EECS151/251A Discussion 7

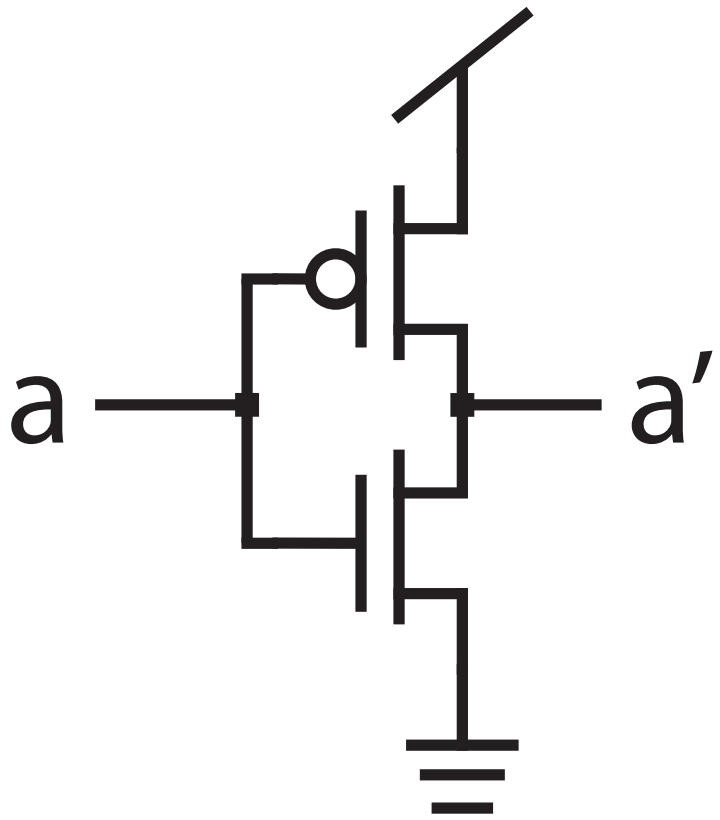
Christopher Yarp

Mar. 8, 2019

Plan for Today

- Deriving Inverter Characteristics
- Practice Problem
- Your Questions

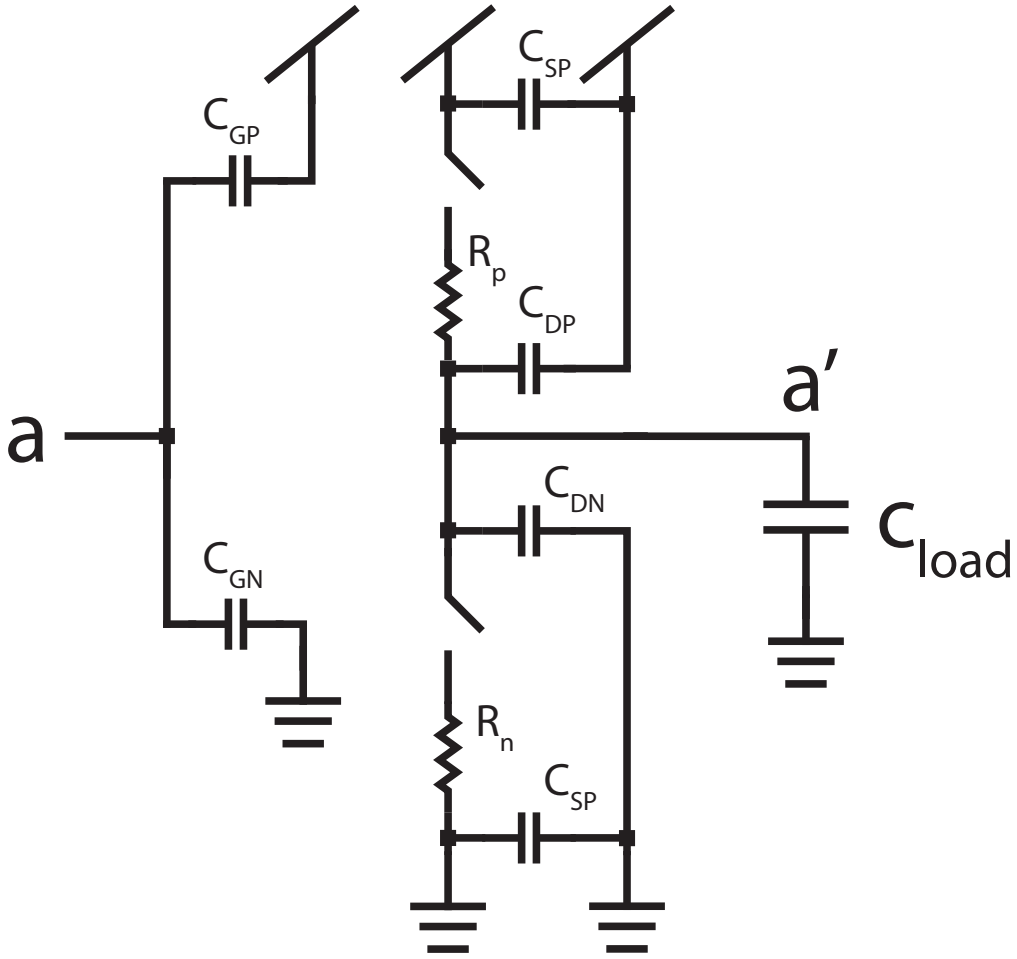
Inverter Characteristics

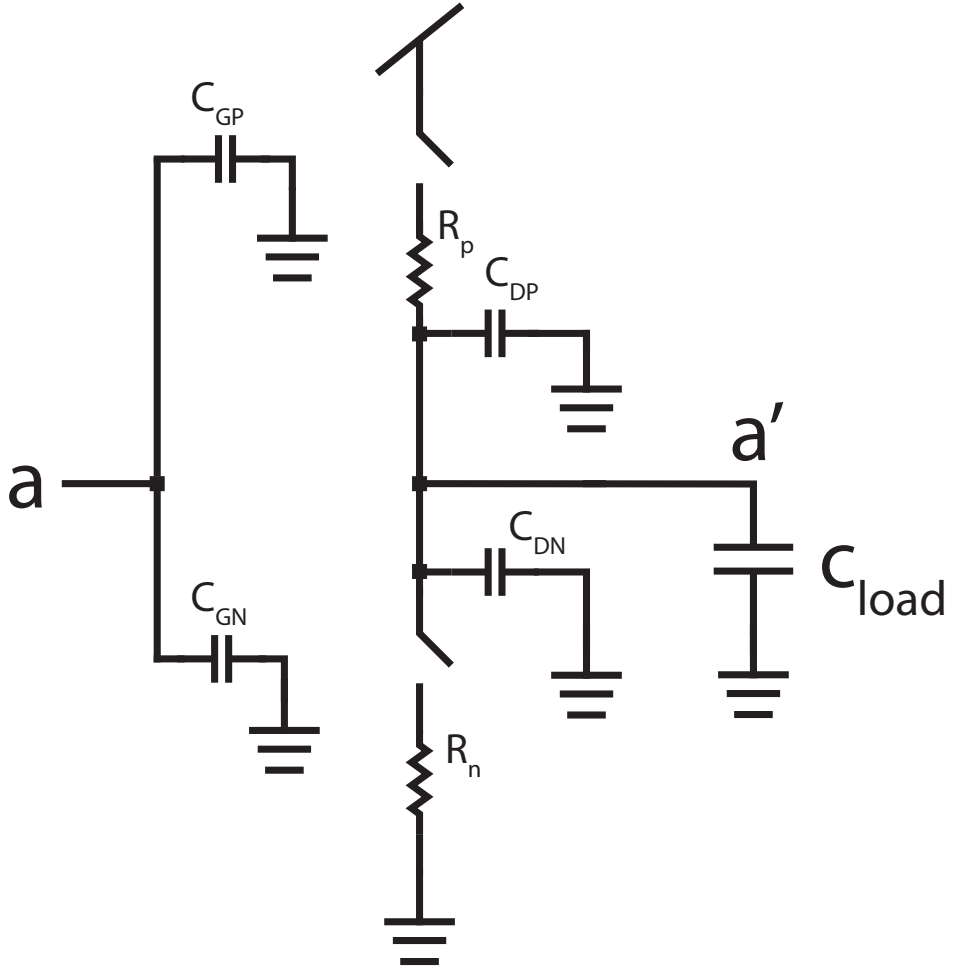


- For this problem we will assume
  - $R_P = 2R_N$ for equal width transistors
  - Let R_{N0} be the resistance of a unit size N-FET
  - $C_{GP} = C_{GN} = C_G$ for equal width capacitors
  - Let C_{G0} be the gate capacitance of a unit size FET
  - Scaling transistor width by a factor W scales the following parameters
    - $R = \frac{R_{orig}}{W}$
    - $C = WC_{orig}$
  - We will size the width of the N-FET by W and the width of the P-FET by 2W
- We will characterize the following aspects of the inverter
  - Internal Capacitance
  - Input Capacitance
  - Gate delay as a function of fanout

