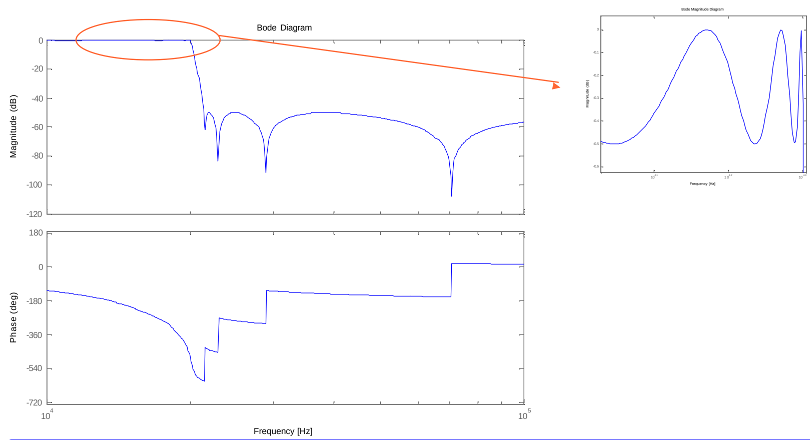
Higher Order Filter Options

- Cascade of Biquads
  - High-Q poles
  - High component sensitivity
- Ladders
  - Low sensitivity (Orchard)
  - Synthesize from LC prototypes
- Digital filters
  - Preferred solution when possible

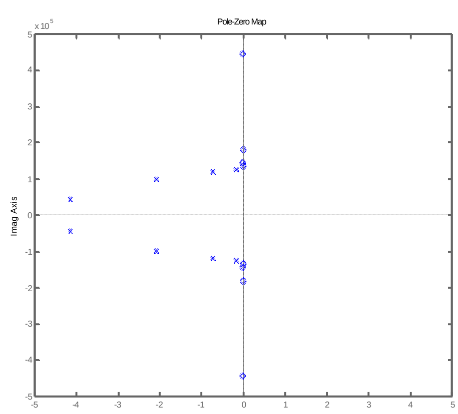
Cascade of Biquads

- LPF with
  f_{pass} = 20 kHz
  f_{stop} = 22.05 kHz
  r_{pass} = 0.5 dB
  r_{stop} = 50 dB
- 8^{th} order Elliptic Filter
- Implementation with Biquads
  Goal: maximize dynamic range
  - Pair poles and zeros
    highest Q poles with closest zeros is a good starting point, but not necessarily optimum
  - Ordering
    lowest Q poles is a good start

Filter Response



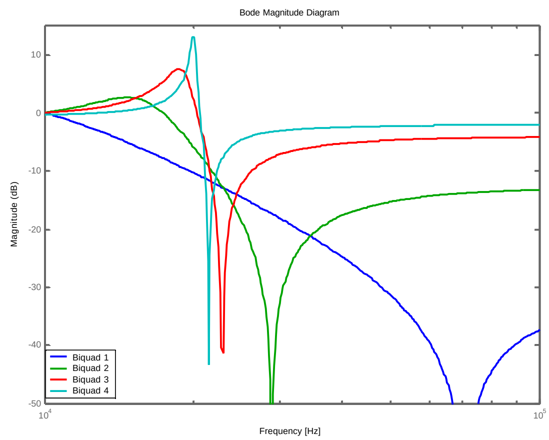
Pole-Zero Map



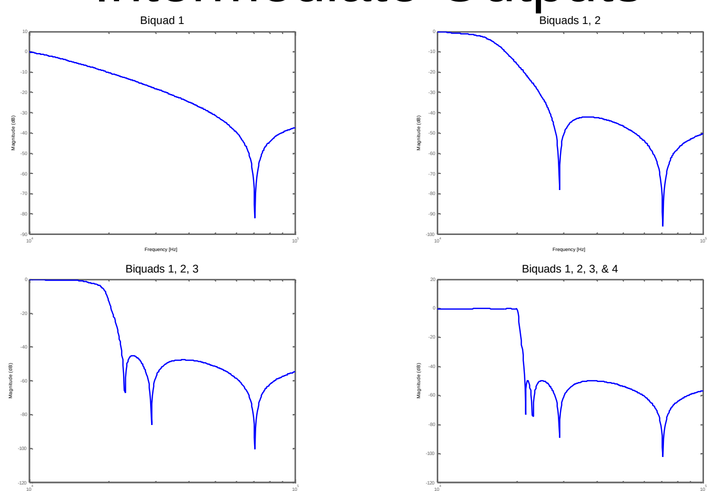
Q_{pole}	f_{pole} [kHz]
38.4389	20.0501
8.2903	19.0959
2.4134	16.0142
0.7130	9.4282

