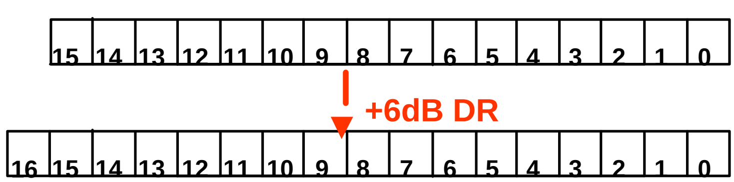
Digital Filters

- Advantages of digital filters
  - Dynamic range
  - No coefficient errors, aging
  - Programmable
  - Always work on first silicon if …
- FIR filters
  - Linear phase
  - Synthesis
- FIR / IIR comparison
- Implementation issues
  - Coefficient rounding
  - Intermediate result dynamic range
  - Limit cycles

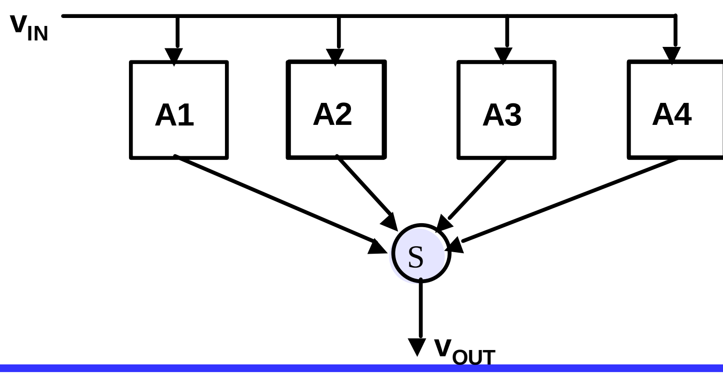
Analog versus Digital DR

- It's much less expensive to add dynamic range to digital circuits than analog circuits
- To double the dynamic range of a digital datapath, we need to add only a bit to an already-wide datapath:



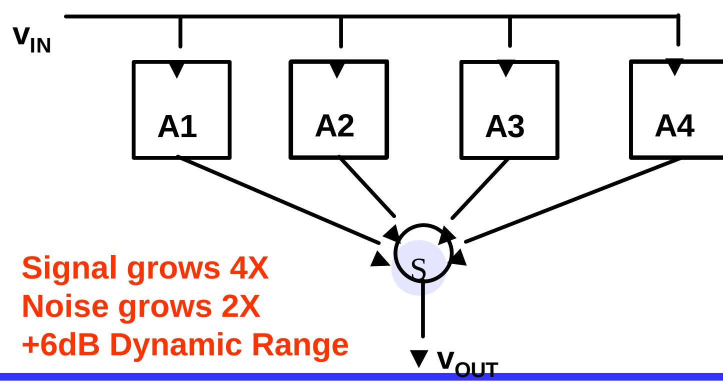
Analog versus Digital DR

For comparison, consider summing the outputs of 4 identical analog circuits with identical inputs:



Analog versus Digital DR

Analog noise is typically uncorrelated in each of the blocks A1-A4:



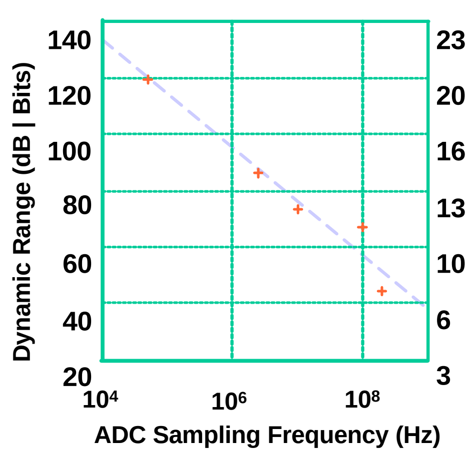
Signal grows 4X
Noise grows 2X
+6dB Dynamic Range

Analog versus Digital DR

- Doubling analog DR is very expensive:
  - 4X the power
  - 4X the area
- Doubling digital DR is relatively cheap,
  - And cost/function decreases by 29%/year (3dB/year)!
- Practical circuits tolerate very little loss of DR due to finite datapath precision in their DSP sections
  - Analog dynamic range is too precious to lose
  - Digital DR loss of 5% (~ 0.4dB) of total noise power is typical
- Why use analog filters at all?