Inverter Parasitics

- The source capacitances are shorted to the voltage ranges and can be ignored
- An equivalent model can be made by tying all capacitors connected to static voltage rails to ground (see Weste & Harris pg 144)





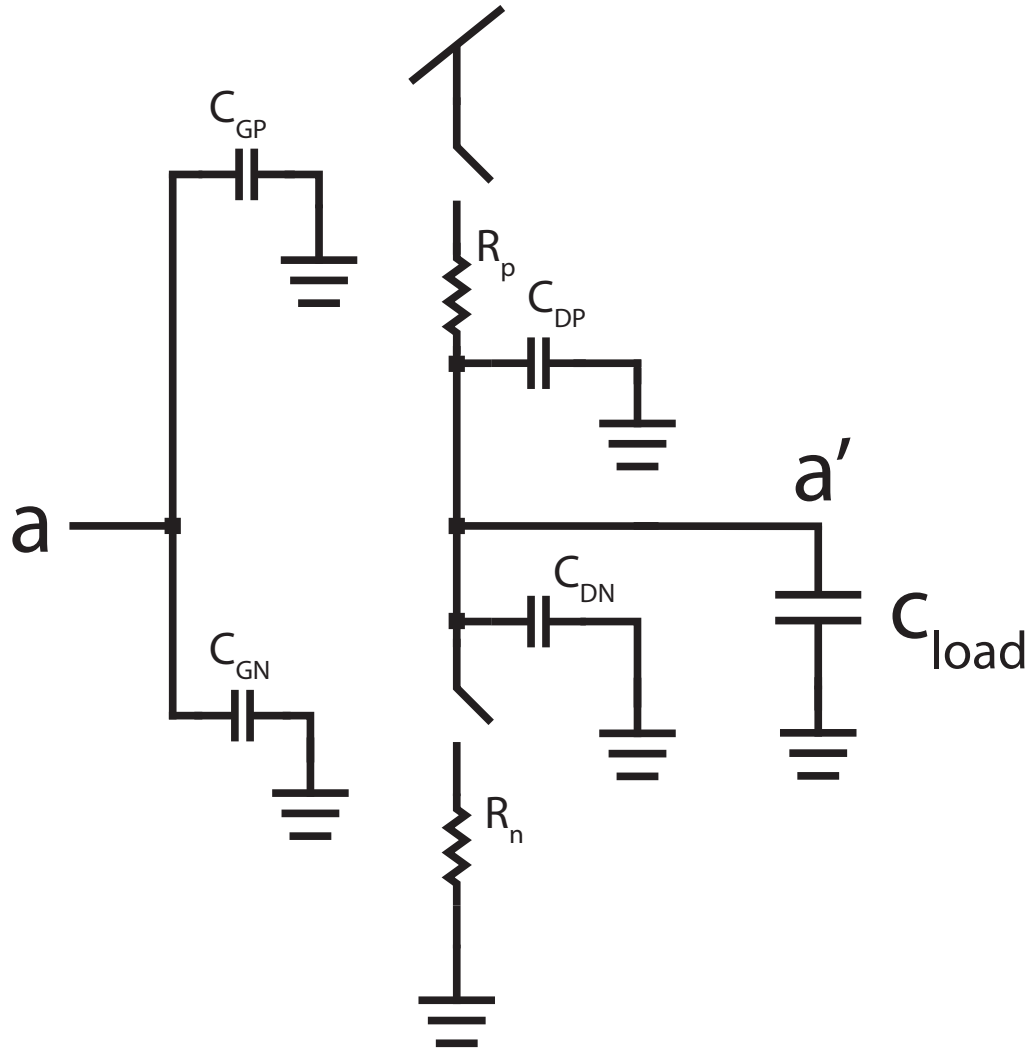
Input Capacitance

- Width N-FET = W
- $C_{GN} = WC_{G0}$

- Width P-FET = 2W
- $C_{GP} = 2WC_{G0}$

- $C_{in} = C_{GP} + C_{GN} = 3WC_{G0}$

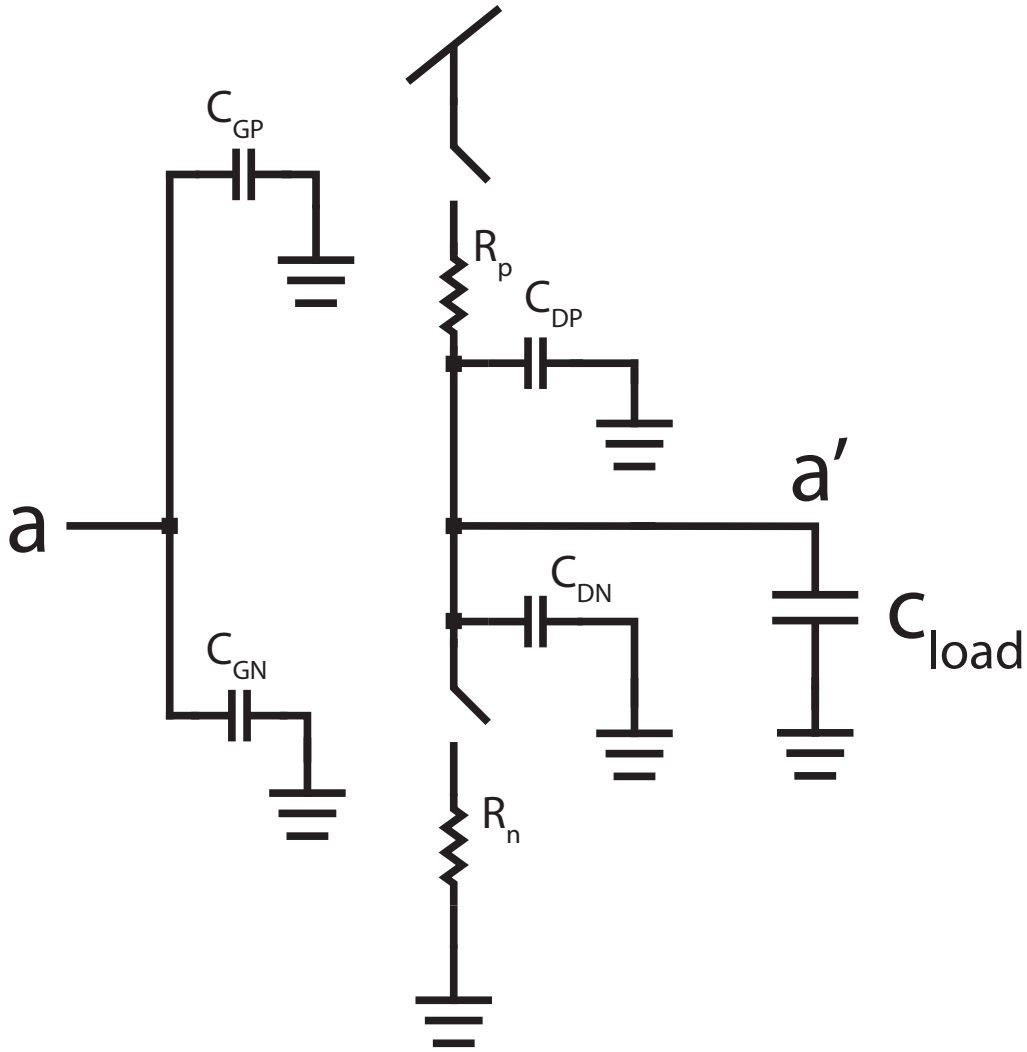
  (Parallel Capacitance)