f_{zero} [kHz]
70.6923
28.7992
22.8585
21.4663

Biquad Response



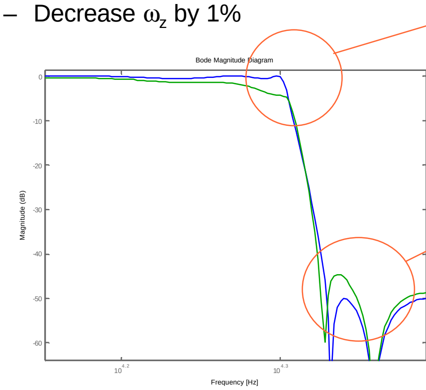
Intermediate Outputs

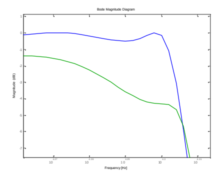


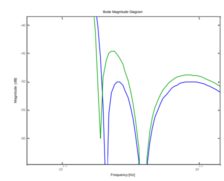
Sensitivity

Component variation in Biquad 4:

- Increase ω_p by 1%
- Decrease ω_z by 1%







High Q poles → High sensitivity in Biquad realizations

Ladder Filters

- Ladder example
  - Table
  - De-normalization
  - State variable synthesis
  - Transmission zeros
- Implementation
  - Tuning
  - Gm-C

LC Ladder Synthesis

- CAD tool
- Filter table
  - A. Zwerev, *Handbook of filter synthesis*, Wiley, 1967.
  - R. Saal, *Handbook of filter synthesis*, AEG-Telefunken, 1979.
  - A. B. Williams and F. J. Taylor, *Electronic filter design*, 3rd edition, McGraw-Hill, 1995.
- Example:
  - f_{corner} = 10MHz, f_{stop} = 20MHz, R_p = 2dB, R_s=25dB
  - 5th order Butterworth (from Matlab)

Filter Table

NORMALIZED FILTER DESIGN TABLES 11.3

TABLE 11-2 Butterworth LC Element Values (Continued)

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.2073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.6845		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.1103		
	Inf	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.2347	
	1.2500	0.3445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2972	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5159	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6598	0.3788	5.4025	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	Inf	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.8817
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	Inf	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

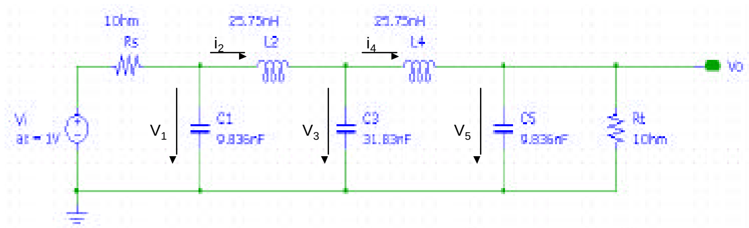
Williams and Taylor, p. 11.3

Denormalization:

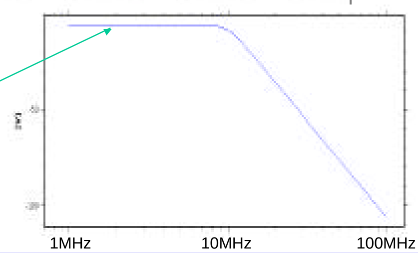
Multiply all L, C with

- $L_r = R/\omega_{corner}$ = 14.1 nH
- $C_r = 1/R/\ \omega_{corner}$ = 14.1 nF
- R is the value of the source and termination resistor (choose both 1Ω for now)

