Small-Signal Operation
MOSFET Small-Signal Model - Summary

- Since gate is insulated from channel by gate-oxide input resistance of transistor is infinite.
- Small-signal parameters are controlled by the Q-point.
- For the same operating point, MOSFET has lower transconductance and an output resistance that is similar to the BJT.

\[
\begin{align*}
I_G &= 0 \\
I_D &= \frac{K_n}{2} (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS})
\end{align*}
\]

Transconductance:
\[
g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \sqrt{2K_n I_D}
\]

Output resistance:
\[
r_o = \frac{1}{g_m} = \frac{1 + \lambda V_{DS}}{\lambda I_D} = \frac{1}{\lambda I_D}
\]

Amplification factor for \(V_{DS} \ll 1\):
\[
\mu = g_m r_o = \frac{1 + \lambda V_{DS}}{\lambda I_D} = \frac{2K_n}{\lambda I_D}
\]
MOSFET Small-Signal Operation

Body Effect in Four-terminal MOSFETs

Drain current depends on threshold voltage which in turn depends on $v_{SB}$.

Back-gate transconductance is:

$$g_{nb} = \frac{\partial i_D}{\partial v_{SB}} = -\frac{\partial i_D}{\partial V_{TN}} \frac{\partial V_{TN}}{\partial v_{SB}}_{Q_{point}} = -g_m \eta$$

$0 < \eta < 3$ is called the back-gate transconductance parameter.

Bulk terminal is a reverse-biased diode. Hence, no conductance from the bulk terminal to other terminals.

Small-Signal Model for PMOS Transistor

- For a PMOS transistor
  $$v_{SG} = V_{GG} - v_{gs}$$
  $$i_D = i_D - i_d$$

- Positive signal voltage $v_{gs}$ reduces source-gate voltage of the PMOS transistor causing decrease in total current exiting the drain, equivalent to an increase in the signal current entering the drain.

- The NMOS and PMOS small-signal models are the same.
Common-Source Amplifiers
Small-Signal Analysis - ac Equivalent Circuit

• ac equivalent circuit is constructed by assuming that all capacitances have zero impedance at signal frequency and dc voltage sources are ac ground.

Common-Source Amplifiers
Small-Signal Equivalent Circuit

• Input voltage is applied to the gate terminal
• Output signal appears at the drain terminal
• Source is common to both input and output signals Thus circuit is termed a Common-Source (C-S) Amplifier.
• The terminal gain of the C-S amplifier is the gain from the gate terminal to the drain terminal

\[ A_{vit}^{CE} = \frac{V_d}{V_g} = -g_m R_L \quad R_L = r_e \| R_D \| R_3 \]
Common-Source Amplifiers
Input Resistance and Signal-Source Gain

Define \( R_{\text{in}} \) as the input resistance looking into the base of the transistor. \( R_{\text{in}} \) is the resistance presented to \( \hat{v}_i \).

The signal source voltage gain is:

\[
A_{CS} = \frac{\hat{v}_o}{\hat{v}_i} = \frac{g_m \hat{I}_D}{R_{\text{ds}}} = \frac{g_m \hat{I}_D}{R_{\text{ds}}} R_L + R_{\text{g}}
\]

“Rule of Thumb” Design Estimate

Typically: \( r_e \gg R_D \) and \( R_L \gg R_D \)

\[
A_{CS} = -g_m R_D = \frac{\hat{I}_D R_D}{V_{GS} - V_{TH}}
\]

\( \hat{I}_D R_D \) represents the voltage dropped across drain resistor \( R_D \).

A typical design point is \( \hat{I}_D R_D = \frac{V_{DD}}{2} \) with \( V_{GS} - V_{TH} = 1 \) V

\[
\therefore A_{CS} = -V_{DD}
\]

Our rule-of-thumb estimate for the C-S amplifier:
the voltage gain equals the power supply voltage.
Note that this is 10 times smaller than that for the BJT.
Common-Source Amplifiers
Voltage Gain Example

• Problem: Calculate voltage gain, input resistance and maximum input signal level for a common-source amplifier with a specified Q-point
• Given data: $K_n = 0.50 \text{ mA/V}^2$, $V_{TN} = 1 \text{ V}$, $\lambda = 0.0133 \text{ V}^{-1}$, Q-point is $(0.241 \text{ mA}, 3.81 \text{ V})$
• Assumptions: Transistor is in the active region. Signals are low enough to be considered small signals.