Internal Capacitance

- $C_D = \gamma C_G$
- Width N-FET = W
- $C_{DN} = W\gamma C_{G0}$
- Width P-FET = 2W
- $C_{DP} = 2W\gamma C_{G0}$
- $C_{int} = C_{DP} + C_{DN} = 3W\gamma C_{G0}$

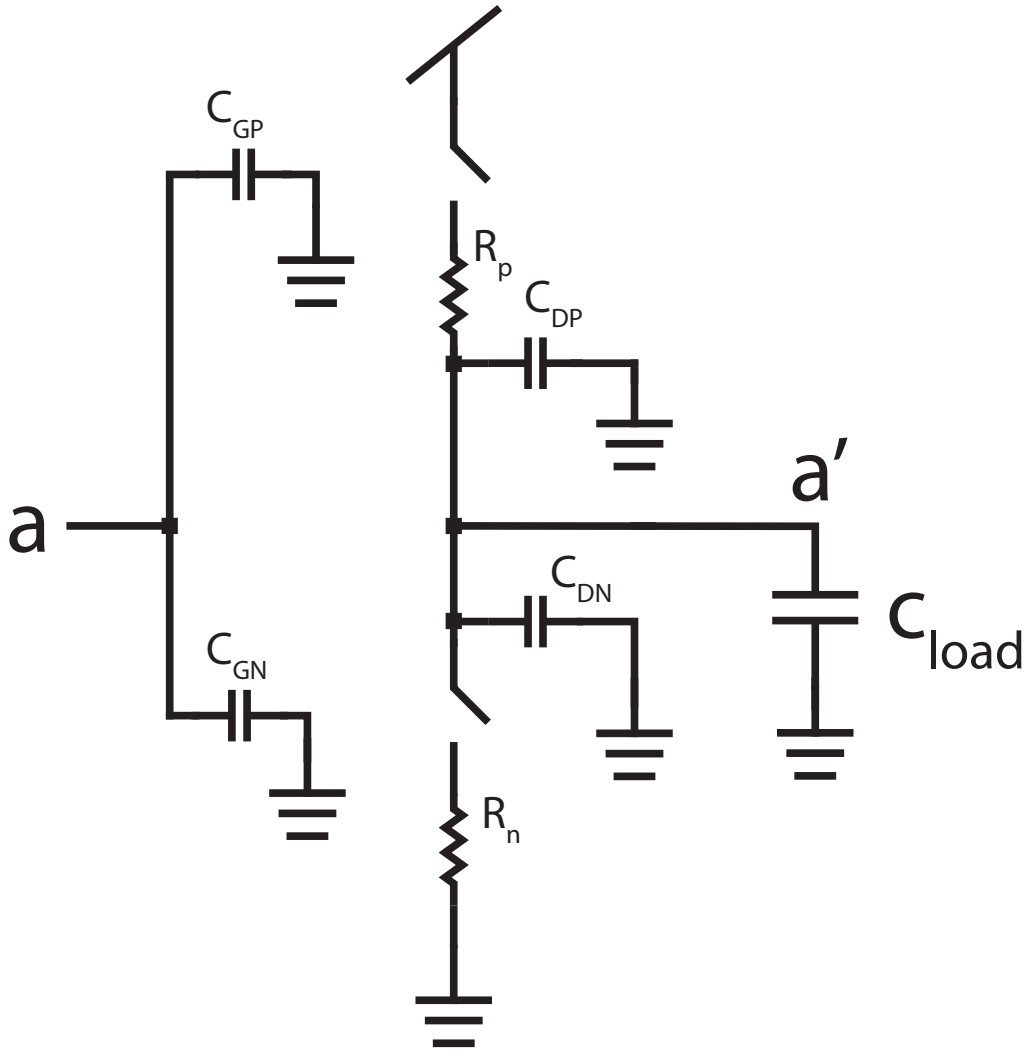
  (Parallel Capacitance)



Output Capacitance

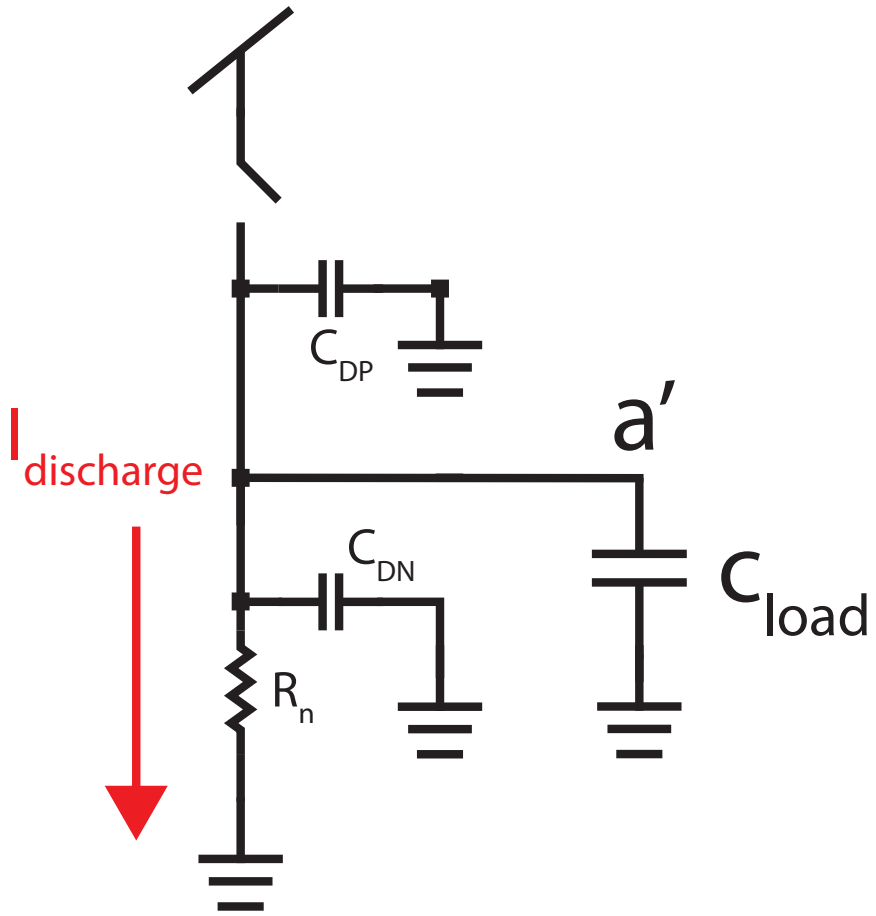
- $C_{out} = C_{int} + C_{load} = 3W\gamma C_{G0} + C_{load}$

  (Parallel Capacitance)



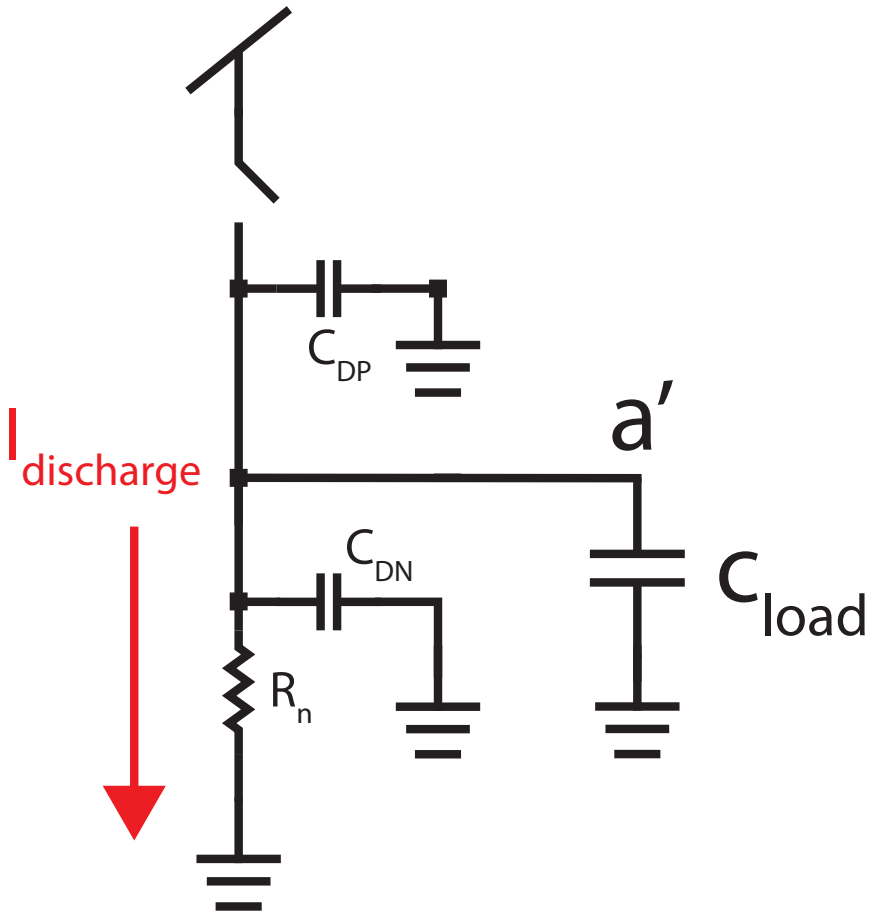
Discharge Time

- How long does it take to discharge the output to $V_{DD}/2$?
- This looks like an RC circuit!
- $R_N = R_{N0}/W$
- $C_{out} = 3W\gamma C_{G0} + C_{load}$
- $V = V_{DD}e^{-t/(R_N C_{out})}$
- Solve for t when $V = 0.5V_{DD}$



