Modeling ΣΔ Nonlinearities

- Many component nonlinearities contribute negligible errors
  - Don't waste CPU cycles modeling the voltage coefficient of every capacitor in the loop
  - Unnecessarily complex models reduce the chance to find relevant problems, and, perhaps, solutions
  - As with all nonidealities, model one at a time
- Expect errors from the 2^{nd} integrator to be reduced by the gain of the 1^{st} integrator
  - Errors further downstream are even less significant

Capacitor Voltage Coefficient

- Ideal capacitor

$$Q = CV$$

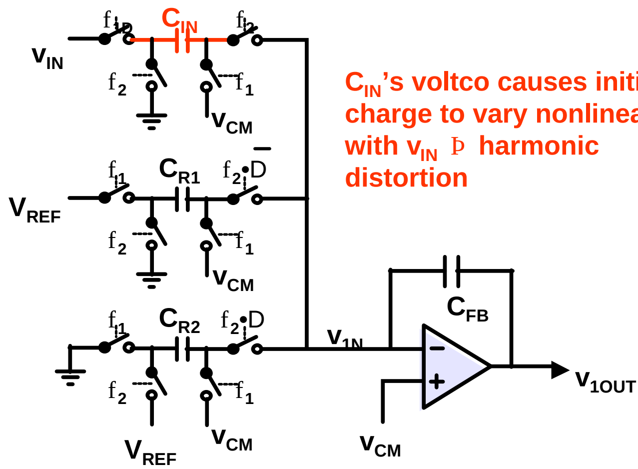
- Practical capacitor (1^{st} order model)

$$Q = C(V)V \qquad \text{with}$$

$$Q(V) = C_o(1 + aV + \ldots)V$$

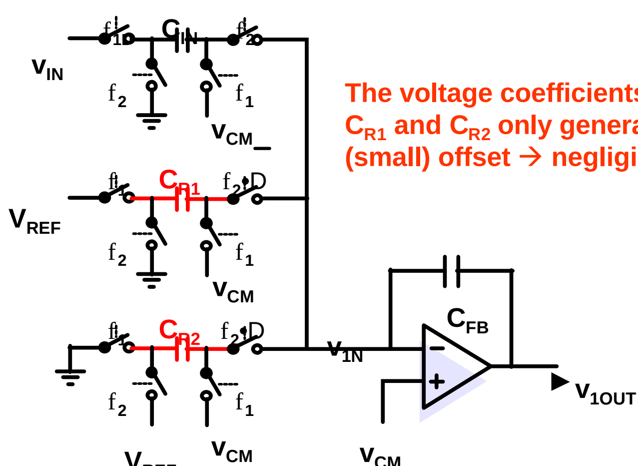
- Typical voltage coefficients
  - Poly-poly capacitors 10 ppm/V
  - Metal-metal capacitors 1 … 10 ppm/V

Integrator 1



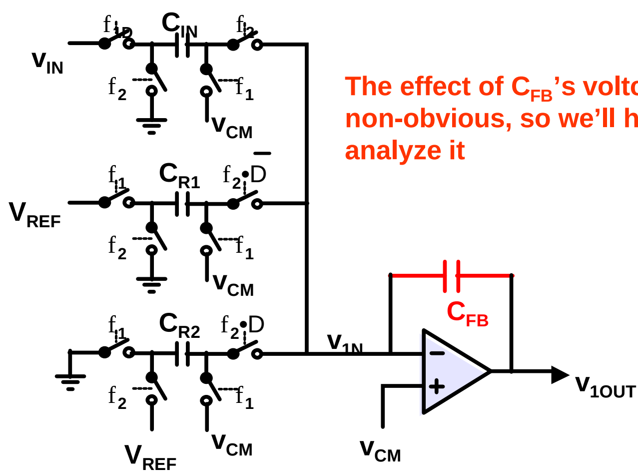
C_{IN}'s voltco causes initial charge to vary nonlinearly with v_{IN} Þ harmonic distortion

Integrator 1



The voltage coefficients of C_{R1} and C_{R2} only generates a (small) offset → negligible

Integrator 1



The effect of C_{FB}'s voltco is non-obvious, so we'll have to analyze it

C_{IN} Voltage Coefficient

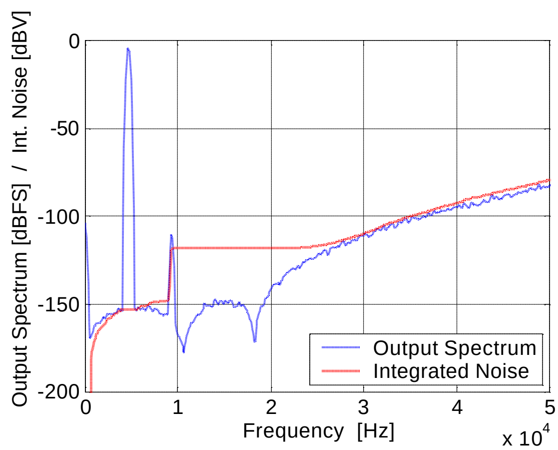
- From charge conservation (V_{CM}=0, C_{R1}=C_{R2}=C_R):

