• VBE Bias Current Source

- Let's develop this from the beginning. The obvious approach is to place a VBE across a Resistor.

\[ I_0 = \frac{V_{BE1}}{R_2} \]  

- Neglecting base current, note: no direct Vcc dependence.

But this circuit has serious problems.

- Among several, \( I_0 \) is a poor current source because \( R_0 = \frac{R_2}{R_2} \) is not large.

- Fix this problem by adding \( Q_2 \) as shown:

\[ I_0 = \frac{V_{BE2}}{R_2} \]  

With \( \beta = \infty \), \( I_{C2} = I_{E2} = V_{BE1} = I_0 \)

- This circuit is better in two ways:

  1) We can use transistors to do something with \( I_0 \) (e.g., mirrors)

  2) Much higher \( R_0 \):

\[ R_0 = \left[ 1 + \left( \frac{g_{m2}R_{d2}}{V_{BE1}} \right) \right] V_{BE2} \]

\[ = \frac{g_{m2}R_{d2}R}{1 + g_{m2}R} \] where \( R = \frac{R_2}{R_1} \)

- But, now we need to find a convenient bias point, \( V_{B2} \), for the base voltage of \( Q_2 \). Grab a point that looks convenient:

\[ I_{REF} \]

\[ V_{B2} \]

\[ I_0 \]

\[ Q_2 \]

\[ +\]

\[ V_{BE2} \]

\[ R_2 \]

1) DC Voltage at \( Q_2 \) not established until base of \( Q_2 \) connected.

4) Source of base current for \( Q_2 \) connected.

Thus, \( V_2 = V_{BE1} + V_{BE2} = 1.4V \)
Now let's find \( \mathcal{S}_{V_{CC}} \) (neglect base currents):

\[
I_{\text{ref}} = \frac{V_{CC} - 2V_{BE}(on)}{R_1} \approx \frac{V_{CC}}{R_1} \quad ; \quad \frac{dI_{\text{ref}}}{dV_{CC}} = \frac{1}{R_1}
\]

\[
I_0 = \frac{V_{BE1}}{R_2} = \frac{V_T \ln \frac{I_{\text{ref}}}{I_{S1}}}{R_2} = \frac{V_T}{R_2} \left( \ln I_{\text{ref}} - \ln I_{S1} \right)
\]

\[
\frac{dI_0}{dV_{CC}} = \frac{V_T}{R_2} \left[ \frac{1}{I_{\text{ref}}} \frac{dI_{\text{ref}}}{dV_{CC}} - \frac{1}{I_{S1}} \frac{dI_{S1}}{dV_{CC}} \right]
\]

\[
\mathcal{S}_{V_{CC}} = \frac{V_{CC}}{I_0} \frac{dI_0}{dV_{CC}} = \frac{V_T}{I_0 R_2} \left[ \frac{V_{CC}}{I_{\text{ref}}} \frac{dI_{\text{ref}}}{dV_{CC}} - \frac{V_{CC}}{I_{S1}} \frac{dI_{S1}}{dV_{CC}} \right]
\]

\[
\mathcal{S}_{V_{CC}} = \frac{V_T}{I_0 R_2} \left[ \mathcal{S}_{V_{CC}} \frac{I_{\text{ref}}}{I_{\text{ref}}} - \mathcal{S}_{V_{CC}} \frac{I_{S1}}{I_{S1}} \right] = \frac{V_T}{V_{BE1}(on)}
\]

Note: \( \mathcal{S}_{I_0} \) is small (\( \approx 0.035 \)) but how to make it smaller (ideally, \( = 0 \))?

Observation: Non-zero \( \mathcal{S}_{I_0} \) comes from \( \mathcal{S}_{I_{\text{ref}}} = 1 \).

Solution: Find another current to replace \( I_{\text{ref}} = \frac{V_{CC}}{R_1} \), that is much less sensitive to \( V_{CC} \) variations; i.e.,

We want a new \( I_{\text{ref}} \) with \( \mathcal{S}_{I_{\text{ref}}} = 0 \).

We have that current already, \( I_0 \).

* Use self-biasing technique:

![Diagrams](image-url)

**Solutions:**

1) Graphical
2) Analytical
Consider a graphical solution:

- Bottom half of circuit: VBE generator
  \[ I_o = \frac{V_{BE1}}{R_2} = \frac{V_{BE1}}{R_2} \]

- Top half of circuit: pnp current mirror
  \[ I_o = I_{ref} \]

- Two possible stable solutions:
  1. Zero current solution. In a practical circuit, avoid this by including a start-up circuit. Once current is flowing, circuit operation moves to \( B \) as desired. \( \int I_o \text{d}V_c = 0 \) ignoring VA Effects.

We now have a general purpose current generator:

- Variation with \( V_c \) with \( V_A = 0 \)

- Let's gain some insight using simple negative feedback
Suppose \( V_c \) is perturbed by an amount, \( \Delta V_c \). How much are the voltage perturbations at nodes 1 and 2? Use small-signal voltage divider analysis:

At node 2:
\[
\Delta V_2 = \frac{R_{o2}}{R_{o2} + R_{o4}} \Delta V_c.
\]

Note the negative feedback loop that now exists relative to what we found in the middle of page 71.