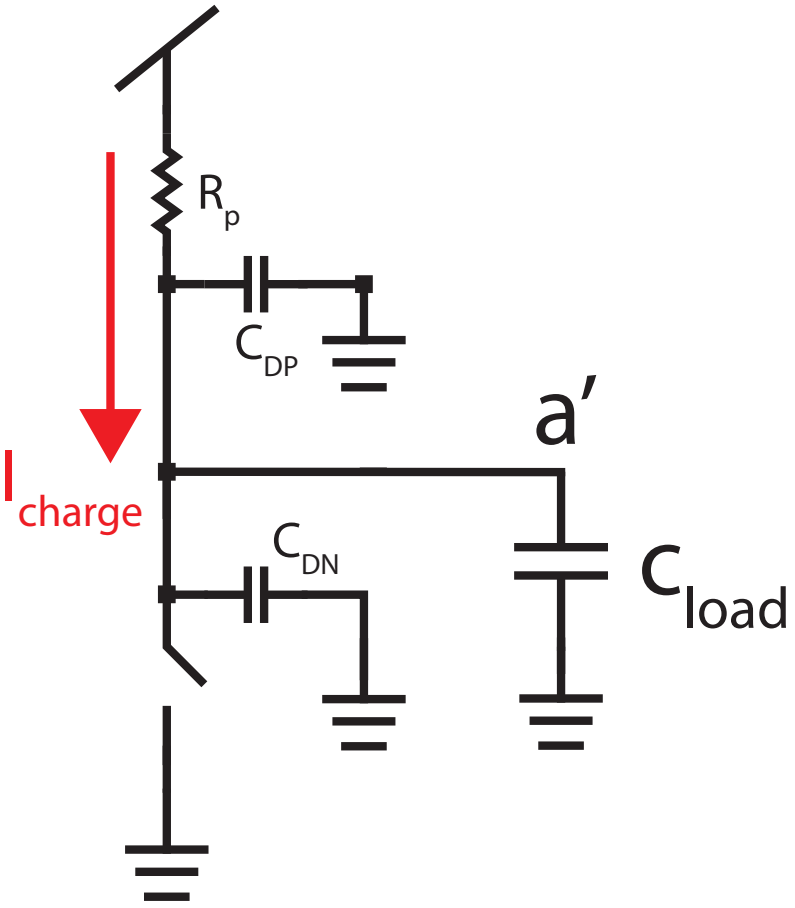
Discharge Time

- Solve for t when $V = 0.5V_{DD}$
  - $0.5V_{DD} = V_{DD}e^{-t/(R_N C_{out})}$
  - $0.5 = e^{-t/(R_N C_{out})}$
  - $\ln(0.5) = -t/(R_N C_{out})$
  - $t = -\ln(0.5)(R_N C_{out})$
  - $t = -\ln(0.5)(R_{N0}/W)(3W\gamma C_{G0} + C_{load})$
  - $t = -\ln(0.5)(3R_{N0}\gamma C_{G0})\left(1 + \frac{C_{load}}{3W\gamma C_{G0}}\right)$
  - Recall that $C_{in} = 3WC_{G0}$
  - $t = -\ln(0.5)(3R_{N0}\gamma C_{G0})\left(1 + \frac{C_{load}}{\gamma C_{in}}\right)$
  - Recall fanout: $f = \frac{C_{load}}{C_{in}}$
  - $t = -\ln(0.5)(3R_{N0}\gamma C_{G0})\left(1 + \frac{f}{\gamma}\right)$
  - $t \approx 0.69(3R_{N0}\gamma C_{G0})\left(1 + \frac{f}{\gamma}\right)$



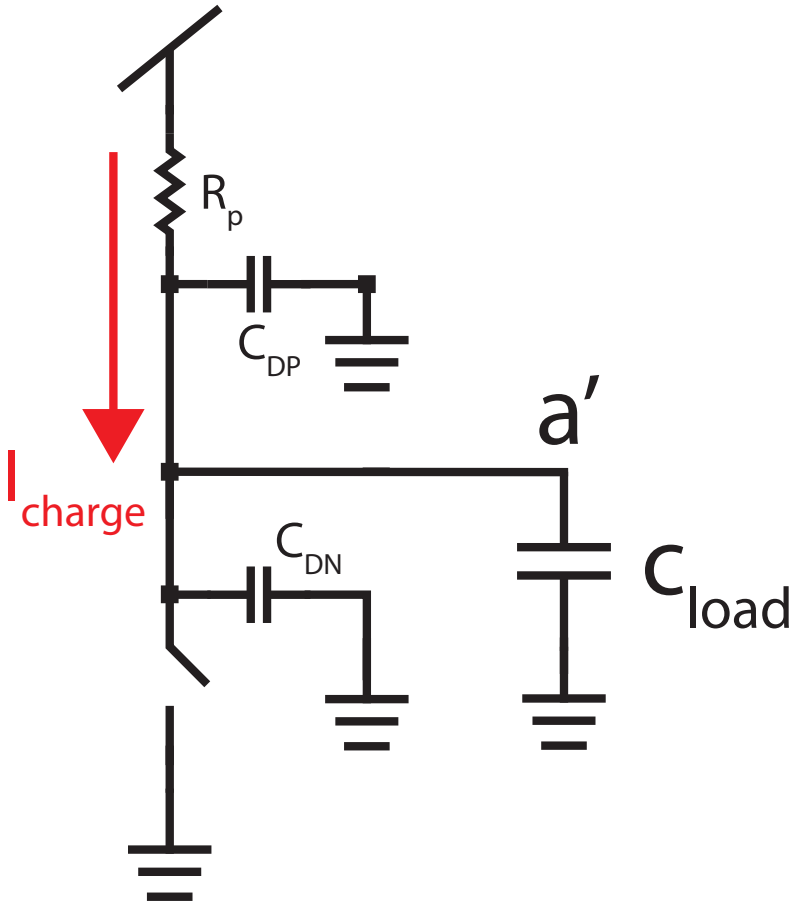
Charge Time

- Similar to discharge time
- How long does it take to charge the output to $V_{DD}/2$?
- This looks like another RC circuit!
- $R_{P0} = 2R_{N0}$
- $R_P = \frac{R_{P0}}{2W} = \frac{2R_{N0}}{2W} = \frac{R_{N0}}{W}$
- $C_{out} = 3W\gamma C_{G0} + C_{load}$ (same as before)
- $V = V_{DD}\left(1 - e^{-t/(R_P C_{out})}\right)$
- Solve for t when $V = 0.5V_{DD}$



Charge Time

- Solve for t when $V = 0.5V_{DD}$
  - $0.5V_{DD} = V_{DD}\left(1 - e^{-t/(R_P C_{out})}\right)$
  - $0.5 = 1 - e^{-t/(R_P C_{out})}$
  - $0.5 = e^{-t/(R_P C_{out})}$
  - $\ln(0.5) = -t/(R_P C_{out})$
  - $t = -\ln(0.5)(R_P C_{out})$
  - $t = -\ln(0.5)\,(R_{N0}/W)(3W\gamma C_{G0} + C_{load})$
  - Note that this is the same expression we had when calculating discharge time
  - $t = -\ln(0.5)(3R_{N0}\gamma C_{G0})\left(1 + \frac{f}{\gamma}\right)$
  - $t \approx 0.69(3R_{N0}\gamma C_{G0})\left(1 + \frac{f}{\gamma}\right)$



