Consider SISO LTI system with input $u(t)$ and output $y(t)$ with transfer function

$$
\frac{Y(s)}{U(s)} = \frac{b_0 s^4 + b_1 s^3 + b_2 s^2 + b_3 s + b_4}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}
$$

(1)

Introduce intermediate function $X(s)$ with

$$
Y(s) = (b_0 s^4 + b_1 s^3 + b_2 s^2 + b_3 s + b_4)X(s)
$$

(2)

and

$$
X(s) = \frac{U(s)}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}.
$$

(3)

Using the inverse Laplace transform on eqn. 3 we have

$$
d^4 x \over dt^4 = u - a_1 d^3 x \over dt^3 - a_2 d^2 x \over dt^2 - a_3 dx \over dt - a_4 x
$$

(4)

This differential equation for $x(t)$ can be solved using a chain of integrators with feedback as shown here:

where $x = x_1, \dot{x}_1 = x_2, \ddot{x}_2 = x_3, \dddot{x}_3 = x_4$ and $\dot{x}_4 = \frac{d^4 x}{dt^4}$.

Referring to the block diagram, it is easy to write the state equation:

$$
\dot{x} = Ax + Bu = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4 & -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t)
$$

(5)

**Output Equation**

Considering that by the inverse Laplace transform of eqn.( 2), the output $y(t)$ is just a linear combination of the derivatives of $x(t)$, we get:

$$
y(t) = b_0 \frac{d^4 x}{dt^4} + b_1 \frac{d^3 x}{dt^3} + b_2 \frac{d^2 x}{dt^2} + b_3 \frac{dx}{dt} + b_4 x .
$$

(6)

The output $y(t)$ is shown in the block diagram.

In terms of the state variable assignment we have $y(t) = b_0 \dot{x}_4 + b_1 x_4 + b_2 x_3 + b_3 x_2 + b_4 x_1$. However, the output equation $y = Cx + Du$ needs to be in terms of the states, thus we need to calculate $\dot{x}_4$ by using eqn.( 4). Hence,

$$
y(t) = b_0 (u - a_1 x_4 - a_2 x_3 - a_3 x_2 - a_4 x_1) + b_1 x_4 + b_2 x_3 + b_3 x_2 + b_4 x_1 .
$$

(7)

By combining terms, we get

$$
y(t) = b_0 u + (b_1 - b_0 a_1) x_4 + (b_2 - b_0 a_2) x_3 + (b_3 - b_0 a_3) x_2 + (b_4 - b_0 a_4) x_1
$$

(8)

then the corrected output equation for controllable canonical form is:

$$
y = Cx + Du = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + b_0 u(t)
$$

(9)