SPICE Verification



-6 dB passband attenuation
due to termination



State Space Description

$$V_1 = \frac{1}{sC_1}\left[\frac{V_i - V_1}{R_s} - i_2\right]$$

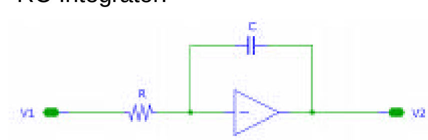
$$i_2 = \frac{1}{sL_2}\left[V_1 - V_3\right]$$

$$V_3 = \frac{1}{sC_3}\left[i_2 - i_4\right]$$

$$i_4 = \frac{1}{sL_4}\left[V_3 - V_5\right]$$

$$V_5 = \frac{1}{sC_5}\left[i_4 - \frac{V_5}{R_t}\right]$$

RC Integrator:



$$\frac{V_2}{V_1} = -\frac{1}{sRC}$$

Normalize

$$V_1 = \frac{1}{sC_1}\left[\frac{V_i - V_1}{R_s} - i_2\right]$$

$$i_2 = \frac{1}{sL_2}[V_1 - V_3]$$

$$V_3 = \frac{1}{sC_3}[i_2 - i_4]$$

$$i_4 = \frac{1}{sL_4}[V_3 - V_5]$$

$$V_5 = \frac{1}{sC_5}\left[i_4 - \frac{V_5}{R_t}\right]$$

$$\longrightarrow \boxed{\begin{aligned} V_2 &= i_2 R^* \\ V_4 &= i_4 R^* \end{aligned}} \longrightarrow$$

$$V_1 = \frac{1}{sC_1}\left[\frac{V_i - V_1}{R_s} - \frac{V_2}{R^*}\right]$$

$$V_2 = \frac{R^*}{sL_2}[V_1 - V_3] = \frac{1}{sC_2 R^*}[V_1 - V_3]$$

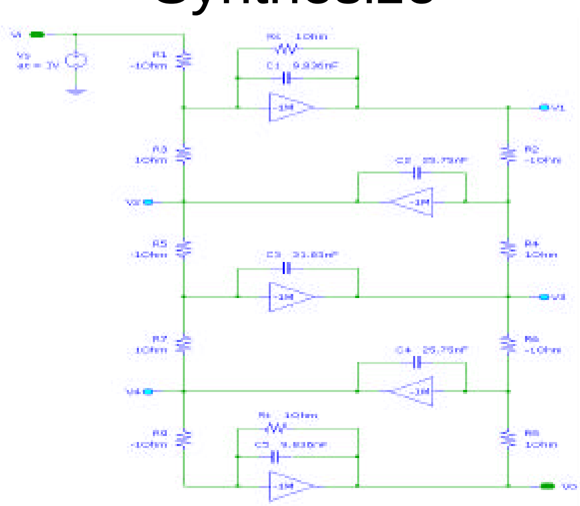
$$V_3 = \frac{1}{sC_3 R^*}[V_2 - V_4]$$

$$V_4 = \frac{R^*}{sL_4}[V_3 - V_5] = \frac{1}{sC_4 R^*}[V_3 - V_5]$$

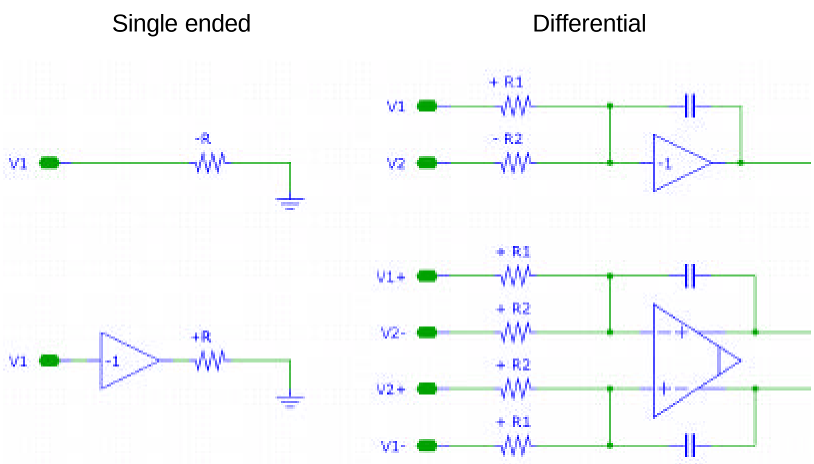
$$V_5 = \frac{1}{sC_5}\left[\frac{V_4}{R^*} - \frac{V_5}{R_t}\right]$$