Notes on the Inverter

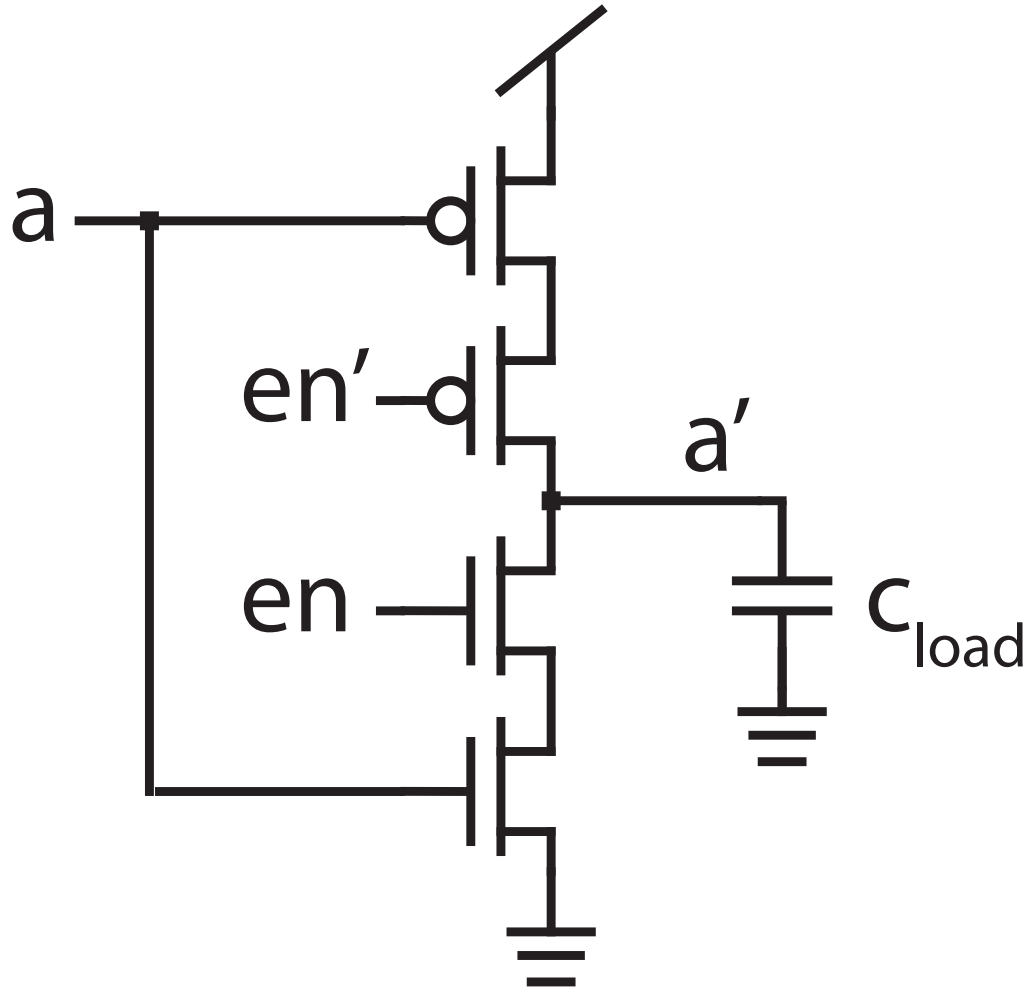
- The sizing of the P-MOS to be 2x the width of the N-MOS was not an accident!
- It caused the charge time and discharge times to be balanced
- Generally, we try to design gates that exhibit balanced charge and discharge time

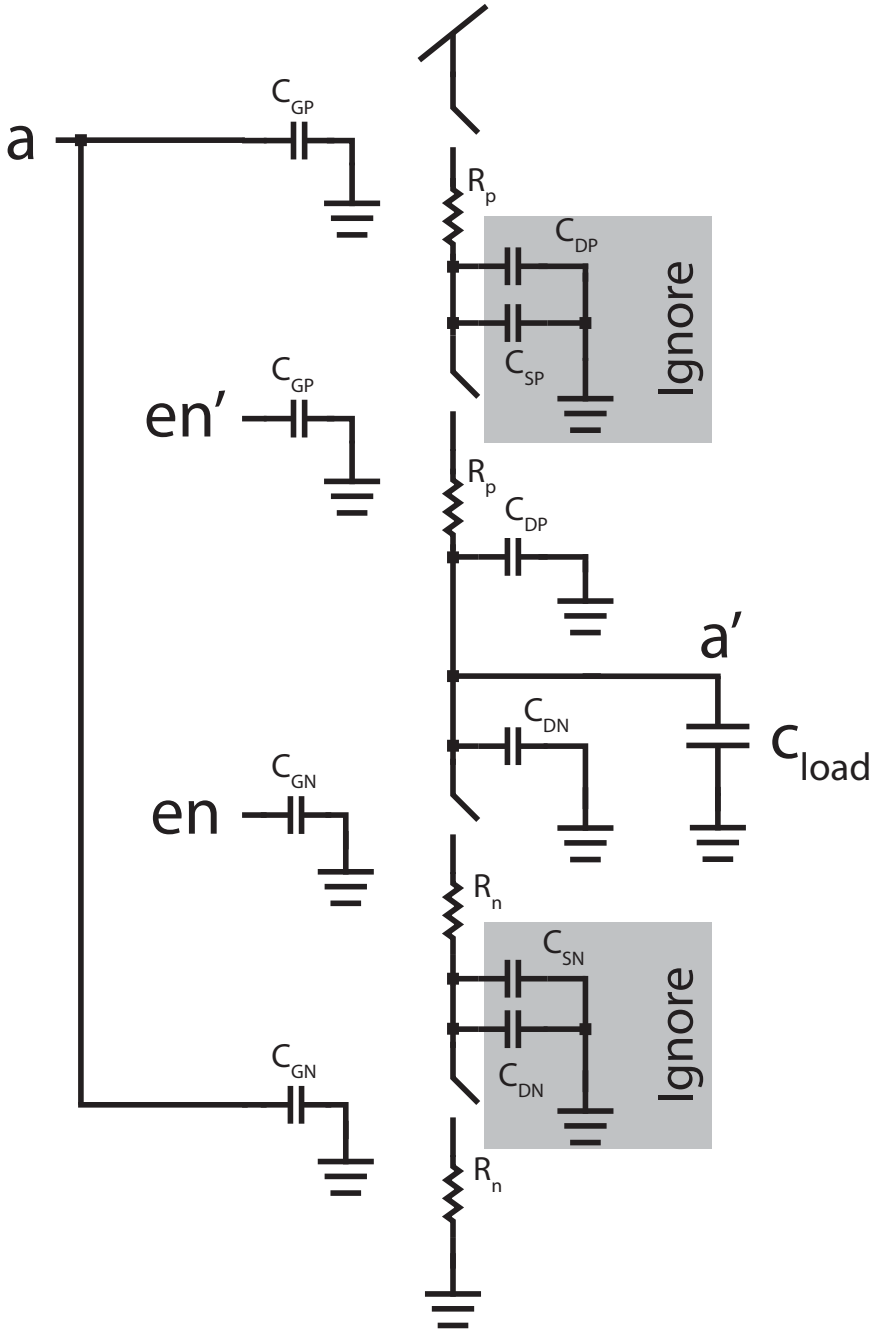
More Complex Gates – Tristate Inverter

- Characterize the delays for a tristate inverter from its enable input
  - Assume both complemented and uncomplemented inputs are available
  - Assume the output has been discharged by another circuit when calculating the charge time
  - Assume the output has been charged to V_{DD} when calculating the discharge time

- You can ignore the capacitance at intermediate nodes between transistors in series for this problem … more on this later

