Today: Hashing (Data Structures Chapter 7).

Next topic: Sorting (Data Structures Chapter 8).
Back to Simple Search: Hashing

- Linear search is OK for small data sets, bad for large.
- So linear search would be OK if we could rapidly narrow the search to a few items.
- Suppose that in constant time could put any item in our data set into a numbered bucket, where # buckets stays within a constant factor of # keys.
- Suppose also that buckets contain roughly equal numbers of keys.
- Then search would be constant time.
Hash functions

• To do this, must have way to convert key to bucket number: a *hash function*.

• Example:
  
  - \( N = 200 \) data items.
  - keys are longs, evenly spread over the range \( 0..2^{63} - 1 \).
  - Want to keep maximum search to \( L = 2 \) items.
  - Use hash function \( h(K) = K \mod M \), where \( M = N/L = 100 \) is the number of buckets: \( 0 \leq h(K) < M \).
  - So 100232, 433, and 10002332482 go into different buckets, but 10, 400210, and 210 all go into the same bucket.
External chaining

- Array of $M$ buckets.
- Each bucket is a list of data items.

```
300 -> 100 -> 1500
```

```
201 -> 1
```

```
1199
```

- Not all buckets have same length, but average is $N/M = L$, the load factor.
- To work well, hash function must avoid collisions: keys that “hash” to equal values.
Open Addressing

• Idea: Put one data item in each bucket.
• When there is a collision, and bucket is full, just use another.
• Various ways to do this:
  - Linear probes: If there is a collision at $h(K)$, try $h(K)+m$, $h(K)+2m$, etc. (wrap around at end).
  - Quadratic probes: $h(K) + m$, $h(K) + m^2$, …
  - Double hashing: $h(K) + h'(K)$, $h(K) + 2h'(K)$, etc.
• Example: $h(K) = K \% M$, with $M = 10$, linear probes with $m = 1$.
  - Add 1, 2, 11, 3, 102, 9, 18, 108, 309 to empty table.

| 108 | 1 | 2 | 11 | 3 | 102 | 309 | 18 | 9 |

• Things can get slow, even when table is far from full.
• Lots of literature on this technique, but
• Personally, I just settle for external chaining.
Filling the Table

• To get (likely to be) constant-time lookup, need to keep #buckets within constant factor of #items.

• So resize table when load factor gets higher than some limit.

• In general, must re-hash all table items.

• Still, this operation constant time per item,

• So by doubling table size each time, get constant amortized time for insertion and lookup

• (Assuming, that is, that our hash function is good).
Hash Functions: Strings

- For String, \"s_0s_1 \cdots s_{n-1}\" want function that takes all characters and their positions into account.

- What’s wrong with \( s_0 + s_1 + \ldots + s_{n-1} \)?

- For strings, Java uses

\[
h(s) = s_0 \cdot 31^{n-1} + s_1 \cdot 31^{n-2} + \ldots + s_{n-1}
\]

computed modulo \(2^{32}\) as in Java int arithmetic.

- To convert to a table index in \(0..N - 1\), compute \(h(s) \% N\) (but don’t use table size that is multiple of 31!)

- Not as hard to compute as you might think; don’t even need multiplication!

```java
int r; r = 0;
for (int i = 0; i < s.length(); i += 1)
    r = (r << 5) - r + s.charAt(i);
```
Hash Functions: Other Data Structures I

• Lists (ArrayList, LinkedList, etc.) are analogous to strings: e.g., Java uses

        hashCode = 1; Iterator i = list.iterator();
        while (i.hasNext()) {
            Object obj = i.next();
            hashCode =
                31*hashCode
                + (obj==null ? 0 : obj.hashCode());
        }

• Can limit time spent computing hash function by not looking at entire list. For example: look only at first few items (if dealing with a List or SortedSet).

• Causes more collisions, but does not cause equal things to go to different buckets.
Hash Functions: Other Data Structures II

- Recursively defined data structures ⇒ recursively defined hash functions.

- For example, on a binary tree, one can use something like

  ```java
  hash(T):
  if (T == null)
    return 0;
  else return someHashFunction (T.label ())
    + 255 * hash(T.left ())
    + 255*255 * hash(T.right ());
  ```

- Can use address of object ("hash on identity") if distinct (≠) objects are never considered equal.

- But careful! Won't work for Strings, because .equal Strings could be in different buckets:

  ```java
  String H = "Hello",
  S1 = H + ", world!",
  S2 = "Hello, world!";
  ```

- Here S1.equals(S2), but S1 != S2.
What Java Provides

- **In class** `Object`, is function `hashCode()`.
- By default, returns address of `this`, or something similar.
- Can override it for your particular type.
- For reasons given on last slide, is overridden for type `String`, as well as many types in the Java library, like all kinds of `List`.
- The types `Hashtable`, `HashSet`, and `HashMap` use `hashCode` to give you fast look-up of objects.

```java
HashMap<KeyType,ValueType> map = 
    new HashMap<KeyType,ValueType> (approximate size, load factor);

map.put (key, value);  // Map KEY -> VALUE.
// VALUE last mapped to by SOMEKEY.
... map.get (someKey) 
    // VALUE last mapped to by SOMEKEY.
... map.containsKey (someKey) 
    // Is SOMEKEY mapped?
... map.keySet ()  // All keys in MAP (a Set)
```
Characteristics

• Assuming good hash function, add, lookup, deletion take $\Theta(1)$ time, amortized.

• Good for cases where one looks up equal keys.

• Usually bad for range queries: “Give me every name between Martin and Napoli.” [Why?]

• But sometimes OK, if hash function is monotonic (i.e., when key $k_1 > k_2$, then $h(k_1) \geq h(k_2)$). For example,
  - Items are time-stamped records; key is the time.
  - Hashing function is to have one bucket for every hour.

• Hashing is probably not a good idea for small sets that you rapidly create and discard [why?]
Comparing Search Structures

Here, $N$ is #items, $k$ is #answers to query.

<table>
<thead>
<tr>
<th>Function</th>
<th>Unordered List</th>
<th>Sorted Array</th>
<th>Bushy Search Tree</th>
<th>“Good” Hash Table</th>
<th>Heap</th>
</tr>
</thead>
<tbody>
<tr>
<td>find</td>
<td>$\Theta(N)$</td>
<td>$\Theta(lg N)$</td>
<td>$\Theta(lg N)$</td>
<td>$\Theta(1)$</td>
<td>$\Theta(N)$</td>
</tr>
<tr>
<td>add</td>
<td>$\Theta(1)$</td>
<td>$\Theta(N)$</td>
<td>$\Theta(lg N)$</td>
<td>$\Theta(1)$</td>
<td>$\Theta(lg N)$</td>
</tr>
<tr>
<td>range query</td>
<td>$\Theta(N)$</td>
<td>$\Theta(k + lg N)$</td>
<td>$\Theta(k + lg N)$</td>
<td>$\Theta(N)$</td>
<td>$\Theta(N)$</td>
</tr>
<tr>
<td>find largest</td>
<td>$\Theta(N)$</td>
<td>$\Theta(1)$</td>
<td>$\Theta(lg N)$</td>
<td>$\Theta(N)$</td>
<td>$\Theta(1)$</td>
</tr>
<tr>
<td>remove largest</td>
<td>$\Theta(N)$</td>
<td>$\Theta(1)$</td>
<td>$\Theta(lg N)$</td>
<td>$\Theta(N)$</td>
<td>$\Theta(lg N)$</td>
</tr>
</tbody>
</table>