

## Experiment #3 – RC Circuits

### I. Introduction

#### A. Capacitors

A capacitor is a passive electronic component that stores energy in the form of an electrostatic field. In its simplest form, a capacitor consists of two conducting plates separated by an electrically insulating material called the dielectric. The capacitance  $C$  of a “parallel-plate capacitor” is directly proportional to the area  $A$  of the plates and the dielectric constant (permittivity)  $\epsilon$  of the insulator, and it is inversely proportional to the separation  $d$  between the plates:

$$C = \frac{\epsilon A}{d} \quad (\text{Eq. 1})$$

The standard unit of capacitance is the Farad (Coulomb/Volt). Practical capacitor values usually lie in the picofarad ( $1 \text{ pF} = 10^{-12} \text{ F}$ ) to microfarad ( $1 \text{ }\mu\text{F} = 10^{-6} \text{ F}$ ) range.

Recall that a current is a flow of charges. When current flows into a capacitor, the charges don't pass through (although to maintain local charge balance, an equal number of the same polarity charges leave the other plate of the device) but instead accumulate in the device, increasing the voltage across the capacitor. The voltage across the capacitor is directly proportional to the charge  $Q$  stored on the conductors:

$$Q = CV \quad (\text{Eq. 2})$$

Since  $Q$  is the integration of current over time, we can write:

$$V = \frac{Q}{C} = \frac{\int i(t) dt}{C} \quad (\text{Eq. 3})$$

Differentiating this equation, we obtain the  $I$ - $V$  characteristic equation for a capacitor:

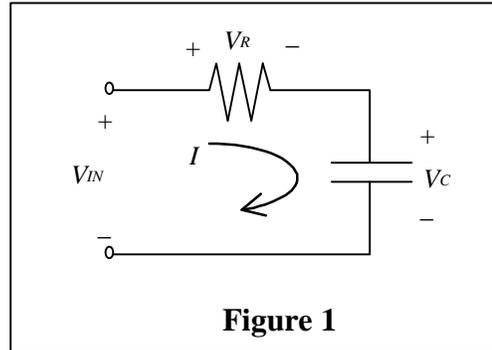
$$i = C \frac{dV}{dt} \quad (\text{Eq. 4})$$

#### B. RC Circuits

An RC (resistor + capacitor) circuit will have an exponential voltage response of the form  $v(t) = A + B \exp(-t/RC)$  where  $A$  and  $B$  are constants that express the final voltage and the difference between the initial voltage and the final voltage, respectively. ( $\exp(x)$  is  $e$  to the  $x$  power, where  $e = 2.718$ , the base of the natural logarithm.) The product  $RC$  is called the **time constant** (whose units are seconds) and is usually represented by the Greek letter  $\tau$ . When the time has reached a value equal to the time constant,  $\tau$ , then  $\exp(-t/RC) = \exp(-1) = 0.3678$ , or about  $5/8$  of the way from the initial value to the final value.

The characteristic “exponential decay” associated with an RC circuit is important to understand, because complicated circuits can oftentimes be modelled simply as resistor and a capacitor. This is especially true in integrated circuits (ICs).

A simple RC circuit is drawn in Figure 1 with currents and voltages defined as shown. Equation 5 is obtained from Kirchoff's Voltage Law, which states that the algebraic sum of voltage drops around a closed loop is zero. Equation 6 is the defining  $I$ - $V$  characteristic equation for a capacitor (as derived above), and Equation 7 is the defining  $I$ - $V$  characteristic equation for a resistor (Ohm's Law).



$$V_{IN} = V_C + V_R \quad (\text{Eq. 5})$$

$$I = C \frac{dV_C}{dt} \quad (\text{Eq. 6})$$

$$V_R = IR \quad (\text{Eq. 7})$$

By substituting Equations 6 and 7 into Equation 5, the following first-order linear differential equation is obtained:

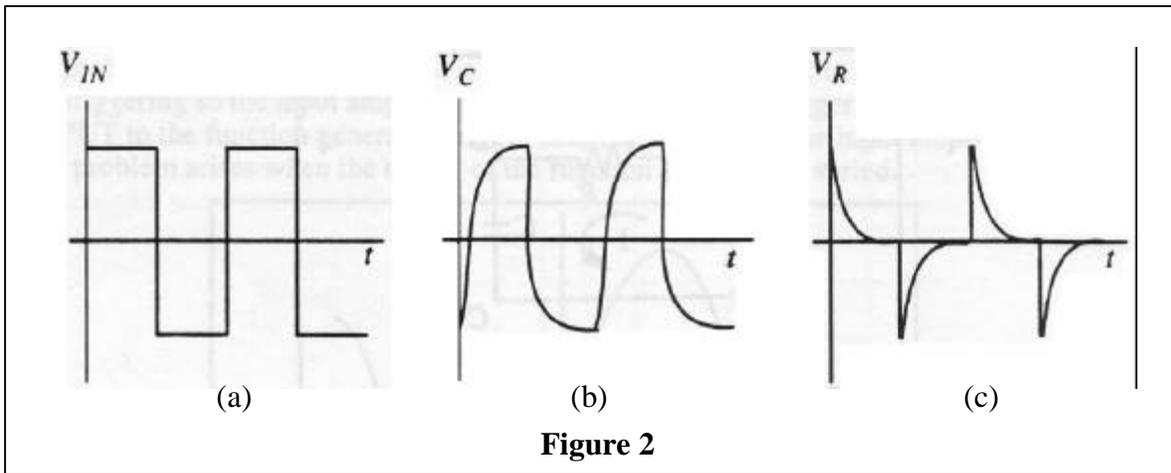
$$V_{IN} = V_C + RC \frac{dV_C}{dt} \quad (\text{Eq. 8})$$