Tristate Inverter Model

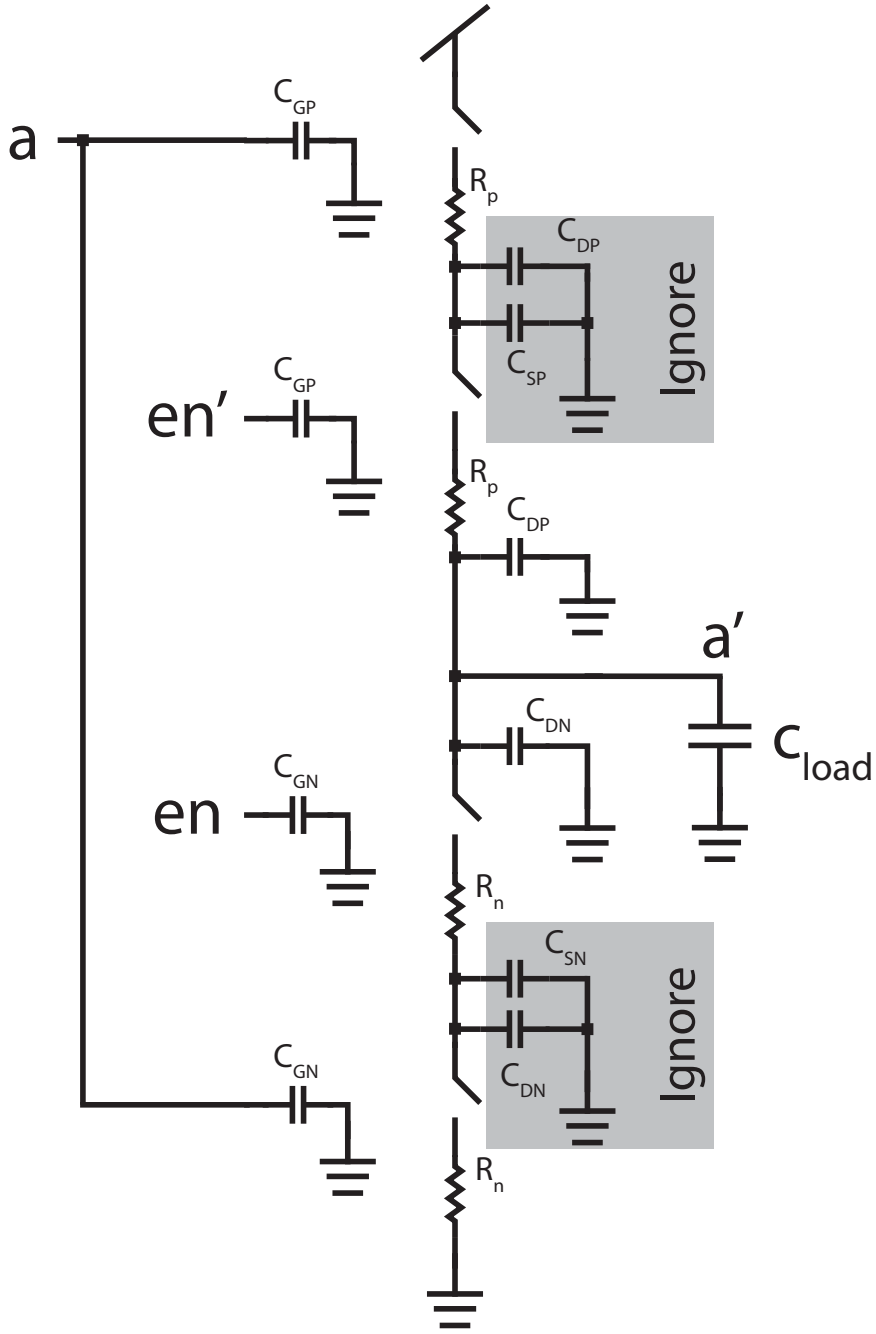




Calculating Resistance

- Width N-FET = W
- $R_N = \frac{R_{N0}}{W}$

- Width P-FET = 2W
- $R_{P0} = 2R_{N0}$
- $R_P = \frac{R_{P0}}{2W} = \frac{2R_{N0}}{2W} = \frac{R_{N0}}{W}$



