High-Swing Cascode current source:

\[ V_{\text{X}} = V_{\text{B2}} - V_{g_{52}} = V_{\text{ov}} \]

\[ V_{B2} = V_{T2} + 2V_{\text{ov}} \]

Use negative feedback technique:

Negative feedback loop is shown:

\[ T = \text{Loop gain} = g_{m5} \left[ r_{05} g_{m6} r_{06} \right] \]

\[ R_i (\text{open-loop}) = r_{05} \cdot g_{m6} r_{06} \]

We have shunt feedback. So,

\[ R_i (\text{closed-loop}) = \frac{R_i (\text{open-loop})}{1 + T} \]

\[ V_{\text{X}} = V_{B2} - V_{g_{52}} = V_{\text{ov}} \]

\[ V_{B2} = V_{T2} + 2V_{\text{ov}} \]

This is our design goal for \( V_{B2} \)

(i) Simple NMOS diode bias for \( V_{B2} \):

Need: \( V_{B2} = V_{g_{52}} + V_{\text{ov}} \)

\[ = V_{T2} + 2V_{\text{ov}} \]

Design \( M9 \) for \( 2V_{\text{ov}} \)

\[ (W/L)_{\text{Q9}} = \#(W/L) \]

From KVL:

\[ V_{\text{X}} = V_{B2} - V_{g_{52}} \]

\[ = (V_{T0} - V_{T2}) + V_{\text{ov}} \]

\[ < 0 \text{ because } V_{g_{52}} = V_{\text{ov}} \]

(ii) Stacked bias generator:

Need: \( V_{B2} = V_{T2} + V_{\text{ov}} \)

\[ V_{B2} = V_{T2} + V_{\text{ov}} \]

\[ M9: \text{Design for} \]

\[ V_{DS9} = V_{\text{ov}}. \text{ Thus,} \]

\[ V_{T10} = V_{T2} \text{ and} \]

\[ V_{\text{X}} = V_{\text{ov}} \]

a) \( M9 \) in triode

b) \( M10 \) in saturation
Design $M_9$ & $M_{10}$:

\[
I_{10} = I = \frac{k_n'}{2} \left( \frac{W}{L} \right)_{10} (V_{gs10} - V_t)^2 = \frac{k_n'}{2} \left( \frac{W}{L} \right)_{10} (V_{t2} + 2V_{ov} - V_{t10})^2
\]

\[
\therefore \left( \frac{W}{L} \right)_{10} = \frac{2I_{10}}{k_n'V_{ov}}
\]

Now, for $M_9$ in triode region: $V_{t2} = V_{t10}$ by design

\[
I_9 = I = k_n' \left( \frac{W}{L} \right)_{9} [ (V_{gs9} - V_t)(V_{bs9}) - \frac{1}{2} V_{gs9}^2 ]
\]

\[
= k_n' \left( \frac{W}{L} \right)_{9} [ (V_{t2} + 2V_{ov} - V_{t0})(V_{ov}) - \frac{1}{2} V_{ov}^2 ]
\]

\[
= k_n' \left( \frac{W}{L} \right)_{9} \left( \frac{3}{2} V_{ov}^2 \right) \text{ (Assume } V_{t0} = V_{t2} \text{)}
\]

\[
\therefore \left( \frac{W}{L} \right)_{9} = \frac{2I_9}{3k_n'V_{ov}^2} = \frac{1}{3} \left( \frac{2I_9}{k_n'V_{ov}^2} \right) = \frac{1}{3} \left( \frac{W}{L} \right)_{10}
\]

---

**Current Source Matching Considerations:**

- Matched current sources are often required in analog IC design — consider the matching of $I_{D1}$ & $I_{D2}$.