If  $V_{IN}$  is a step function at time  $t=0$ , then  $V_C$  and  $V_R$  are of the form:

$$V_C = A + Be^{-t/RC} \quad (\text{Eq. 9})$$

$$V_R = A' + B'e^{-t/RC} \quad (\text{Eq. 10})$$

If a voltage difference exists across the resistor (*i.e.*  $V_R \neq 0$ ), then current will flow (Eq. 7). This current flows through the capacitor and causes  $V_C$  to change (Eq. 6).  $V_C$  will increase (if  $I > 0$ ) or decrease (if  $I < 0$ ) exponentially with time, until it reaches the value of  $V_{IN}$ , at which time the current goes to zero (since  $V_R = 0$ ). For the square-wave function  $V_{IN}$  as shown in Figure 2a, the responses  $V_C$  and  $V_R$  are shown in Figure 2b and Figure 2c, respectively.

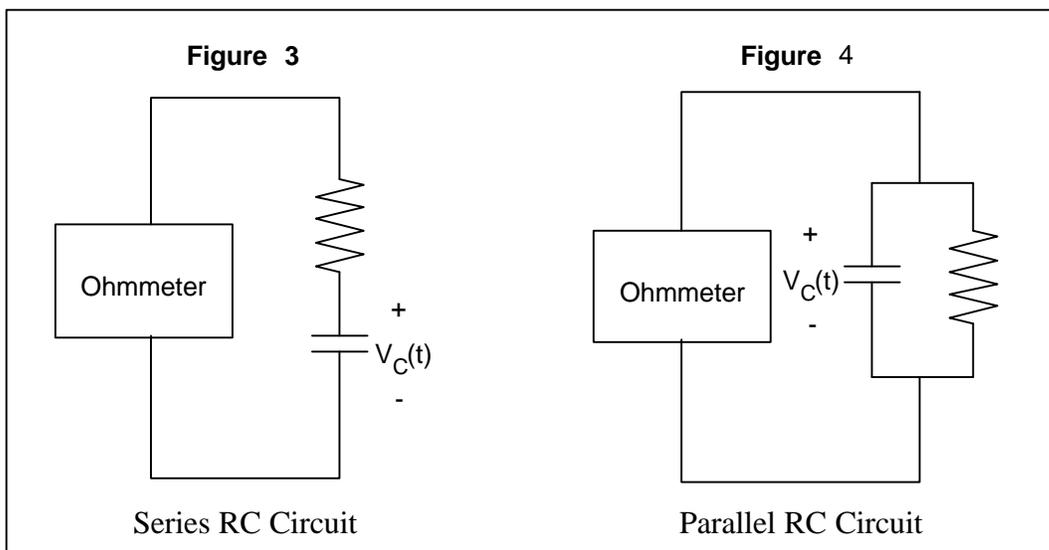


Note that if the frequency of the square wave  $V_{IN}$  is too high (*i.e.* if  $f \gg 1/RC$ ), then  $V_C$  and  $V_R$  will not have enough time to reach their asymptotic values. If the frequency is too low (*i.e.* if  $f \ll 1/RC$ ), the decay time will be very short relative to the period of the waveform and thus the exponential decay will be difficult to observe. As a rough guideline, the period of the square wave should be chosen such that it is approximately equal to  $10RC$ , in order for the responses shown in Figure 2b-c to be readily observed on an oscilloscope.

## II. Hands On

### A. Determining the RC Circuit Configuration

In this part of the experiment, you will make ohmmeter measurements to see if you can discover a method to determine if a resistor and capacitor are connected in series or in parallel.



(a) Get a resistor ( $1k\Omega$ ) and capacitor ( $1\mu F$ ) from your TA.

Recall that an ohmmeter has a built-in current source that sends a small current into the circuit under test. The ohmmeter reads the voltage across the circuit under test and determines the resistance of the circuit using Ohm's Law.

- (b) Build the circuit shown in Figure 3. Note that the ohmmeter's current source keeps on charging up the capacitor. (For small values of capacitance, the capacitor will be fully charged almost instantly.)

**Question 1:** Are you able to measure the value of the resistor? If not, explain the reason why you cannot make the measurement.