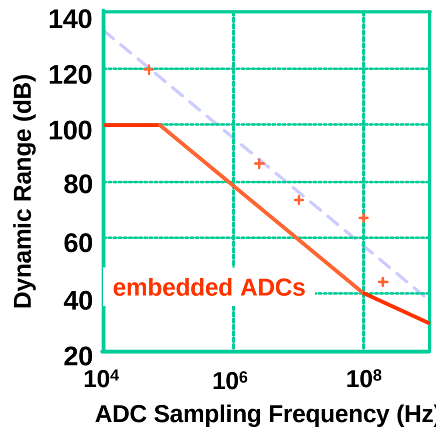
ADC Dynamic Range

- The figure shows the DR of the best standalone ADCs in 2000
- Dynamic range decreases as converter bandwidth increases
- From 1975-1995, ADC performance at any sampling frequency improved by 2dB/year



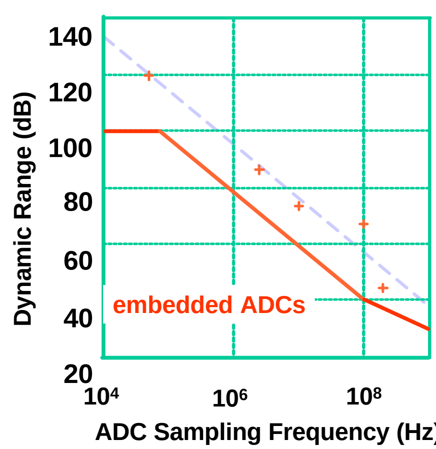
ADC Dynamic Range

- ADCs embedded in IC “Systems on a Chip” (SoCs) have less DR than the best standalone ADCs
- The embedded ADC performance level is shown in red
- Analog-digital crosstalk and design risk issues limit embedded ADC DR to about 100dB
- 1 GHz, 30dB DR levels are much more forgiving and the performance gap narrows



ADC Dynamic Range

- Minimization of analog signal processing is a key goal of mixed-signal IC architecture
- However, analog signal processing is almost unavoidable “above the red line”



Practical Constraints

- Only few ADC design teams in the world can produce "green line" dynamic range
- If your SoC architecture requires one of those teams to succeed, think again!
- Mixed-signal SoC architectures fail when their architects choose to ignore long-established, empirically-proven performance scaling laws

FIR Filters

- Only finite zeros
- Linear phase if coefficients are symmetric
- Implement with delays, multipliers, adders
- Lack of good analog delays prevents widespread use of analog FIR filters
- Good synthesis tools (e.g. Remez-Exchange algorithm)

FIR Filter Phase Response

- Consider the Nth-order FIR filter with transfer function:

$$H(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{N-2} z^{2-N} + a_{N-1} z^{1-N} + a_N z^{-N}$$

- Suppose the filter coefficients are symmetric about the middle term, i.e.:

$$H(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_2 z^{2-N} + a_1 z^{1-N} + a_0 z^{-N}$$

FIR Filter Phase Response

$$H(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_2 z^{2-N} + a_1 z^{1-N} + a_0 z^{-N}$$

$$= a_0 (1+z^{-N}) + a_1 (z^{-1}+z^{1-N}) + a_2 (z^{-2} + z^{2-N}) + \ldots$$

$$= a_0 z^{-N/2}(z^{N/2}+z^{-N/2}) + a_1 z^{-N/2} (z^{-1+N/2}+z^{1-N/2}) +$$

$$+ a_2 z^{-N/2} (z^{-2+N/2} + z^{2-N/2}) + \ldots$$

$$= z^{-N/2}[\, a_0(z^{N/2}+z^{-N/2}) + a_1(z^{-1+N/2}+z^{1-N/2}) +$$

$$a_2(z^{-2+N/2} + z^{2-N/2}) + \ldots]$$

FIR Filter Phase Response

- The term in brackets [] is a sum of cosine terms with no phase shift:

$$H(e^{jwT}) = e^{-jwNT/2}\ [\ 2a_0\cos(wNT/2)\ +\ \text{more real cos terms}]$$

- The constant group delay of the symmetric coefficient FIR filter is obvious:

$$q(w) = -\ wNT/2 \qquad t_{GR} = NT/2$$

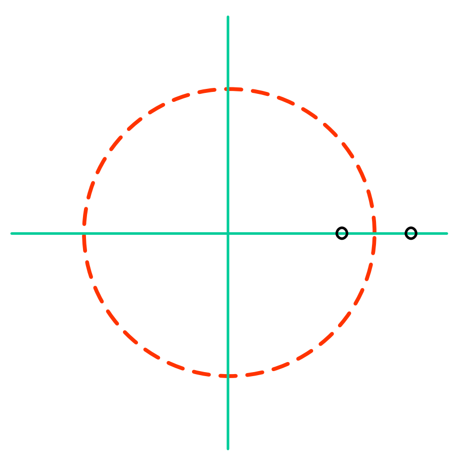
half the filter impulse response duration