**Example Application:**

- Source-coupled pair with source degeneration:

**Simple Differential Pair**

- Consider mismatched currents as follows:

(A) (W/L) mismatches:

1. $I_1 = \frac{k_n'}{2} \left( \frac{W}{L} \right)_1 (V_{gs} - V_t)^2$

2. $I_2 = \frac{k_n'}{2} \left( \frac{W}{L} \right)_2 (V_{gs} - V_t)^2$

Note: 

- neglect channel-length and other mismatches for now:
Average Values:
\[ I_0 = \frac{I_{D1} + I_{D2}}{2} \]
\[ (W/L) = \frac{(W/L) + (W/L)}{2} \]
\[ \Delta (W/L) = (W/L) - \frac{(W/L)}{2} \]

Difference Values:
\[ \Delta I_D = I_{D1} - I_{D2} \]

Combining those:
\[ I_{D1} = I_0 + \frac{\Delta I_D}{2} \]
\[ I_{D2} = I_0 - \frac{\Delta I_D}{2} \]
\[ (W/L) = (W/L) + \frac{\Delta (W/L)}{2} \]
\[ (W/L)_2 = (W/L) - \frac{\Delta (W/L)}{2} \]

So, let's rewrite from previous page using those:
\[ I_1 = I_D + \frac{\Delta I_D}{2} = \frac{k}{2} \left[ (W/L) + \frac{\Delta (W/L)}{2} \right] \frac{V_{OV}^2}{I_D} \]
\[ = \frac{k}{2} \left( \frac{W}{L} \right) \frac{V_{OV}^2}{I_D} + \frac{\Delta (W/L)}{2} \frac{V_{OV}^2}{I_D} \]

Finally,
\[ \frac{\Delta I_D}{I_D} = \frac{\Delta (W/L)}{(W/L)} \]

(B) \( V_T \) Mismatches:
\[ I_1 = \frac{k}{2} \left( \frac{W}{L} \right) \left( V_{T1} - V_{T2} \right)^2 \]
\[ I_2 = \frac{k}{2} \left( \frac{W}{L} \right) \left( V_{T3} - V_{T4} \right)^2 \]

Using, \[ I_1 + \frac{\Delta I}{2} = \frac{k}{2} \left( \frac{W}{L} \right) \left( V_{T1} - V_{T2} - \frac{\Delta V_T}{2} \right)^2 \]
\[ = \frac{k}{2} \left( \frac{W}{L} \right) \left( V_{OV} - \frac{\Delta V_T}{2} \right)^2 \]
\[ = \frac{k}{2} \left( \frac{W}{L} \right) \left( V_{OV} - \frac{\Delta V_T}{2} \right)^2 \]
\[ = \frac{k}{2} \left( \frac{W}{L} \right) \left( V_{OV}^2 - V_{OV} \frac{\Delta V_T}{2} + \frac{\Delta V_T^2}{4} \right) \]
\[ \rightarrow \Delta V_T^2 \text{ are negligible} \]

\[ \frac{\Delta I_D}{I_D} = \frac{2 \Delta V_T}{V_{OV}} \]
\[ \text{(3) \, \text{Mismatch:}} \]

\[ I_D + \frac{\Delta V_D}{2} = \frac{1}{2} (k' + \frac{\Delta k'}{k'}) (V_O)^2 \]

\[ \Rightarrow \quad \frac{\Delta I_D}{I_D} = \frac{1}{2} \frac{k' (V_O)^2}{I_D} + \frac{\Delta k'}{k'} \frac{(V_O)^2}{I_D} \]

\[ \Rightarrow \quad \Delta I_D = \frac{1}{2} \frac{k' (V_O)^2}{I_D} \Delta k' \frac{(V_O)}{k'} \]

Finally, \[ \frac{\Delta I_D}{I_D} = \left[ \frac{\Delta k'}{k'} \right] \]

\[ \text{(1) \, \text{Mismatch:}} \quad \lambda = \frac{\lambda_1 + \lambda_2}{2} ; \quad \Delta \lambda = \lambda_1 - \lambda_2 \]

Using \( \lambda \) with \( \lambda_1 \) term:

\[ I_D + \frac{\Delta V_D}{2} = \frac{k' (V_O - V_T)^2}{2} \left[ 1 + \frac{(\lambda + \Delta \lambda)}{2} \right] V_O \]

\[ = \frac{k' (V_O)^2}{2} \left( 1 + \lambda V_O \right) + \frac{k' (V_O)^2}{2} V_O \frac{\Delta \lambda V_O}{2} \]

\[ \Rightarrow \quad \frac{\Delta I_D}{I_D} = \left[ \Delta \lambda V_O \right] \]

• Putting this all together assuming statistically independent random variations:

\[ \frac{\Delta I}{I} = \frac{\Delta (W/L)}{(W/L)} + 2 \frac{\Delta V_T}{V_O} + \frac{\Delta k'}{k'} + V_O \Delta \lambda \]