• Analysis:

\[
g_m = 2K_n I_D (1 + \lambda V_{DS}) = 0.503 \text{ mS}
\]

\[
r_o = \frac{\lambda^{-1} + V_{DS}}{I_D} = 328 \text{ k}\Omega
\]

\[
R_C = R_1 R_2 = 892 \text{ k}\Omega
\]

\[
R_L = r_o R_D R_3 = 17.1 \text{ k}\Omega
\]

\[
A_v = \frac{-g_m R_L}{r_o R_D} = \frac{-0.503 \times 892}{829} = -0.8099 \approx -0.81
\]

\[
V_{GS} - V_{IN} = \frac{2I_D}{K_n} = 0.982 \text{ V}
\]

\[
\therefore |v| \leq 0.2(0.982V) = 0.197 \text{ V}
\]

Check the rule-of-thumb estimate: $A_v^{CS} = -V_{G0} = -12 \text{ V}$ (ballpark estimate)
For comparable bias points, output resistances of C-S and C-E amplifiers are similar.

Apply test source $v_i$ and find $i_i$ (with $v_i = 0$)

$v_{be} = 0 \rightarrow g_m v_{be} = 0$

$\therefore R_{out} = \frac{v_i}{i_i} = R_C |_{r_i}$

$R_{out} \approx R_C$ for $r_i \gg R_C$

$v_{gs} = 0 \rightarrow g_m v_{gs} = 0$

$\therefore R_{out} = \frac{v_i}{i_i} = R_D |_{r_i}$

$R_{out} \approx R_D$ for $r_i \gg R_D$

For comparable bias points, output resistances of C-S and C-E amplifiers are similar.
**TABLE 13.3**  
Small-Signal Parameter Comparison

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>BIPOLAR TRANSISTOR</th>
<th>MOSFET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transconductance $g_m$</td>
<td>$I_c \over V_T$</td>
<td>$2I_D \over V_{GS} - V_{TH} \approx \sqrt{2K_nI_D}$</td>
</tr>
<tr>
<td>Input resistance $r_x$</td>
<td>$g_m \over I_c$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Output resistance $r_o$</td>
<td>$V_{BE} + V_{CE} \over I_c$</td>
<td>$V_A \over I_D \approx \frac{1}{\lambda} \sqrt{\frac{2K_n}{I_D}}$</td>
</tr>
<tr>
<td>Intrinsic voltage gain $\mu_f$</td>
<td>$V_{BE} + V_{CE} \over V_T$</td>
<td>$2 \left( \frac{1}{V_T} + V_{GS} \right) \over V_{GS} - V_{TH} \approx \frac{1}{\lambda} \sqrt{\frac{2K_n}{I_D}}$</td>
</tr>
</tbody>
</table>

Small-signal requirement:  
- $V_{BE} \leq 0.005$ V  
- $V_{CE} \leq 0.2(V_{GS} - V_{TH})$

For active region expressions, use Table 13.3:

**BJT:**  
$I_c = I_f \exp \left( \frac{V_{BE}}{V_T} - 1 \right) \left[ 1 + \frac{V_{CE}}{V_T} \right] V_T = \frac{kT}{q}$

**MOSFET:**  
$I_D = \frac{K}{2} (V_{GS} - V_{TH}) (1 + \lambda V_{DS})$  
$K = \mu_n C_{ox} \frac{W}{L}$

**JFET:**  
$I_D = I_{DS} \left( 1 - \frac{V_{GS}}{V_T} \right) (1 + \lambda V_{DS})$  

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**Common-Emitter / Common-Source Amplifiers Summary**

<table>
<thead>
<tr>
<th>COMMON-EMITTER AMPLIFIER</th>
<th>COMMON-SOURCE AMPLIFIERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terminal gain $A_{vT}$</td>
<td>$-g_m R_T$</td>
</tr>
<tr>
<td>Voltage gain $A_v$</td>
<td>$-10V_{CC}$</td>
</tr>
<tr>
<td>Input resistance $R_{in}$</td>
<td>$R_D \over V_T$</td>
</tr>
<tr>
<td>Output resistance $R_{out}$</td>
<td>$R_C | R_T | R_D$</td>
</tr>
<tr>
<td>Input signal phase</td>
<td>0.005 V</td>
</tr>
</tbody>
</table>
Amplifier Power Dissipation

Static power dissipation in amplifiers is found from their dc equivalent circuits.

(a) Total power dissipated in BJT:

\[ P_D = V_{CE} I_C + V_{BE} I_B \]

Total power supplied is:

\[ P_S = V_{CC} (I_C + I_B) \]

(b) Total power dissipated in MOSFET:

\[ P_D = V_{DS} I_D \]

Total power supplied is:

\[ P_S = V_{DD} (I_D + I_S) \]

Amplifier Signal Range

\[ V_{CE} = V_{CE} - V_m \sin \omega t \] where \( V_m \) is the output signal. Active region operation requires \( V_{CE} \geq V_{BE} \). So:

\[ V_m \leq V_{CE} - V_{BE} \]

Also:

\[ V_m(t) = I_c R_c - V_m \sin \omega t \geq 0 \]

\[ \therefore V_m \leq \min \left[ I_c R_c \left( V_{CE} - V_{BE} \right) \right] \]

Similarly for MOSFETs:

\[ V_M \leq \min \left[ I_D R_D \left( V_{DS} - (V_{GS} - V_{TN}) \right) \right] \]