V_{01}} = \frac{V_{02}}{V_{02}} \]

**Diff. Mode Analysis**

Assume a clue, we only diff. input:

\[ V_{oc} \]

Total current thru \( I_{Re} \) const.

\[ \rightarrow V_{es} = \text{const. or input changes} \]

\[ \rightarrow 0 \text{ ohm as an ironwire ground!} \rightarrow V_0 = 0 \text{ V (always)} \]

\[ \text{if we can ground } 0 \text{, and they have} \]

A [Differential Half Ops].

\[ \text{Note: Can really only do this for a purely symmetrical opamp!} \]
Common-Mode Analysis

By inspection:

\[
\frac{N_{C}}{\Delta V_C} = \frac{N_{D}}{\Delta V_D} \Rightarrow \frac{A_{DM}}{A_{DC}} = -g_{m}\frac{R_C}{2R_e}
\]

S.S. parameters determined
\( I_C = \frac{I_{EE}}{2} \)

Common-Mode Analysis

Assume a pure CM input \( \Delta V_C \) the inputs together

By symmetry, \( I_{X}=0 \), thus, really have the equivalent of an open ch. hpa

\( \Rightarrow \) can split to ch. into CM half-ckts.

\( \Rightarrow \) S.S. CM Half-Ch.:

\[
R_{CM} = \frac{N_{CM}}{N_{C}} \approx \frac{g_{m}\frac{R_C}{2R_e}}{1+g_{m}(2R_e)} \approx -\frac{R_C}{2R_e}
\]

What small-signal CMRR \( \Rightarrow \) want

\[
R_{FE} = \text{logs!}
\]

Common-Mode Rejection Ratio = CMRR

\[
\frac{A_{DM}}{A_{DC}} = -g_{m}\frac{R_C}{1+g_{m}(2R_e)} \Rightarrow \text{CMRR} = \frac{1+2g_{m}R_e}{R_C}
\]

Having looked at S.S. parameters, now move to large signal performance. Here, we'll be particularly interested in its linear range of the BOP.
Find $I_{C1} + I_{C2}$:

\[
EVL: \quad V_{C1} - V_{B1} + V_{B2} - V_{C2} = 0
\]

\[
I_{C1} = I_{CT} \exp \left( \frac{V_{B1}}{V_T} \right), \quad V_{B1} = V_T \ln \left( \frac{I_{C1}}{I_{CT}} \right)
\]

\[
V_{C1} = V_T \ln \left( \frac{I_{C1}}{I_{CT}} \right) - V_{C2} = 0 \quad \Rightarrow \quad \ln \frac{I_{C1}}{I_{CT}} = \frac{V_{C1} - V_{C2}}{V_T} = \frac{V_{C1}}{V_T}
\]

\[
\frac{I_{C1}}{I_{CT}} = \exp \left( \frac{V_{C1}}{V_T} \right)
\]

(1)

\[I_{EE} = I_{C1} + I_{C2} = \frac{1}{2} (I_{C1} + I_{C2}) \quad (2)
\]

Combine (1) \& (2) to get:

\[
I_{C1} = \frac{\alpha I_{EE}}{1 + \exp \left( \frac{V_{C1}}{V_T} \right)}, \quad I_{C2} = \frac{\alpha I_{EE}}{1 + \exp \left( \frac{V_{C1}}{V_T} \right)}
\]

\[(3)
\]

Find $V_{O1}$:

\[
V_{O1} = V_{C1} - I_{C1}R_C
\]

\[
V_{O2} = V_{C2} - I_{C2}R_C
\]

\[
V_{O1} = V_{O1} - V_{O2} = (I_{CE} - I_{C1}) R_C \quad \Rightarrow \quad \text{using (3)}
\]

\[
\begin{align*}
&= \alpha P_{BB} R_C \left\{ \frac{1}{1 + \exp \left( \frac{V_{O1}}{V_T} \right)} - \frac{1}{1 + \exp \left( -\frac{V_{O1}}{V_T} \right)} \right\} \\
&\times \frac{\exp \left( \frac{V_{O1}}{V_T} \right)}{\exp \left( -\frac{V_{O1}}{V_T} \right)} \times \frac{\exp \left( \frac{-V_{O1}}{2V_T} \right)}{\exp \left( \frac{V_{O1}}{2V_T} \right)}
\end{align*}
\]

\[
= \alpha P_{BB} R_C \left\{ \frac{\exp \left( \frac{-V_{O1}}{2V_T} \right) - \exp \left( \frac{V_{O1}}{2V_T} \right)}{\exp \left( -\frac{V_{O1}}{2V_T} \right) + \exp \left( \frac{V_{O1}}{2V_T} \right)} \right\}
\]

\[
= \alpha P_{BB} R_C \left\{ \frac{\exp \left( -\frac{V_{O1}}{2V_T} \right) - \exp \left( \frac{V_{O1}}{2V_T} \right)}{\exp \left( -\frac{V_{O1}}{2V_T} \right) + \exp \left( \frac{V_{O1}}{2V_T} \right)} \right\} = \alpha P_{BB} R_C \frac{\sinh \left( \frac{V_{O1}}{2V_T} \right)}{\cosh \left( \frac{V_{O1}}{2V_T} \right)}
\]

\[
\begin{align*}
\sinh u &= \frac{1}{2} (e^u - e^{-u}) \\
\cosh u &= \frac{1}{2} (e^u + e^{-u})
\end{align*}
\]

\[
\Rightarrow \quad V_{O1} = \alpha P_{BB} R_C \frac{\sinh \left( \frac{V_{O1}}{2V_T} \right)}{\cosh \left( \frac{V_{O1}}{2V_T} \right)}
\]