Coefficient Symmetry

- Three classes of zero groupings produce symmetric coefficients and linear phase

- The first is real axis zeroes at r and 1/r:

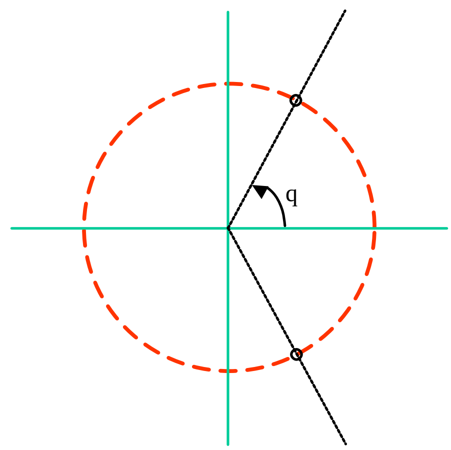
$$H(z) = z^{-2}-(r+1/r)z^{-1}+1$$



Coefficient Symmetry

- Conjugate pairs of unit circle zeroes provide linear phase:

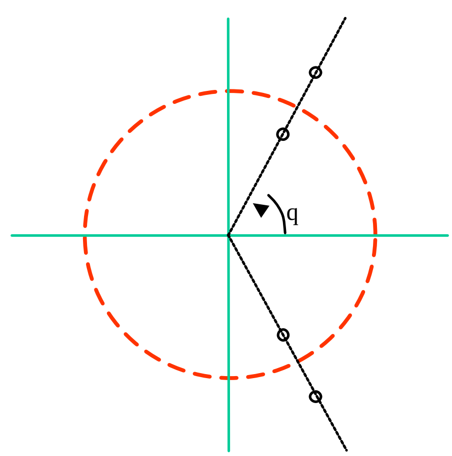
$$H(z) = z^{-2} - 2z^{-1}\cos q + 1$$



Coefficient Symmetry

- Finally, groups of four zeroes at $re^{\pm j\theta}$ and $(1/r)e^{\pm j\theta}$ provide linear phase
- The filter coefficients for these 4 zeroes are:

$$1$$
$$-2(r+1/r)\cos q$$
$$4+r^2+1/r^2$$
$$-2(r+1/r)\cos q$$
$$1$$



FIR Filter Phase Response

- Another interesting case involves antisymmetric filter coefficients:

$$H(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots - a_2 z^{2-N} - a_1 z^{1-N} - a_0 z^{-N}$$

- It's easy to show that

$$H(e^{j\omega T}) = e^{-j\omega NT/2} e^{j\pi/2} \left[2a_0 \sin(\omega NT/2) + \text{more sin terms} \right]$$

FIR Filter Phase Response

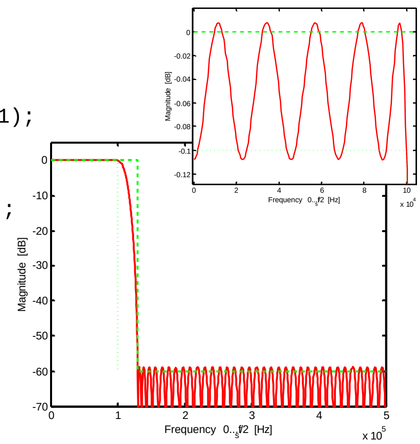
- For the antisymmetric coefficient case

$$\theta(\omega) = \frac{\pi}{2} - \omega NT/2 \qquad \tau_{GR} = NT/2$$

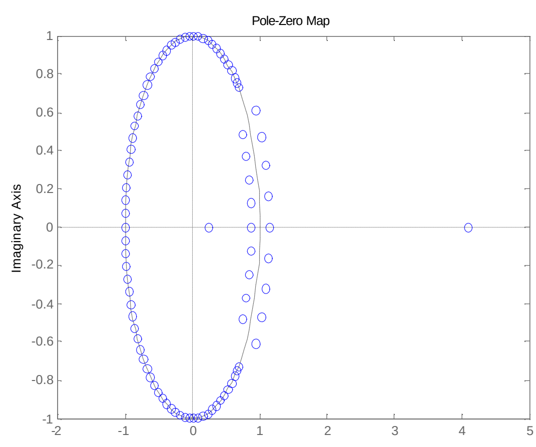
- It's still linear phase, but with the frequency independent 90° phase shift characteristic of differentiators