Note: Random quantities so signs not important:

\[ \Rightarrow \quad \frac{\Delta I}{I} = \frac{\Delta (W/L)}{(W/L)} + 2 \frac{\Delta V_T}{V_O} + \frac{\Delta k'}{k'} + V_O \Delta \lambda \]
For analog circuits, \((W/L) \gg 1 \implies W \gg L\)

\[
\frac{\Delta I}{I} = \frac{\Delta W}{W} + \frac{\Delta L}{L} + 2\frac{\Delta V_T}{V_{DS}} + \frac{\Delta V_T}{V_{DS}} + V_{DS} \Delta L
\]

Due to edge effect variations, \(\frac{\Delta W}{W} \ll \frac{\Delta L}{L}\).

- This is one reason why \(L > L_{min}\) in analog.
  Similar reasoning for last term as \(\Delta L\) variations are correlated to \(\Delta L\) variations.

- Intro to Opamps - see attached
**Ideal Op Amps**

**Properties of Ideal Op Amps:**
1. $R_{in} = \infty$ (leads to $i = 0$)
2. $R_o = 0$
3. $A_v = \infty$ (leads to $V_{+} = V_{-}$, assuming $V_o$ finite)

**Why?**
Because if $V(V_{+} - V_{-}) = V_o$ is finite,

$$\Rightarrow \frac{V_{+} - V_{-}}{V_o} = \text{Virtual short cut.}$$

**Big Assumption! ($V_o$ finite)**

**How can we assume this?**
Only when there is an appropriate negative feedback path.

**Negative Feedback**

![Diagram showing negative feedback](image)

- Negative feedback acts to oppose or subtract from input.
- $S_o = a S_e$ 
- $S_e = S_o - S_{in}$

**Overall Transfer Function**

$$\frac{S_o}{S_{in}} = \frac{a}{1 + a b}$$

**In Summary:**
1. Neg. FB can insure $S_o$ finite even with $a \approx 0$.
2. Gain dependent (or overall T.F.) depends only on external components, e.g., $b$.
3. Overall (closed-loop) gain $\frac{S_o}{S_{in}}$ is independent of amplifier gain $a$

---

*Very important!* As you'll see, when designing amplifiers using transistors, it's easy to get large gain, but it's hard to get an overall gain.

**i.e.,** if you're starting with $a = 50,000$, you might get $9,000 \approx 50,000$ instead.
Properties of Ideal Op Amps:

1. \( R_m = \infty \) leads to \( i_+ = 0 \)
2. \( R_o = 0 \)
3. \( A = \infty \) leads to \( N_p = N_- \), assuming \( N_e \) finite

Why? Because for
\[ \frac{N_p}{N_-} \cdot \frac{N_e}{N_e} = \frac{N_p}{N_e} \cdot \frac{N_e}{N_-} \]
\[ \Rightarrow \frac{V_p}{V_-} = \text{virtual short ckt.} \]
\[ \text{virtual ground} \]

Big assumption! \( (N_e \) finite)

How can we assume this? \( \Rightarrow \) only when there is an appropriate negative feedback path!

Negative Feedback

\[ S_i -> S_o \]

Negative feedback acts to oppose a substantial input.

\[ S_o = a S_e \]
\[ S_o = S_i - \beta S_o \]

\[ S_o = a S_i \Rightarrow S_o \frac{1}{1+a} \Rightarrow S_o \frac{a}{1+ab} \]

\[ \text{Overall transfer function} \]
\[ \Rightarrow S_o \frac{a}{1+ab} \]

\[ \frac{a}{a+1} \Rightarrow S_o \frac{a}{1+ab} \]

In Summary:

1. Net \( FB \) can incur \( S_o \) finite even with \( a \) on.
2. \( FB \) gain dependent (or overall T.F.) dependent only on external components. (e.g., \( \beta \))
3. Overall (closed-loop) gain \( \frac{S_o}{S_i} \) is independent of amplifier gain \( a \).

\( \Rightarrow \) very important! As you'll see, when designing amplifiers using transistors, it's easy to get large gain, but it's hard to get an exact gain.

i.e., if you're starting in \( a \approx 50,000 \), you might get
\[ 57,000 \approx 60,000 \text{ instead} \]