From our knowledge of the Taylor series for

\[
\tanh x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots
\]

This is fairly linear for small $V_{O1}$, but gets non-linear

abruptly when $V_{O1}$ approaches a threshold value!
In the above curve, the \( \frac{V_{out}}{V_{in}} \) Xrn function is really only linear for \( \text{Vol} \approx V_T \rightarrow \text{beyond} \ V_T \), start to enter the nonlinear region of curve \( \rightarrow \text{causes signal distortion: e.g., phone breaking up, television static} \).

To linearize, add emitter degeneration (same trick as used before for single transistors amplifiers). Needed in common to handle large input signals into distortion.

\[
\text{Adm} = \frac{-\beta \cdot R_e}{1 + \beta \cdot R_e} \rightarrow \text{s.s. gain reduced, but the linear range is increased.}
\]

If \( I_{RE} \) large, then this can form large supply voltages.

\[
\text{If } I_{RE} \text{ large, then this can be } V_{in} < V_T \text{ if this absorbs some of the input voltage!}
\]

**Alternative Draining Technique if Need Large DC Currents**:

Some S.S. performance use the need to drop a DC voltage across \( R_e \rightarrow \text{gutsy stuff.} \)

Can now drain \( V_{in} \) to \( V_{RE} \).
MOS Source-Coupled Pair

Assume: $V_{gs}$ and $V_{ds}$ are identical.

Find $\Delta I_d = I_{ds} - I_{de} = f(V_{ds})$.

-same approach as $V_{ds} = f(A_{ds}) \rightarrow$ haven't to get $\Delta I_d = f(V_{ds})$

$$I_{ds} = \frac{1}{2} \mu C_{ox} \left( \frac{W}{L} \right) (V_{gs} - V_{th})^2 \Rightarrow V_{gs} = V_{th} + \frac{2I_{ds}}{k}$$

$$V_{gs} = V_{th} + \frac{2I_{ds}}{k}$$

$$\therefore \quad V_{ds} = V_{gs} - V_{th} = \frac{2I_{ds}}{k} - \frac{2V_{th}}{k}$$

Define:

$$\Delta I_d = I_{ds} - I_{de} \quad \Rightarrow \quad I_{ds} = I_{d} + \frac{\Delta I_d}{2}$$

$$I_{de} = \frac{I_{d} + I_{ds}}{2}$$

$$I_{d} = \frac{I_{ds} - \frac{\Delta I_d}{2}}{2}$$

$$V_{ds} = \frac{\sqrt{2}}{k} I_{ds} - \frac{\sqrt{2}}{k} (\frac{\Delta I_d}{2})^2$$

$$\Rightarrow V_{ds} = \frac{\sqrt{2}}{k} I_{ds} - \sqrt{I_{ds} - (\frac{\Delta I_d}{2})^2}$$

$$\frac{k}{2} V_{ds}^2 = 2I_{ds} - \frac{\sqrt{2}}{k} (\frac{\Delta I_d}{2})^2$$

$$\Rightarrow \text{new expression to get } \Delta I_d \text{ (algebra)}$$

Solve for $\Delta I_d$:

$$\Delta I_d = \frac{k}{2} V_{ds} \left( \frac{2I_{ds}}{k} - V_{ds}^2 \right)^{\frac{1}{2}}$$

$$\frac{1}{2} \mu C_{ox} \left( \frac{W}{L} \right) V_{th} \left( \frac{2I_{ds}}{k} - V_{ds}^2 \right)^{\frac{1}{2}}$$

$$\text{Large Signal Equation for Differentiated Output Current}$$

Valid so long as the devices stay saturated:

$$V_{gs} \leq \frac{2I_{ds}}{k} \Rightarrow \frac{2I_{ds}}{k} \geq \frac{2I_{ds}}{k} (V_{gs} - V_{th})$$

If true then input devices are both saturated

Thus, to extend the linear input range:

1. $I_{ce} \uparrow \rightarrow (V_{gs} - V_{th}) \uparrow$
2. $W \uparrow$
3. $L \uparrow$

To choose $V_{ge}$:

$$V_{ge} = \frac{2I_{ds}}{k} - \frac{2I_{ds} \Delta I_d}{k} = \frac{V_{ds}}{2}$$

When $V_{ds} = V_{gsa} - V_{th}$ then $V_{gs}$ will cut-off

$$V_{gsa} = \frac{2I_{ds}}{k} \frac{2I_{ds} \Delta I_d}{k} = \frac{V_{ds}}{2}$$

Then plug in $\Delta I_d$ and solve for $V_{ds}$
Common-Mode Input Range - Range of input voltages in which all devices remain in saturation.

Low End - must keep M5 saturated