With \( V_{b2} = \) constant bias voltage, \( R_o = \left[ \frac{1 + (g_{m2} + R_2)}{1 + g_{m} R} \right] R_2 \)

Break feedback loop at point X (very high impedance to right but much less to left).

i) Estimate by inspection:
Loop gain = \( T = \frac{V_A}{V_B} = \frac{V_B}{V_A} \)

ii) Determine \( R_{o2} \) with loop broken:
But this is just \( R_o \) as found before.

iii) This is series negative feedback:
\[
R_{o2} = (1 + T) R_o \approx T R_o = \frac{g_{m1}}{g_{o1}} R_o \approx \text{very high resistance}
\]

Apply same approach at 1 except now shunt feedback:
\[
R_c \text{ (open-loop)} = R_{o1} \Rightarrow \frac{V_{o1}}{V_{b1}} = \frac{1}{(1 + T) \frac{g_{m1} R_{o1}}{g_{m1}}} \approx \text{very low resistance}
\]
Thus, $\Delta V_2 = \frac{R_{O2} \Delta V_{cc}}{R_{O2} + R_4}$  \textbf{Note:} $R_4 = \frac{1}{g_{m4}} = \frac{1}{g_{m4}} (\text{Diode-connected device})$

$= \frac{g_{m1} R_{O1} R_0 \Delta V_{cc}}{g_{m1} R_{O1} R_0 + \frac{1}{g_{m4}}} \approx \frac{g_{m1} R_{O1} R_0 \Delta V_{cc}}{1 + g_{m1} g_{m4} R_{O1} R_0} = \Delta V_{cc}$

and $\Delta V_1 = \frac{R_{O1}}{R_{O1} + R_3}$

$= \frac{1}{g_{m1}} \frac{1}{1 + g_{m1} R_3} = \frac{\Delta V_{cc}}{g_{m1} R_2} \text{ (Small Voltage Change at 0)}$

- **Key point:** look at accuracy of pnp current mirror versus $\Delta V_{cc}$ perturbations:

  $\Delta V_{Ec4} = \Delta V_{cc} - \Delta V_2 = 0$

  $\Delta V_{Ec3} = \Delta V_{cc} - \Delta V_{cc} = (g_{m1} R_3 - 1) \Delta V_{cc} \approx \Delta V_{cc}$

  ![Graph](image)

  Some change in $I_4 = I_{\text{ref}}$ due to Early effect.

  - Improve using pnp cascode current mirror $\frac{V_{cc} + \Delta V}{I_{cc} + \Delta I}$

  - $R_3 \gg R_0$ so $\Delta V_1 = 0$

  - But $R_3 \approx g_{m3} R_0 R_5$

  So $\Delta I_{\text{ref}} = \frac{\Delta V_{cc}}{g_{m3} R_0 R_5}$ is very small so $\frac{\Delta I_{\text{ref}}}{\Delta V_{cc}} \approx 0$
$I_0 = 0$ !!! What about temperature sensitivity?

Define Fractional Temperature Coefficient:

$$TCF = \left[ 1 - \frac{\frac{dI_0}{dT}}{I_0} \right] \quad \text{where} \quad I_0 = \frac{V_{BE1}}{R_2}$$

$$= \frac{R_2}{V_{BE1}} \frac{d}{dT} \left( \frac{V_{BE1}}{R_2} \right) = \frac{R_2}{V_{BE1}} \left( \frac{1}{R_2} \frac{dV_{BE1}}{dT} + \frac{V_{BE1}}{R_2} \frac{dR_2}{dT} \right)$$

$$\therefore TCF = \frac{\frac{1}{V_{BE1}} \frac{dV_{BE1}}{dT} - \frac{1}{R_2} \frac{dR_2}{dT}}{TCF/R_2}$$

Can we cancel?

- Typical $TCF$ values:
  - Diffused $R_2 \approx 1000 - 1500 \text{ ppm/°C}$
    (Note: $1000 \text{ ppm} = \frac{1000}{1,000,000} = 0.1 \%$)
  - Polysilicon $R_2 \approx 500 \text{ ppm/°C}$
  - $V_{BE1} \approx -3300 \text{ ppm/°C}$

So, back to our $V_{BE1}/R_2$ reference:

$$TCF = \frac{TCF}{V_{BE1}} - \frac{TCF}{R_2}$$

$$= -3300 \text{ ppm/°C} - 1000 \text{ ppm/°C} = -4300 \text{ ppm/°C}$$

(i) Commercial Temperature Range = 0-70°C

$\implies \sim 30 \%$ Variation in $I_0$

(ii) Military Temperature Range = -55 to 125°C

$\implies \sim 77 \%$ Variation in $I_0$
• Consider a current generator with a positive TCF.

\[ \frac{\partial I_o}{\partial T} \]

- **KVL Loop:**
  \[ I_o = \frac{V_{BE1} - V_{BE2}}{R_2} \]
  \[ I_o = \frac{\Delta V_{BE}}{R_2} \]

- \[ I_{e1} = I_{e2} = I_o \Rightarrow \]

\[ TCF = \frac{1}{I_o} \frac{\partial I_o}{\partial T} \]

\[ = \frac{R_2}{V_T \ln(N)} \frac{\partial}{\partial T} \left( \frac{V_T}{R_2} \ln(N) \right) \]

\[ = \frac{R_2}{V_T \ln(N)} \ln(N) \left[ \frac{1}{R_2} \frac{DV_T}{DT} - \frac{V_T}{R_2} \frac{DR_2}{DT} \right] \]

\[ = \frac{V_T}{R_2} \frac{DV_T}{DT} - \frac{V_T}{R_2} \frac{DR_2}{DT} = +3300 \text{ ppm/°C} \]

- Better than \[ \frac{V_{BE1}}{R_2} \] reference but \( \neq 0 \)

**Solution:** Silicon Bandgap Reference

**Conceptual:**

- \[ V_{BE} \approx \frac{1.205 \text{ V}}{D} \]

- \[ T = 300 \text{°K} \]

**2404A Students:**

4.4.2 - 4.4.3 6M

Chap. 11 Razavi