Linear Phase FIR Example

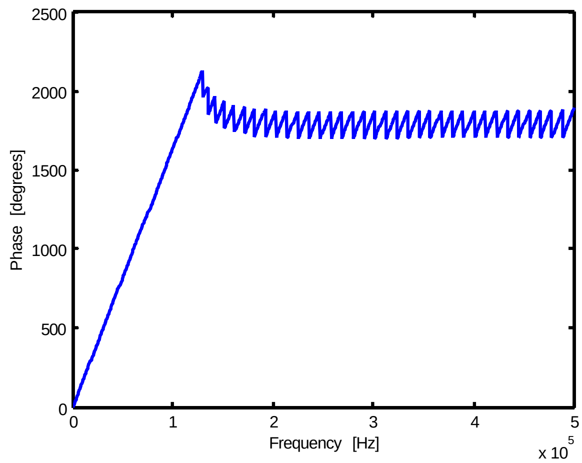
```
fs = 1e6;
Fp = 0.10*fs;   Fs = 0.13*fs;
Rp = 0.1;       Rs = 60;
x = (10^(Rp/20)1)/(10^(Rp/20)+1);
y = 10^(-Rs/20);
[N,fo,ao,W]=remezord( …
        [Fp Fs],[1 0],[x y],fs);
b = remez(N, fo, ao, W);
Hr = tf(b, 1, 1/fs);
Hr = Hr / 10^(rpass/40);
```



z-Plane



Phase Response



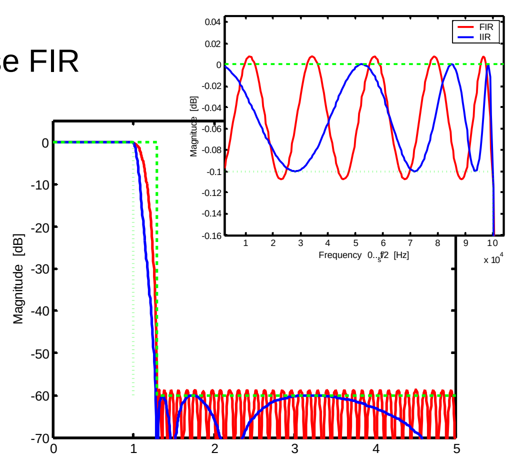
Linear?

FIR / IIR Comparison

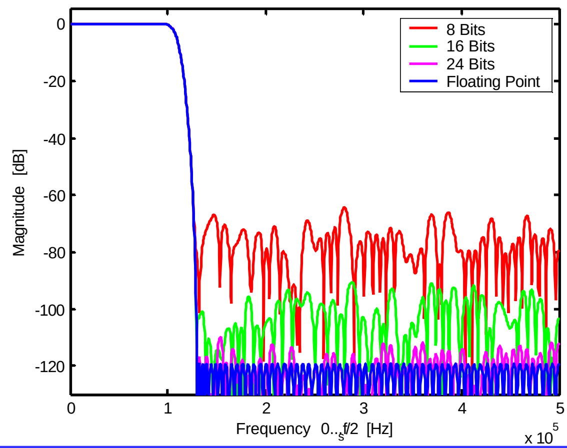
91st order linear phase FIR

or

7th order elliptic IIR



FIR Coefficient Rounding



FIR Coefficient Precision

- Finite precision FIR filters add transfer functions of two filters
  - The infinite precision FIR filter
  - A rounding error FIR filter
- The infinite precision FIR dominates the passband response
- The rounding error FIR filter sets stopband attenuation when the infinite precision FIR response is much smaller

FIR Coefficient Precision

- Random rounding errors transform to white “stopband noise”
  - Stopband attenuation increases by about 6dB for each bit of coefficient precision
- If you don’t like the highest bump in the stopband response, generate a new pattern of rounding error
  - Use slightly different dc gain
  - Or slightly different Parks-McClellan (remez) bands
- Trial and error can improve filter stopbands by several dB at a given coefficient precision

