Homework 1

1. Deriving Laplace Transforms
   Derive the Laplace transforms of the following time functions using the definition of the Laplace transform. Do not simply just look them up in a table.
   
   (a) $\delta(t) = \text{impulse}$
   
   (b) $\sin(\omega t)u(t)$

2. Using Laplace Transform Pairs
   Using Laplace transform pairs in Table 2.1 and theorems in Table 2.2 in the book of Nise, derive the Laplace transforms for the following time function:
   
   (a) $e^{-at}\cos(\omega t)u(t)$

   Solve the following differential equation using Laplace transforms. Assume all forcing functions are zero prior to $t = 0^-$. (Hint: you will need to use partial fraction decomposition)
   
   (a) $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 5\sin 3t$
   
   $x(0) = 4, x'(0) = 1$

4. Differential Equation To Transfer Function in Laplace Domain
   A system is described by the following differential equation (see below). Find the expression for the transfer function of the system, $Y(s)/X(s)$, assuming zero initial conditions.
   
   (a) $\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x$

5. Transfer Function Review
   Write the corresponding differential equation for the following transfer function:
   
   (a) $\frac{X(s)}{F(s)} = \frac{s + 3}{s^3 + 11s^2 + 12s + 18}$

6. Electrical Networks
   Find the transfer function, $G(s) = V_L(s)/V(s)$ for the following network:
\[ \nu(t) \pm 2 \ \text{H} \ \nu_L(t) \]