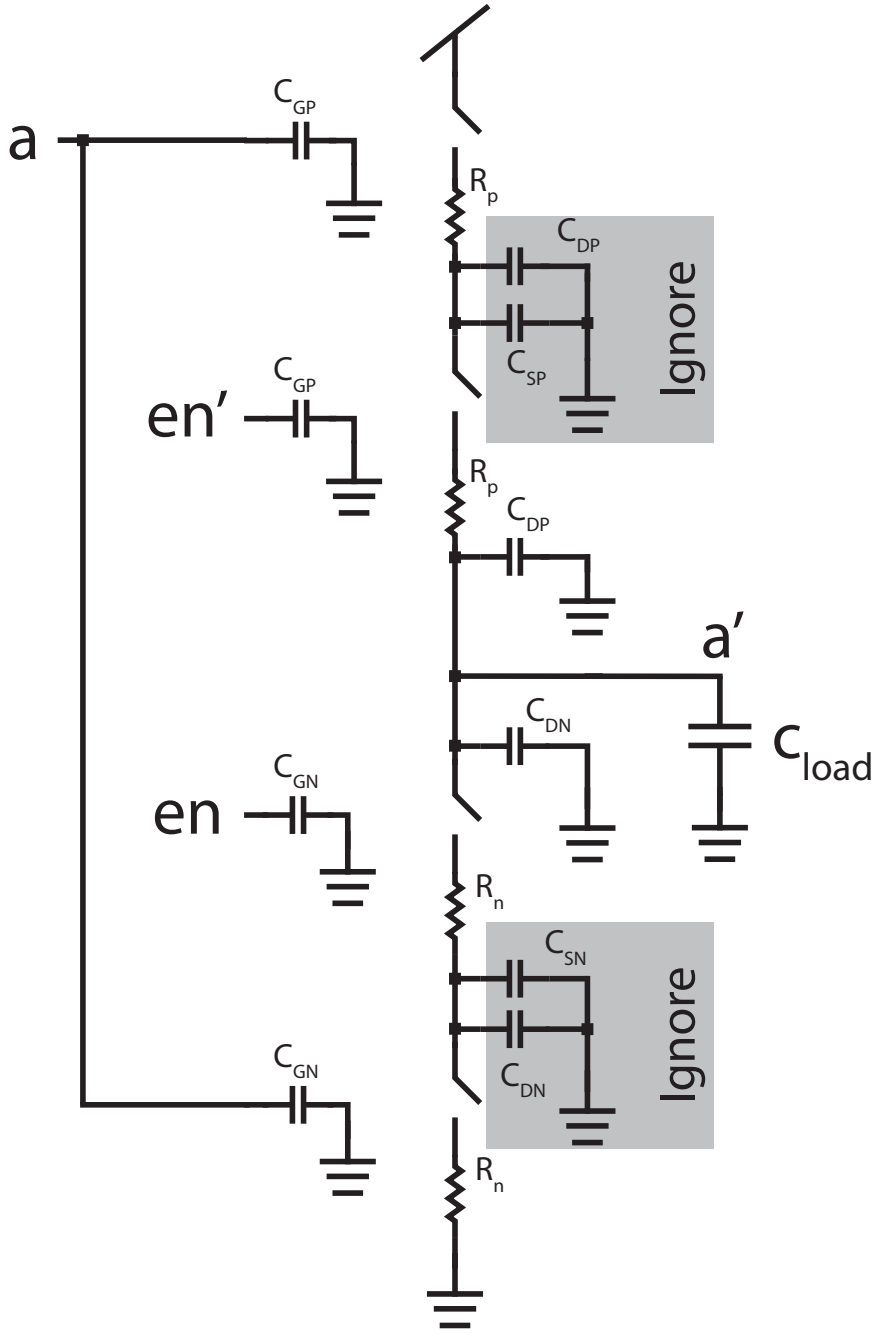
Calculating Capacitance

- $C_D = \gamma C_G$

- Width N-FET = W
- $C_{DN} = W\gamma C_{G0}$

- Width P-FET = 2W
- $C_{DP} = 2W\gamma C_{G0}$

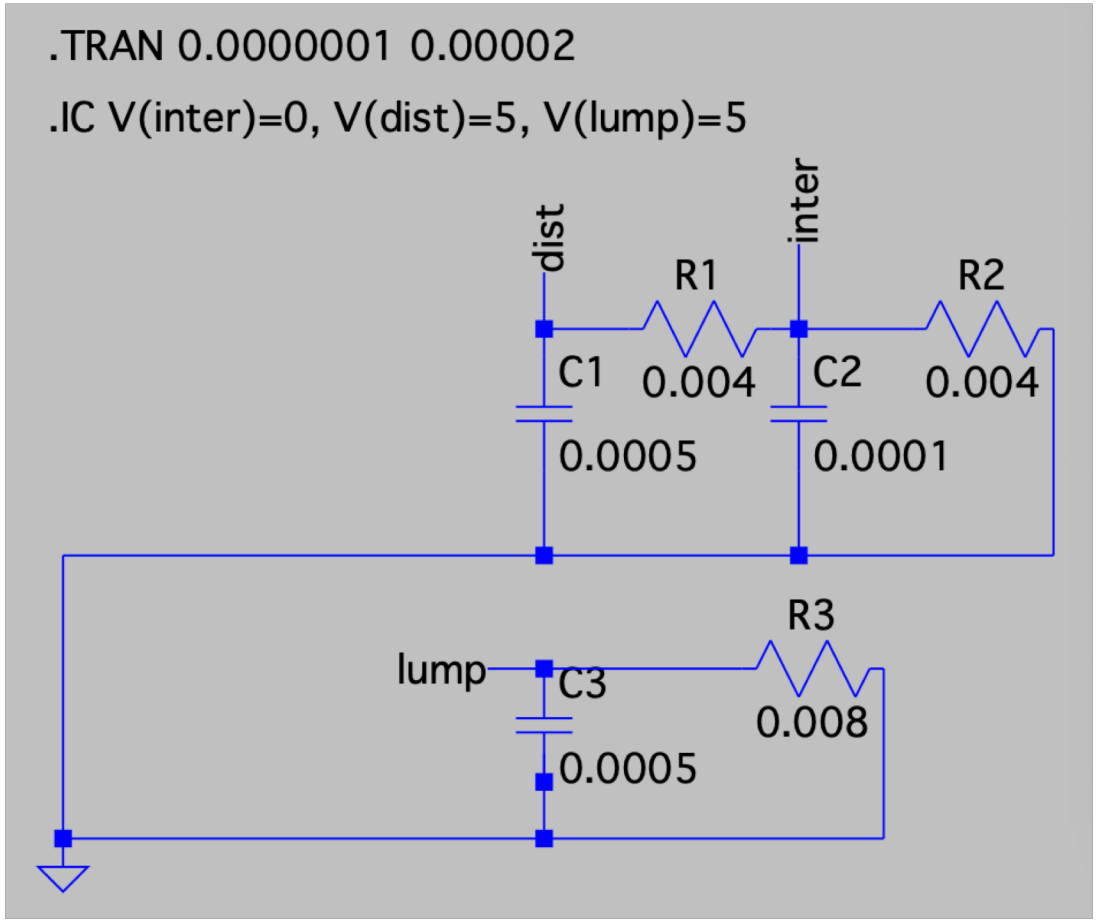
- $C_{int} = C_{DP} + C_{DN} = 3W\gamma C_{G0}$
- $C_{out} = C_{int} + C_{load} = 3W\gamma C_{G0} + C_{load}$

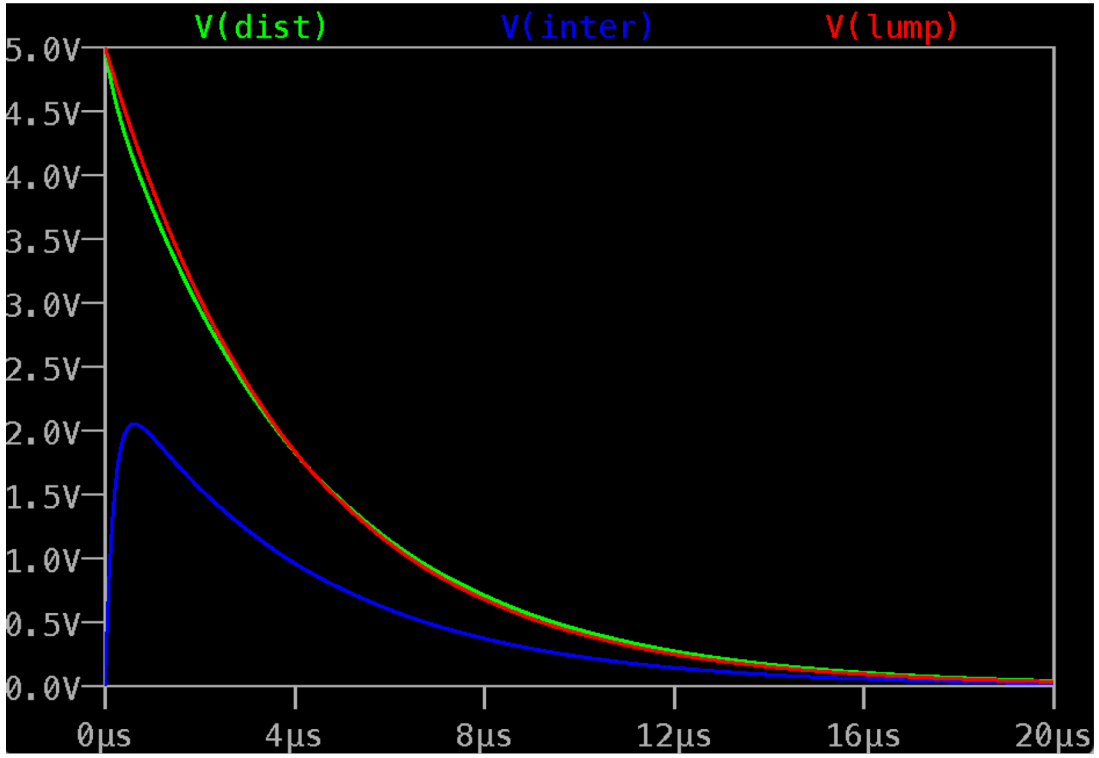


Ignoring the Intermediate Node Capacitance

- When we have transistors in series, there exists some parasitic capacitance at the intermediate node they share
- This capacitance stops us from using simple first order RC charge/discharge equations because we now need to account for charge that is stored in the intermediate node capacitance
- For small intermediate node capacitance values, ignoring this second order effect will not change the calculated propagation delay by too much*
- For a more accurate first order approximation that does not completely ignore the intermediate node capacitance, see Weste & Harris pg. 150.

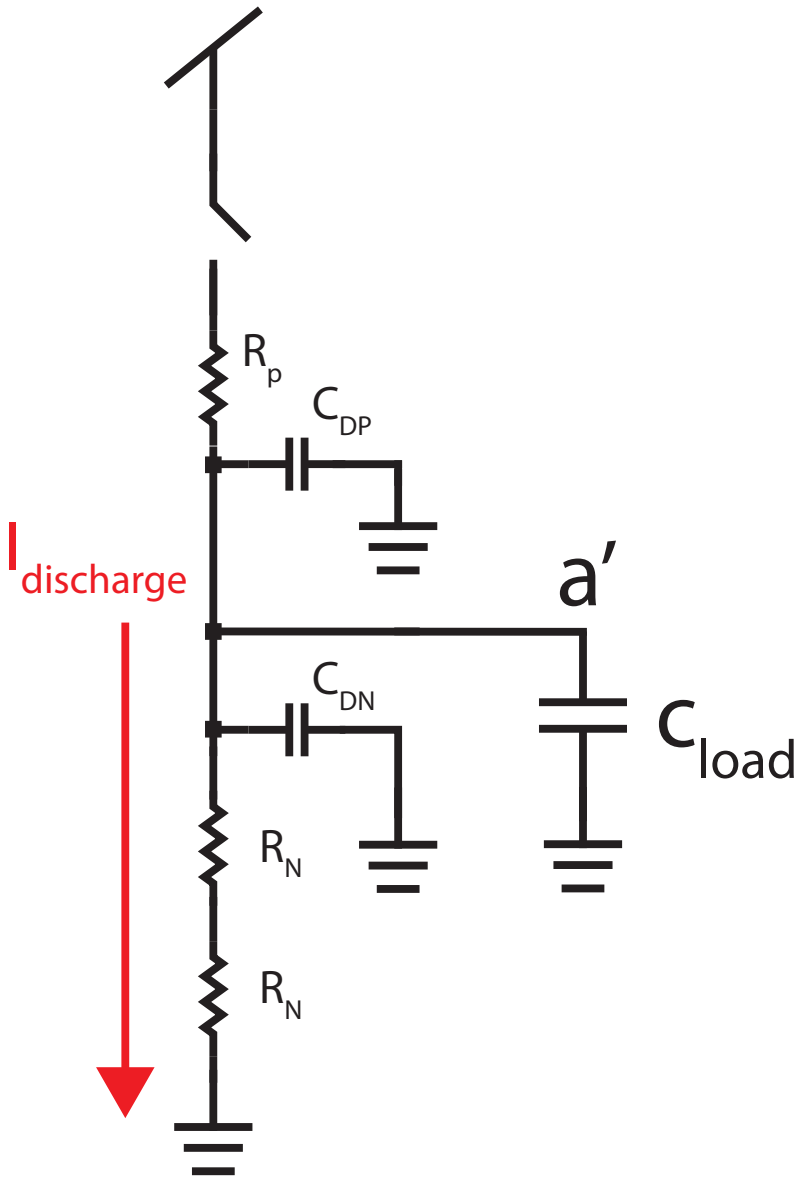
Quick aside to (LT)SPICE!