with $C_2 = \frac{L_2}{(R^*)^2}$ and $C_4 = \frac{L_4}{(R^*)^2}$

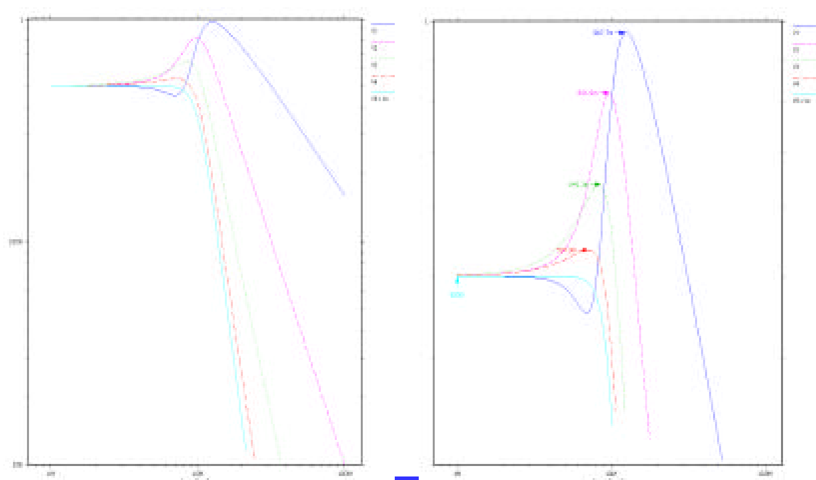
Synthesize



Negative Resistors

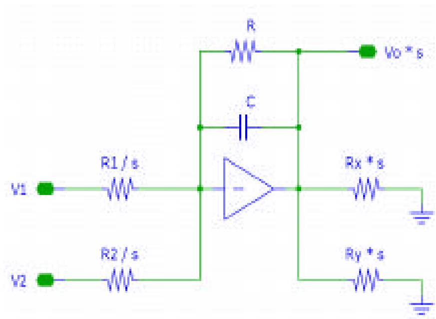


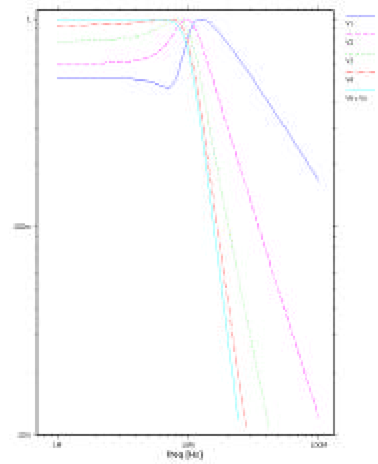
Frequency Response



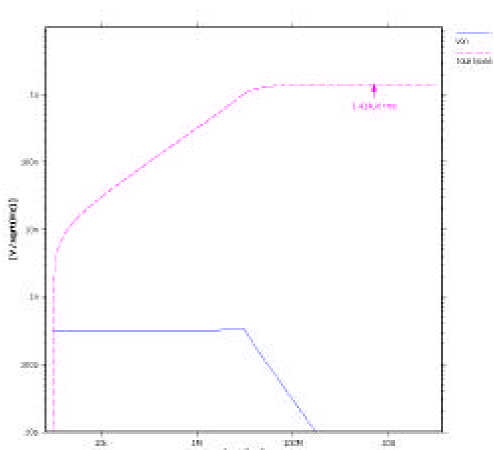
Scale Node Voltages

Scale V_o by factor “s”





Noise



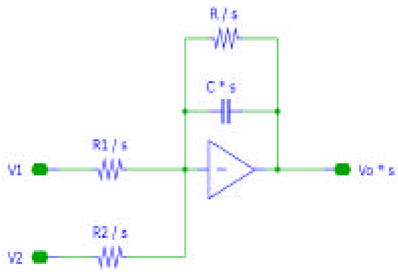
Total noise: 1.4 μV rms (noiseless opamps)

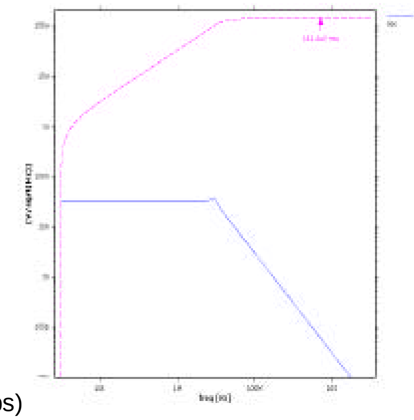
That’s excellent, but the capacitors are very large (and the resistors small).