Filter Dynamic Range

- Digital filters need more numeric dynamic range than the signals they process
  - They must not overload
  - They must not surprise you with quantization noise
- Digital multiplier/accumulators are multiplexed
  - Difference equations are added up term-by-term, giving us “intermediate transfer functions” to worry about
  - Intermediate overload is as bad as overload
- Let’s look at an IIR bandstop filter example…

2^{nd}-Order Bandstop Filter

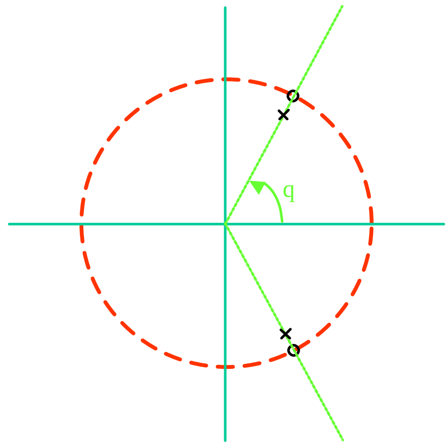
- Bandstop filters have transfer functions:

$$H(z)=\frac{z^{-2}-z^{-1}(2\cos\Theta)+1}{r^2z^{-2}-z^{-1}(2r\cos\Theta)+1}$$

$$\Theta\approx\frac{2pf_P}{f_s}$$

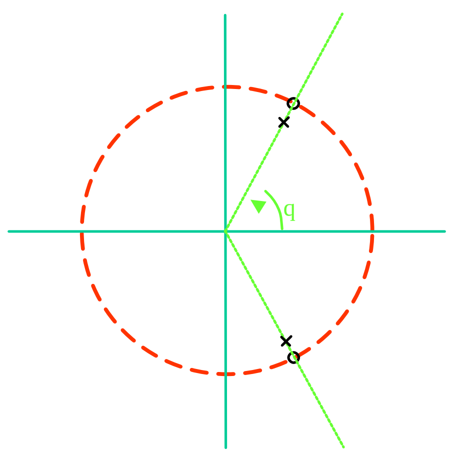
$$r\approx 1-\frac{pf_P}{Qf_s}$$

- Their gains are close to unity at both dc and $f_s/2$



2^{nd}-Order Bandstop Filter

- Bandstop design specifications:
  - f_S=1MHz
  - f_P=20kHz
  - Q_P=100



Direct Form Realization

$$\frac{Y(z)}{X(z)} = \frac{z^{-2} - z^{-1}(2\cos\Theta) + 1}{r^2 z^{-2} - z^{-1}(2r\cos\Theta) + 1}$$

$$Y(z)\left[r^2 z^{-2} - z^{-1}(2r\cos\Theta) + 1\right] = X(z)\left[z^{-2} - z^{-1}(2\cos\Theta) + 1\right]$$

$$y_{k-2} r^2 z^{-2} - y_{k-1}(2r\cos\Theta) + y_k = x_{k-2} - x_{k-1}(2\cos\Theta) + x_k$$

Note: Direct form realizations are not ideal for higher order IIR filters. Lattice filters (and variants), which simulate LC ladders, are less susceptible to finite coefficient precision and dynamic range.

2nd-Order Bandstop Filter

- We can build a direct form bandstop this way (method #1):

$$y_k = \underbrace{\underbrace{\underbrace{x_k - x_{k-1}(2\cos\Theta)}_{\text{intermediate result 1}} + x_{k-2}}_{\text{intermediate result 2}} + y_{k-1}(2r\cos\Theta)}_{\text{intermediate result 3}} - y_{k-2} r^2 z^{-2}$$

- Or this way (method #2):