$$V_{1OUT}(k) = \underbrace{V_{1OUT}(k-1)}_{\text{integration}}$$

$$\underbrace{+\frac{C_{IN}}{C_{FB}}V_{IN}(k-1) + a\frac{C_{IN}}{C_{FB}}V_{IN}^{2}(k-1)}_{\text{converter input}}$$

$$\underbrace{-D\frac{C_R}{C_{FB}}V_{REF}}_{\text{1-bit feedback}}$$

C_{IN} Voltage Coefficient

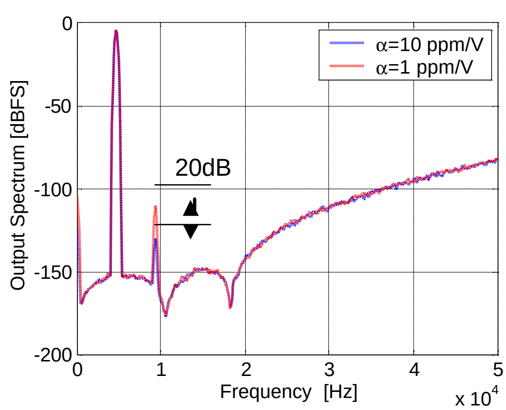


- $V_{in} = V_{FS} = 1V$
- Spectrum scaled for V_{FS} → 0dB (window lowers peak)
- Noise integral excludes DC, fundamental
- $\alpha = 10$ ppm/V

- 2nd harmonic at –103dB dominates noise!

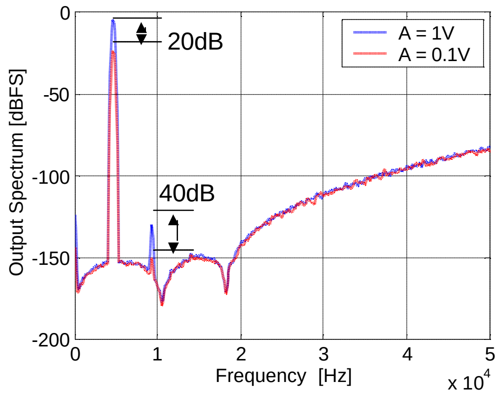
- Let's characterize it …

C_{IN} Voltage Coefficient



2nd harmonic increases 1dB per 1dB increase of α

C_{IN} Voltage Coefficient



2^{nd} harmonic increases 2dB per 1dB increase of the input signal amplitude

C_{FB} Voltage Coefficient

- Let's look next at the voltage coefficient of the feedback capacitor in the 1^{st} integrator
- We "turn off" all other nonidealities – C_{IN} voltage coefficients, noise, etc.
  - Makes it easier to find the effect of C_{FB} on the modulator
  - Downside: we miss potential interactions between nonidealities
  - Often they are negligible: nonidealities (like voltage coefficients) produce small errors … linear superposition applies
  - Of course it's a good idea to run a complete verification at the end
  - And we'll get to diagnose the "real thing" soon enough … without the insight gained from such idealized simulations it's next to impossible to diagnose a complex chip
- Evaluating the effect of the C_{FB} voltage coefficient requires solving a quadratic equation, see next slide …

C_{FB} Voltage Coefficient

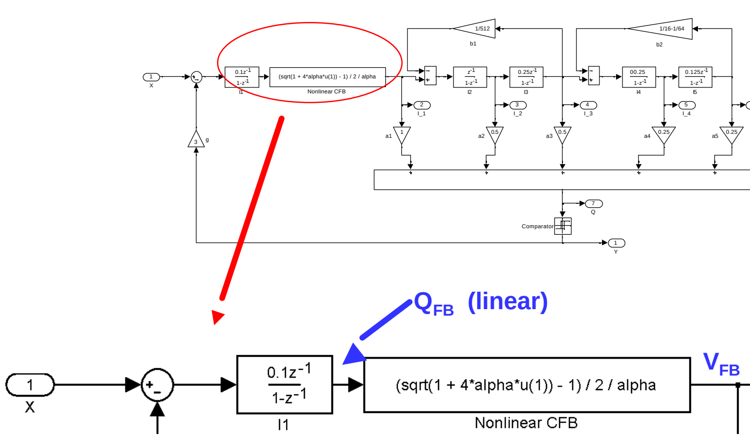
$$V_{OUT1} = \frac{Q_{FB}}{C_{FB1}(V_{OUT1})}$$

$$= \frac{-1+\sqrt{1+\frac{4aQ_{FB}}{C_{FB1}}}}{2a}$$

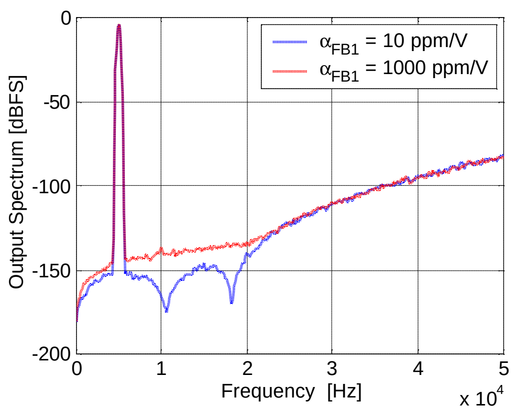
$$Q_{FB}(k) = Q_{FB}(k-1) + C_{IN}V_{IN} + DC_RV_{REF}$$

