1 Derivation of the Gain of a Cascode Stage

I went over a derivation of the gain of a cascode stage during discussion that was incorrect (the end result was correct, but the methodology was flawed). Here I will present a method that still considers the stages as a cascade of a common-emitter and common-base stage, but does so correctly. Consider the cascode configuration in Figure 1.

First, let’s consider the common-emitter stage formed by $Q_2$. We know that a common-emitter amplifier has a gain of $A_{v1} = \frac{v_x}{v_{in}} = -g_{m2}R_C$, where $R_C$ is the equivalent resistance seen at the collector. We just have to find this equivalent $R_C$. We can split the problem by considering the resistance seen looking “up” from node $x$ and the resistance seen looking “down” from node $x$.

Looking “down”, we just have $r_{o2}$. Looking “up”, we need to draw a small-signal model to derive the resistance. See Figure 2 for the appropriate small-signal model.

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Observe that the loop containing the dependent current source is isolated. By this I mean that it cannot contribute any current at the emitter node since it must draw that same amount of current back out (if this
isn’t clear, write KCL at the emitter and you’ll observe the current source cancelling itself out). This just leaves the test source and the resistor $r_{\pi_1}$ to ground, meaning the equivalent resistance looking “up” from node $x$ is just $r_{\pi_1}$. Thus, we have the gain for the first stage:

$$A_{v1} = \frac{v_x}{v_{in}} = -g_{m2} (r_{\pi_1} \parallel r_{o2})$$

Now let’s take a look at the second stage. We need to compute $A_{v2} = \frac{v_{out}}{v_x}$ for this common-base stage. The small-signal model is shown in Figure 3. First, let’s write a KCL equation at the output.

![Small-signal model for finding the gain of the common-base stage of the cascode](image)

Figure 3: Small-signal model for finding the gain of the common-base stage of the cascode

$$g_{m1}v_{\pi_1} + \frac{v_{out} - v_x}{r_{o1}} = 0$$

Noting that $v_{\pi_1} = -v_x$, we can rewrite this KCL equation and solve for $v_{out}/v_x$ as follows:

$$-g_{m1}v_x + \frac{v_{out} - v_x}{r_{o1}} = 0$$

$$v_x \left( g_{m1} + \frac{1}{r_{o1}} \right) = \frac{v_{out}}{r_{o1}}$$

$$A_{v2} = \frac{v_{out}}{v_x} = 1 + g_{m1}r_{o1}$$

Now we can find $A_v = v_{out}/v_{in} = A_{v1}A_{v2}$ as follows:

$$A_v = -g_{m2} (r_{\pi_1} \parallel r_{o2}) (1 + g_{m1}r_{o1})$$

$$\approx -g_{m1}g_{m2}r_{o1} (r_{o2} \parallel r_{\pi_1})$$

This is the same expression as derived in lecture, as expected.