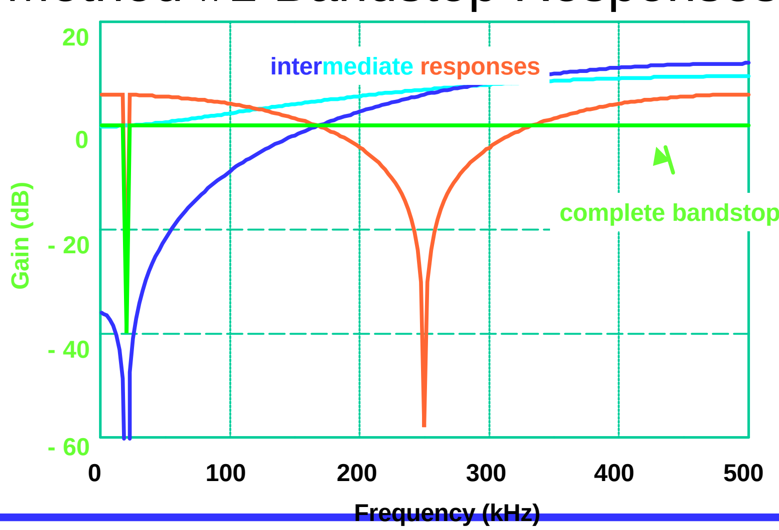
$$y_k = y_{k-1}(2r\cos\Theta) - y_{k-2} r^2 z^{-2} + x_k - x_{k-1}(2\cos\Theta) + x_{k-2}$$

- The order of addition matters if we overload in the middle!

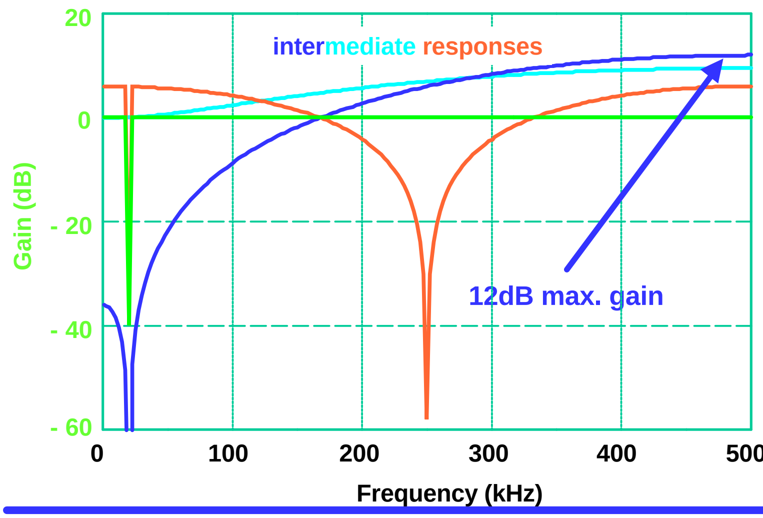
2nd-Order Bandstop Filter

- Proceeding from left to right, the difference equation generates 3 intermediate transfer functions plus the complete bandstop transfer function
- All 4 of these transfer function magnitude responses for method 1 are shown on the next slide

Method #1 Bandstop Responses



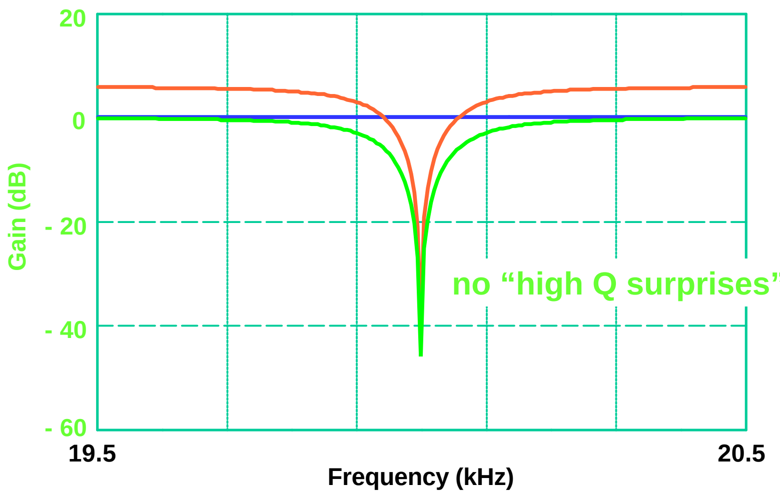
Method #1 Bandstop Responses



Method #1 Bandstop Responses

- The complete bandstop filter never exceeds unity gain for sinusoidal inputs
- Intermediate gains exceed 12dB
  - That's 2-bits above the input MSB
- Let's examine the area of the notch in more detail …