Suppose our application calls for 140 μV rms …

Scale to Meet Noise Target

Scale capacitors and resistors
to meet noise objective

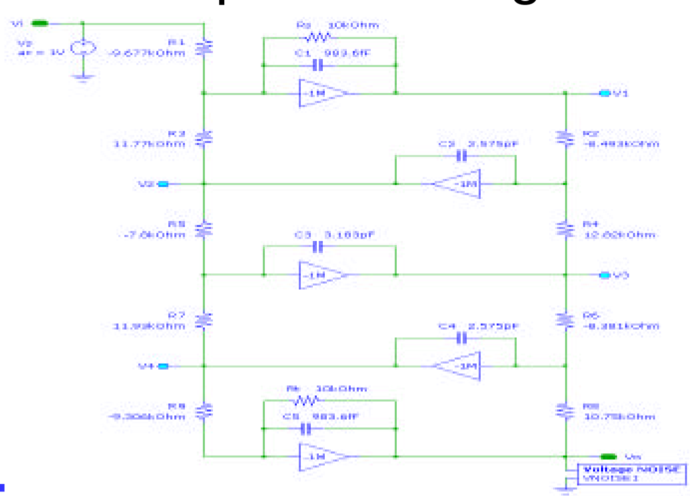




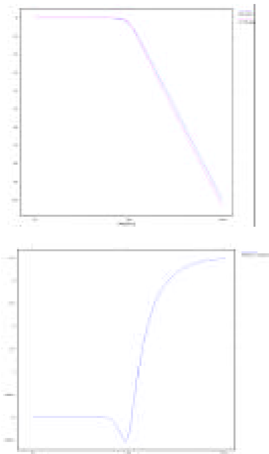
s = 10^{-4}

Noise: 141 μV rms (noiseless opamps)

Completed Design

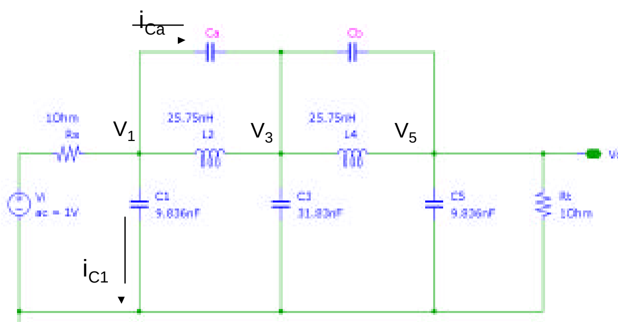


Sensitivity



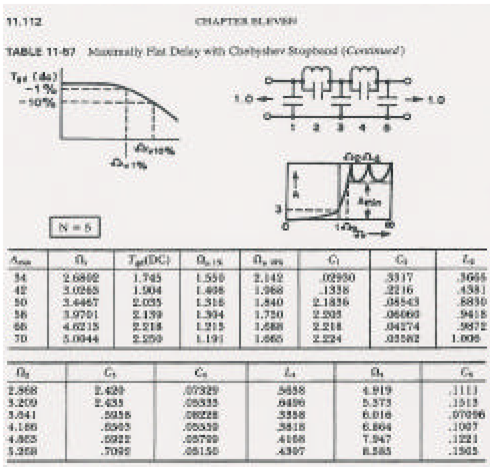
- C_1 made (arbitrarily) 50% (!) larger than its nominal value
- 0.5 dB error at band edge
- 3.5 dB error in stopband
- Looks like very low sensitivity
- More analysis needed (Monte Carlo?)

Transmission Zeros



$$i_a = sC_a(V_1 - V_3)$$
$$i_{C1} = sC_1V_1$$
$$\rightarrow i_a + i_{C1} = s(C_a + C_1)\left[V_1 - V_3\frac{C_a}{C_a + C_1}\right]$$

Filter Table



11.112 CHAPTER ELEVEN

TABLE 11-57 Maximally Flat Delay with Chebyshev Stopband (*Continued*)