(same as output from linear integrator)

C_{FB} Voltage Coefficient

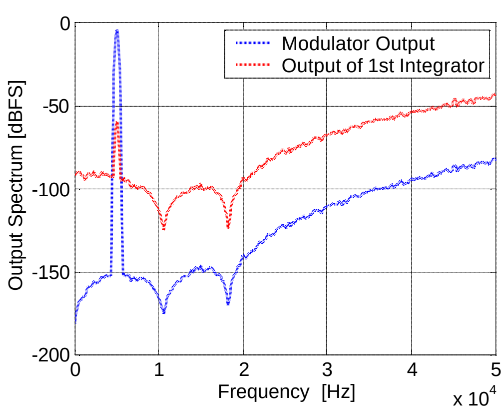


C_{FB} Voltage Coefficient



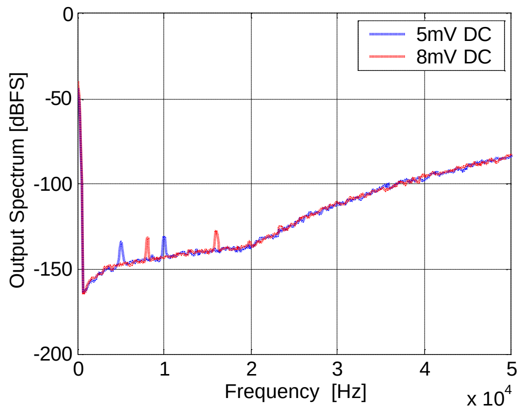
- Effect less pronounced than for C_{IN}
- Noise remains zero at DC
- First order noise for large α
- Nonlinearities operating on shaped noise change the shape of the noise …
  – No linear model can predict this
- No harmonics … why?

1^{st} Integrator Output



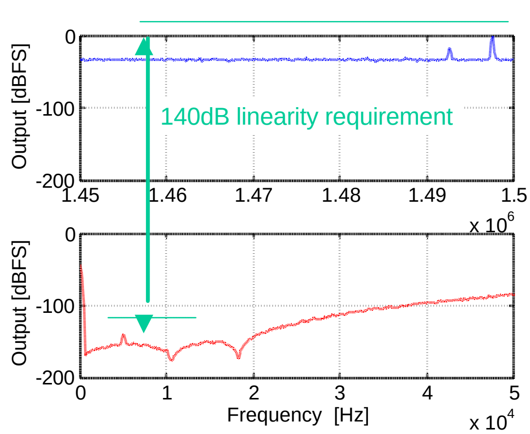
- The input signal appears much attenuated at the output of the 1^{st} Integrator (by its gain)
- This signal appears across C_{FB} … and since it contains no strong tones it produces no harmonics
- Or does it?

DC Input



- For α = 1000 ppm/V, tones produced by C_{FB} are much larger than native tones, but move with the same velocity as native tones (1kHz/mV)
- Where are these tones coming from?

C_{FB} Voltage Coefficient



- Tones appear near $f_s/2$, as expected
- Apparently these are "folded" to the base-band
- How?

α = 10 ppm/V

Quantization Noise Nonlinearity

- Native tones at a frequency f_D close to fs/2 have much higher power than in-band tones

$$f_D = \frac{f_s}{2} - \frac{f_d}{2}$$

- When this tone passes a nonlinearity in the modulator loop filter, it produces distortion

$$\sin^2(2\pi f_D t) = \frac{1}{2} - \frac{1}{2}\cos[2\pi(2 f_D)t]$$

Quantization Noise Nonlinearity

$$\sin^2(2\pi f_D t) = \frac{1}{2} - \frac{1}{2}\cos[2\pi(2 f_D)t]$$

- In the sampled data system,

$$2 f_D = f_s - f_d$$

maps to f_d

- Small nonlinearities applied to aggressively shaped quantization noise can produce big tone problems …

Out-of-Band Tones

- "In principle", the digital filter removes out-of-band tones
  - Except their distortion components, caused by nonlinearities in the modulator loop filter
  - Except components that are mixed down to DC due to noise in the DAC reference
- The C_{FB1} voltage coefficient adds only a small nonlinearity to the quantization noise path
  - Fortunately this nonlinearity is applied to the integral of the quantization noise and sees only a small signal component
  - Nonlinearities in the amplifier are much more serious
  - Including those in the model is left as an exercise …
- Maintaining extremely high levels of linearity in H(z) is the biggest transistor-level design challenge of ΣΔ modulators