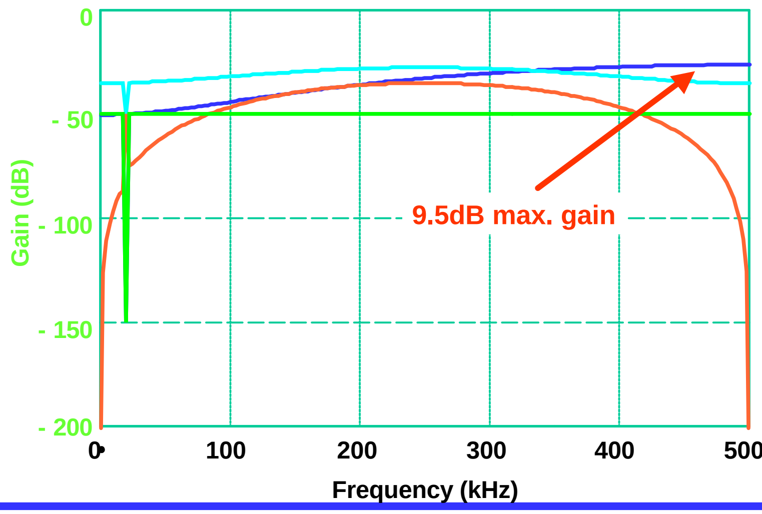
Method #1 Bandstop Responses



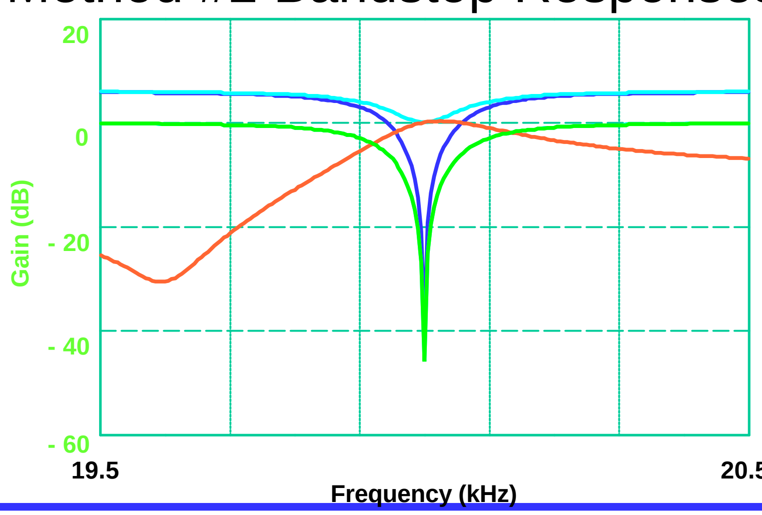
Method #2 Bandstop Responses

- Next, we’ll examine all the intermediate transfer functions for the method #2 difference equation
- The following figures show significantly different intermediate frequency responses and somewhat lower maximum intermediate gains

Method #2 Bandstop Responses



Method #2 Bandstop Responses



Avoiding Overload

- Sinusoidal steady-state responses for intermediate results are easy to compute and provide useful insight, but sinusoidal inputs are “never” worst cases for overload
- Absolute values of filter impulse response coefficients can give worst-case conditions for overload and intermediate overload

Digital Filter Models

- The order of arithmetic operations in digital signal processing matters
- **Digital filter models must be “cycle true”**
- Unanticipated filter overloads are inexcusable design errors
  – Real-world chip developments must never be late-to-market because of such easy-to-avoid errors!

Biquad Quantization Noise

- Suppose we build our bandstop difference equation with B-bit registers and a BxB = 2B hardware multiplier:

$$y_k = Gx_k - Gx_{k-1}(2\cos\Theta) + Gx_{k-2} + y_{k-1}(2r\cos\Theta) - y_{k-2}r^2z^{-2}$$

- Build up the difference equation leaving partial results in a 2B-bit accumulator
  - Accumulate y(k)'s with a minimum number (i.e. 1) of rounding operations
  - If each of the 5 products above is rounded to B-bits, you'll have 5X more quantization noise power
- Output noise from rounding operations can be large for high Q digital biquads

Digital Filter Models

- Datapath rounding operations can degrade digital filter dynamic ranges by surprisingly large amounts
- **Digital filter models must be "bit true"**
- Bit true and cycle true models require that filter models (and modelers) provide exact test vectors for integrated digital filters