Discharge Time

- How long does it take to discharge the output to $V_{DD}/2$?
- This looks like another RC circuit!
- Except, this time, we have series resistance we need to consider
- $R = 2R_N = 2R_{N0}/W$
- $C_{out} = 3W\gamma C_{G0} + C_{load}$
- $V = V_{DD}e^{-t/(RC_{out})} = V_{DD}\ e^{-t/(2R_NC_{out})}$
- Solve for t when $V = 0.5V_{DD}$



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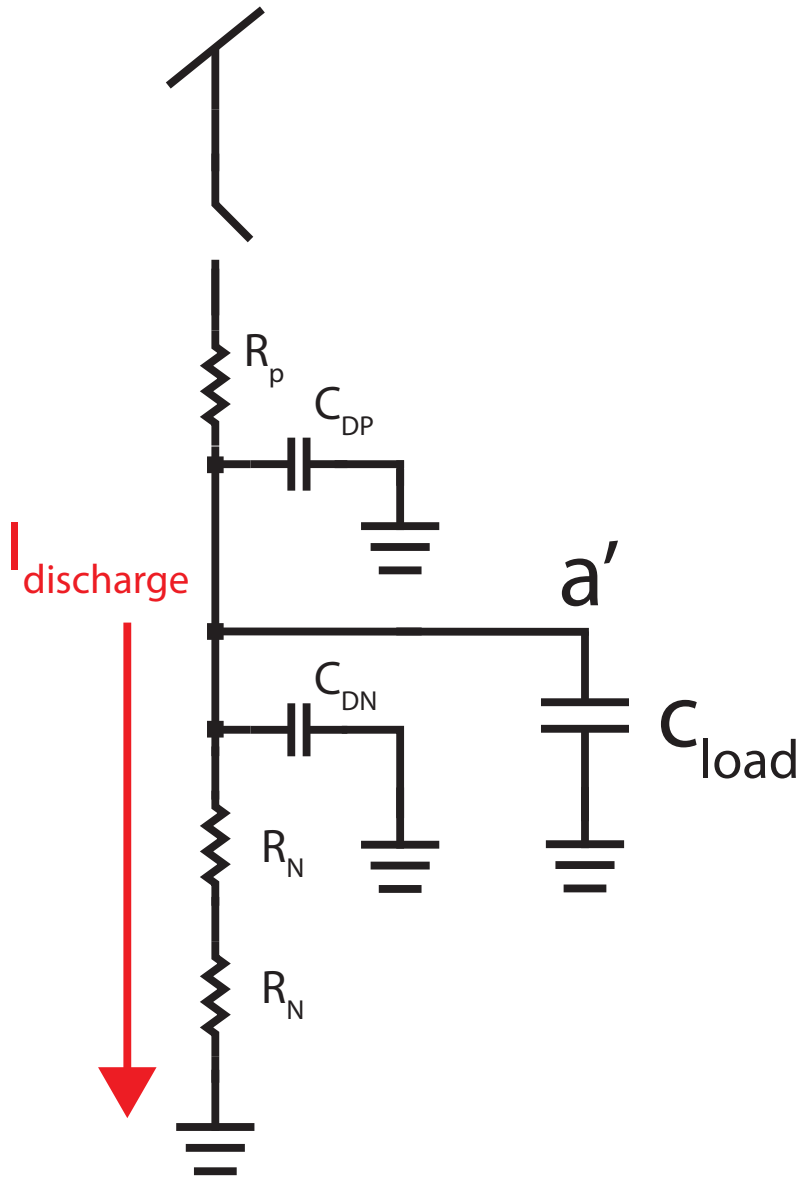
  - $t = -\ln(0.5)(2R_{N0}/W)(3W\gamma C_{G0} + C_{load})$
  - $t = -\ln(0.5)(6R_{N0}\gamma C_{G0})\left(1 + \frac{C_{load}}{3W\gamma C_{G0}}\right)$

Compared to the inverter, this term is 2x larger



Charge Time

- How long does it take to charge the output to $V_{DD}/2$?
- This looks like another RC circuit!
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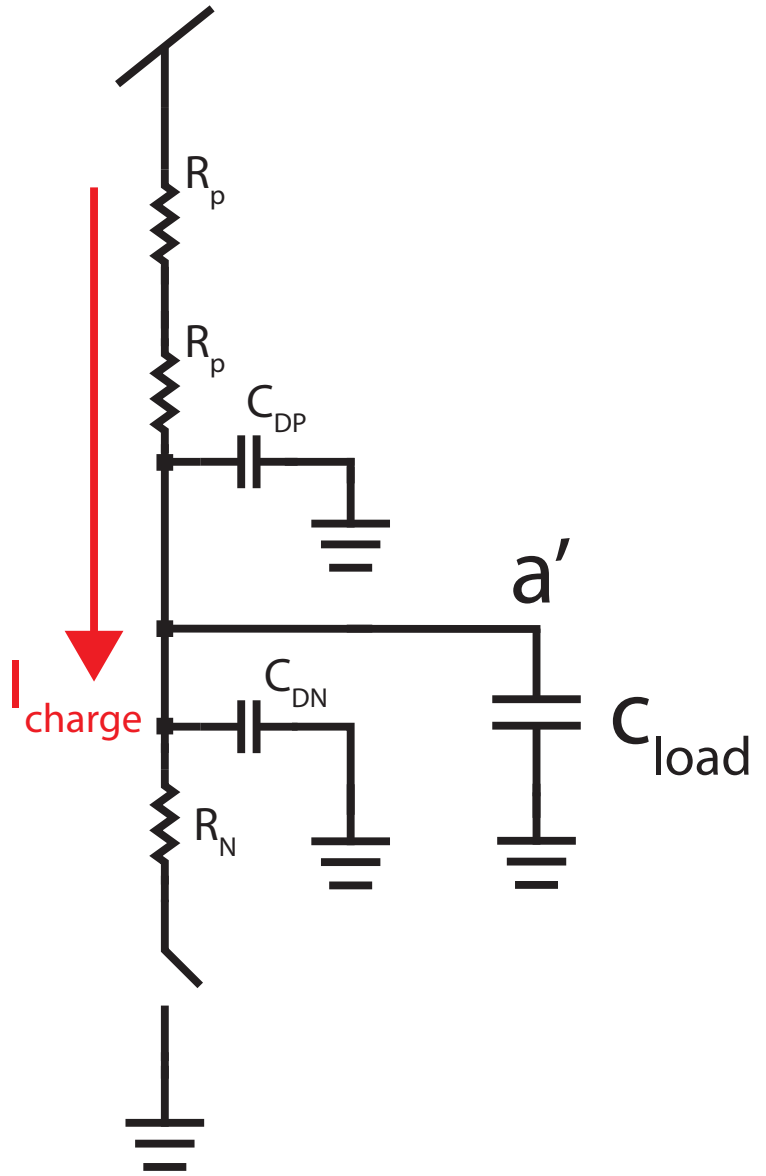

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  - Note that this is the same expression we had when calculating discharge time
  - $t = -\ln(0.5)(6R_{N0}\gamma C_{G0})\left(1 + \frac{C_{load}}{3W\gamma C_{G0}}\right)$

Compared to the inverter, this term is 2x larger