# EECS 151 Disc 12 

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## Contents

- Wallace tree
- Signed multiplication
- Multiplication by a constant
- Clocks
- Packaging


## Announcement

- HW1 1 is not as short as we expected
- However, each problem is quite simple ( $\sim 5 \mathrm{~min}$ )
- Due May 8 Monday, solution will be posted just after deadline
- Unfortunately, we don't have time to do review for the final


## 7x7 Wallace Tree (Unsigned Multiplier)



## 7x7 Wallace Tree: After 1st Stage

$$
\begin{aligned}
& \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc
\end{aligned}
$$

$$
\begin{aligned}
& \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \\
& 0000000 \\
& \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc
\end{aligned}
$$

## 7x7 Wallace Tree: Following Stages

(There were some bugs in last week's slide)


The MSB adder can be just an XOR, since the result of $7 \times 7$ does not exceed 14 bits.

## Berkeley

## Signed Multiplication

$$
\begin{aligned}
& B=\{b 3, b 2, b 1, b 0\}=b 0+(2 * b 1)+(4 * b 2)-(8 * b 3) \\
& \begin{aligned}
A * B & =A * b 0+A *(2 * b 1)+A *(4 * b 2)-A *(8 * b 3) \\
& =A * b 0+(A \ll 1) * b 1+(A \ll 2) * b 2-(A \ll 3) * b 3
\end{aligned}
\end{aligned}
$$

- One more left shift each time
- Subtract the partial product for the MSB of B
- Sign extend partial products properly ( 2 N is safe, but you can save a little by using some techniques)


## Berkeley

## Straightforward Signed Multiplier




## Baugh-Wooley's Algorithm

- Replace sign extension with negation and minus one at the MSB

○ aa...axyz = a’xyz-1000 because
■ 00...0xyz = $1 x y z-1000$
■ $11 \ldots 1 x y z=0 x y z+11 \ldots 1000=0 x y z-1000$

- Also apply 2's complement to the last partial product
- -aaxyz = a'a'x'y'z' + $1=a x{ }^{\prime} y^{\prime} z^{\prime}-1000+1$
- Precompute constants

○ $-1000-10000-100000-1000000+1000=-1110000=+10010000$

## Berkeley

## Baugh-Wooley's Signed Multiplier



## Multiplication by a Constant

- Multiplication by a constant is just shift-and-add
- $75 \mathrm{X}=1001011 \mathrm{X}=(\mathrm{X} \ll 6)+(\mathrm{X} \ll 3)+(\mathrm{X} \ll 1)+\mathrm{X}$
- CSD representation utilizes subtraction

○ 01...11 to 10...01' (1' here means minus one)

- 11 ' to 01 , and 1 ' 1 to $01^{\prime}$

■ $75=1001011=1001101^{\prime}=10101^{\prime} 01^{\prime}$

- $75 \mathrm{X}=(\mathrm{X} \ll 6)+(\mathrm{X} \ll 4)-(X \ll 2)-X$
- Doesn't help in this case


## Constant Coefficient Multiplication

- Break down the constant and cascade shift-and-add
- $75=3 * 5 * 5$
- $(3 * 25): 3$ is $(2+1)$ and 25 is $(16+8+1)$-> 3 adders
$\bigcirc$ (5*15): 5 is $(4+1)$ and 15 is (16-1) -> 2 adders (one used as a subtractor)
- $Y=(X \ll 2)+X$
- $Z=(Y \ll 4)-Y$
- Then, $Z=75 \mathrm{X}$


## Clock Skew

- Clock skew ... adjust delay of clock signal to reduce max delay


$$
(50-\mathrm{a}) \mathrm{ps}
$$

$(10+a) p s$
Each stage has 30 ps delay when $\mathrm{a}=20$.

## Hold Time Constraint

Near the end of cycle:


If R 1 is triggered first:


And if delay of logic is too small:


R2 cannot fetch 0 correctly:


## Berkeley

## Clock Constraints

- Setup time constraint
- T > clock_to_q + setup + max_delay + 2*jitter
- Hold time constraint
- clock_to_q + min_delay > hold + 2*jitter


## Packaging

- C4 (solder balls) works better (smaller R) than bond wires
- Still, need to consider inductive effects
- V_diff $=\mathrm{L}$ * dl/dt should be less than 0.1 V
- Decoupling capacitors makes current spike smoother (low pass)
- ESD protection using diodes