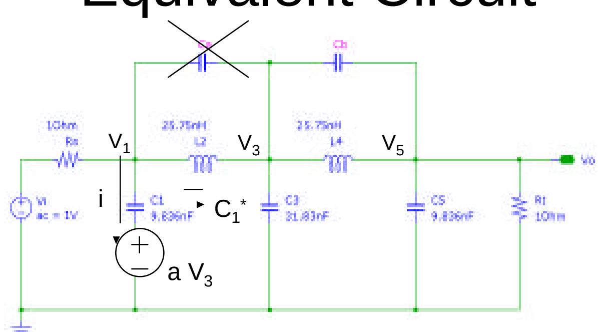
N = 5

A_{min}	Ω_s	T_{gd}(DC)	$\Omega_{-1\%}$	$\Omega_{-10\%}$	C_1	C_2	L_2
34	2.6802	1.745	1.550	2.142	.02950	.3317	.3665
42	3.0265	1.904	1.408	1.968	.1338	.2216	.4381
50	3.4467	2.035	1.316	1.840	2.1836	.08543	.6830
58	3.9701	2.139	1.304	1.730	2.205	.06060	.9418
66	4.6215	2.218	1.215	1.688	2.218	.04274	.9872
70	5.0044	2.250	1.191	1.665	2.224	.03562	1.006

Ω_2	C_3	C_4	L_4	Ω_4	C_5
2.868	2.420	.07329	.5658	4.919	.1111
3.209	2.431	.05333	.6496	5.373	.1513
3.641	.5938	.08228	.3358	6.016	.07096
4.186	.5505	.05539	.3818	6.864	.1007
4.863	.5922	.05799	.4168	7.947	.1221
5.268	.7090	.05150	.4307	8.585	.1505

- 5^{th} order Chebychev II
- Williams & Taylor, p. 11.112
- 50dB stopband attenuation

Equivalent Circuit



$$i = i_a + i_{C1} = s(C_a + C_1)\left[V_1 - V_3 \frac{C_a}{C_a + C_1}\right]$$

$$= sC_1^*(V_1 - aV_3)$$

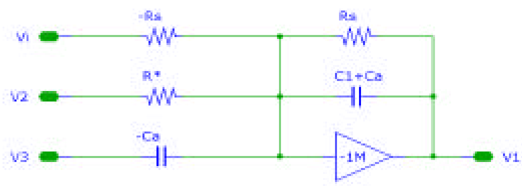
$$\text{with:} \quad C_1^* = C_a + C_1 \qquad \text{and} \qquad a = \frac{C_a}{C_a + C_1}$$

Realization with Integrator

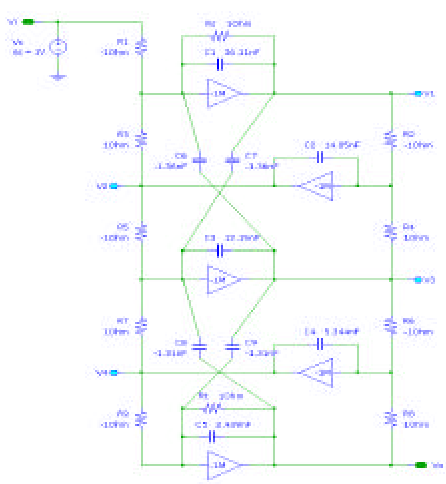
$$V_1 = \frac{1}{s(C_a + C_1)}\left[\frac{V_i - V_1}{R_s} - i_{L2}\right] + aV_3$$

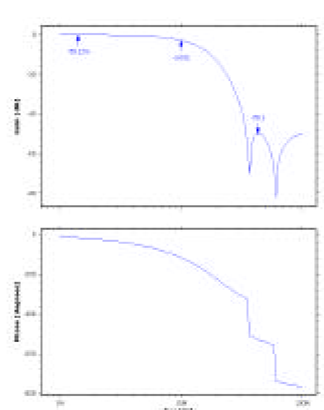
$$= \frac{1}{s(C_a + C_1)}\left[\frac{V_i - V_1}{R_s} - i_{L2}\right] + \frac{C_a}{C_a + C_1}V_3$$

$$= \frac{1}{s(C_a + C_1)}\left[\frac{V_i - V_1}{R_s} - \frac{V_2}{R^*} + sC_aV_3\right]$$



Active RC Simulation





Summary

Higher Order Filter Realization

- Cascade of Biquads
  - High sensitivity often problem for N>4
- Ladder Filters
  - Based on LC prototypes
  - Low sensitivity
  - Active RC simulation retains low sensitivity
  - Many implementation choices:
    Active RC, Gm-C, MOSFET-C