Limit Cycles

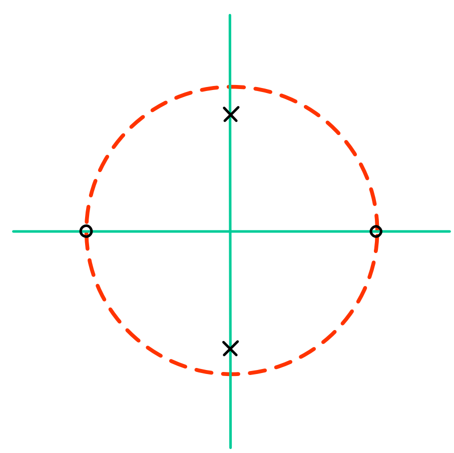
- A disadvantage of digital IIR filters relative to digital FIR filters is that their responses get strange as they settle in response to transients
- As settling error approaches rounding error, offsets and oscillations can occur
  - Non-zero offsets lead to “dead zones”
  - Oscillations are called “limit cycles”
- A combination of rounding (or truncation) and feedback is required for limit cycles

Limit Cycles

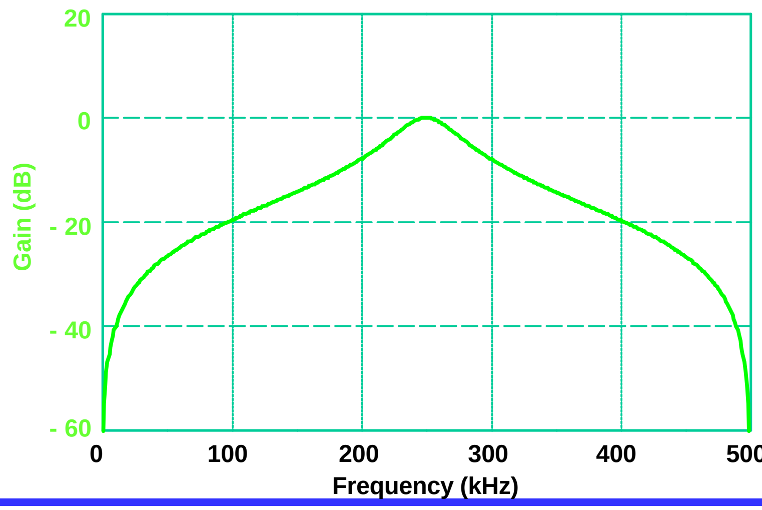
- We’ll look for limit cycles in the bandpass filter:

$$H(z) = \frac{0.125\,(z^{-2}-1)}{z^{-2} + 0.75}$$

- Note that this filter passes frequencies near $f_S/4$



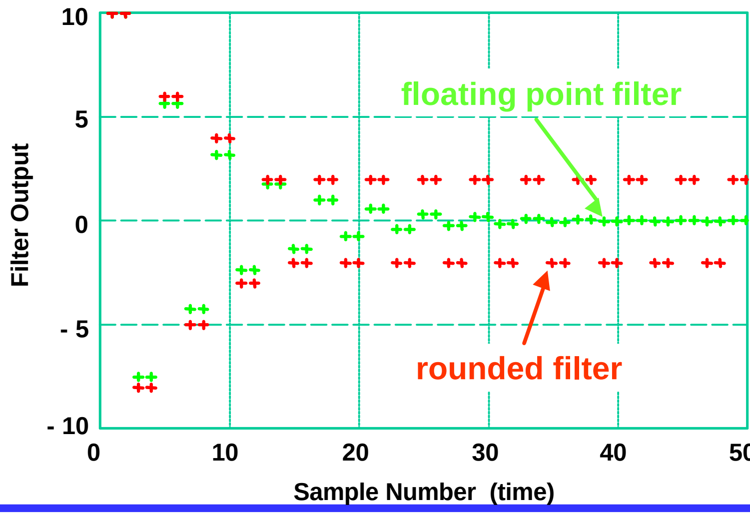
Bandpass Magnitude Response



Bandpass Transient Response

- Let's examine the bandpass filter's response to the initial condition y(1)=y(2)=10
- The bandpass filter output <u>should</u> decay to 0
  – The floating point filter output does
  – The fixed point filter output doesn't
  – Let's take a look…

Bandpass Transient Response



Limit Cycles

- This bandpass filter limit cycle oscillation occurs right at $f_s/4$
  - Right in the middle of the filter passband
  - Could this be a low-level input to the filter at $f_s/4$?
- IIR filter designers must evaluate and be wary of limit cycle oscillations

Digital Filter Models

- Bit true and cycle true digital filter models allow simulation and evaluation of:
  - Overload and intermediate overload
  - Quantization noise
  - Limit cycles and dead zones
  - Finite precision coefficient effects

- Spending time and money on silicon without such models is crazy!