- (c) Build the circuit shown in Figure 4. Note that the capacitor stops charging when the current through the resistor is equal to the current from the ohmmeter.

**Question 2:** Explain how you got your ohmmeter reading for the circuit in Figure 4. Why does it take some time before the ohmmeter's reading stabilizes?

**Question 3:** Can you determine the RC circuit configuration (series or parallel) using an ohmmeter? If so, how?

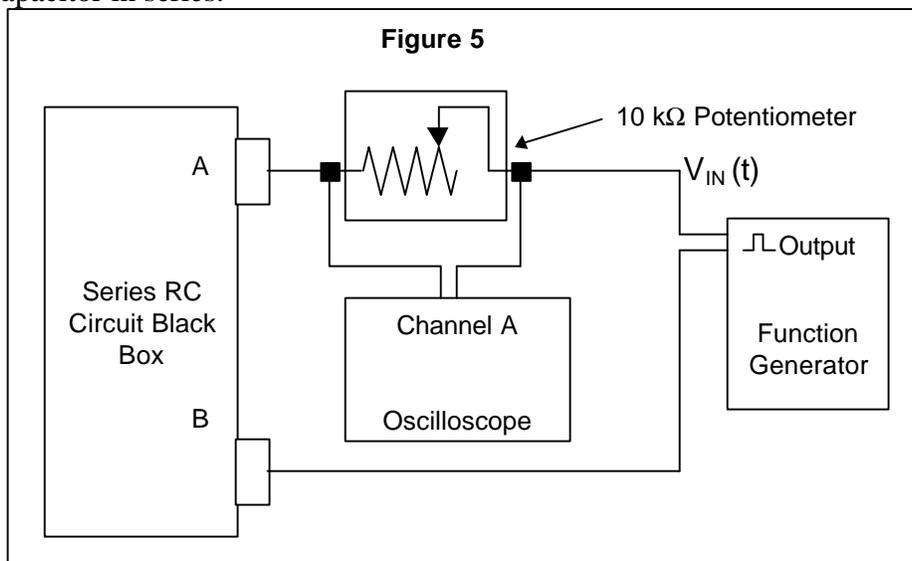
## B. Identifying Physical Values in a Series RC Circuit Black Box and a Parallel RC Circuit Black Box

The TA will give you two black boxes. One contains a series RC circuit and the other contains a parallel RC circuit. Determine the basic resistor-capacitor configuration in each black box using an ohmmeter.

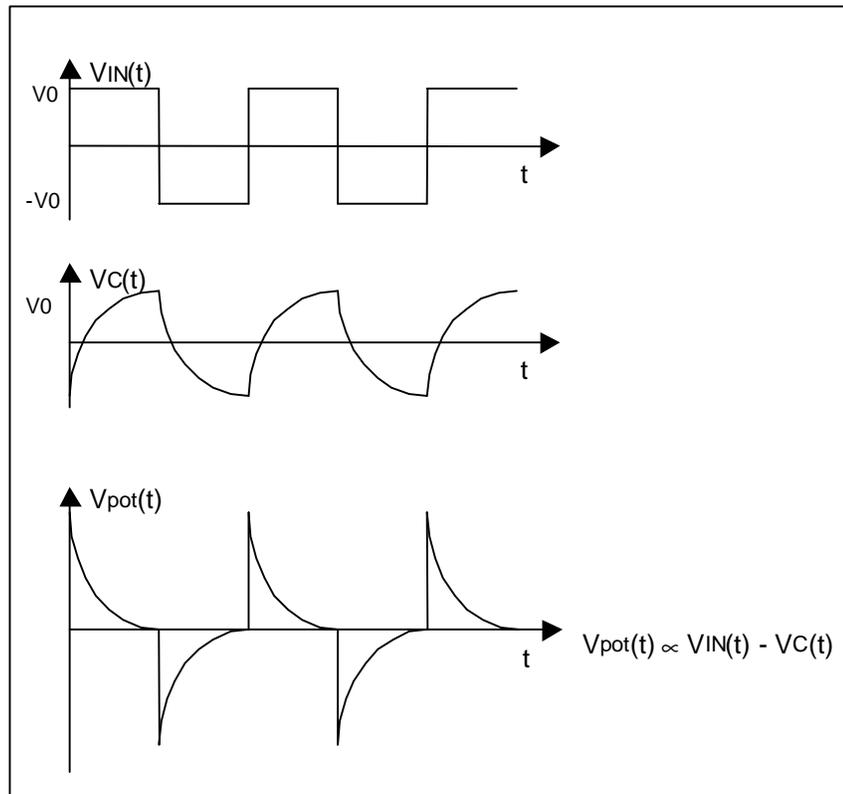
**Question 4:** Which box contains a series RC circuit? Which box contains a parallel RC circuit? (Identify the color of the binding posts.)

### 1) Series RC Circuit Black Box

- (a) Construct the following circuit (Figure 5) for the black box that contains the resistor and capacitor in series.



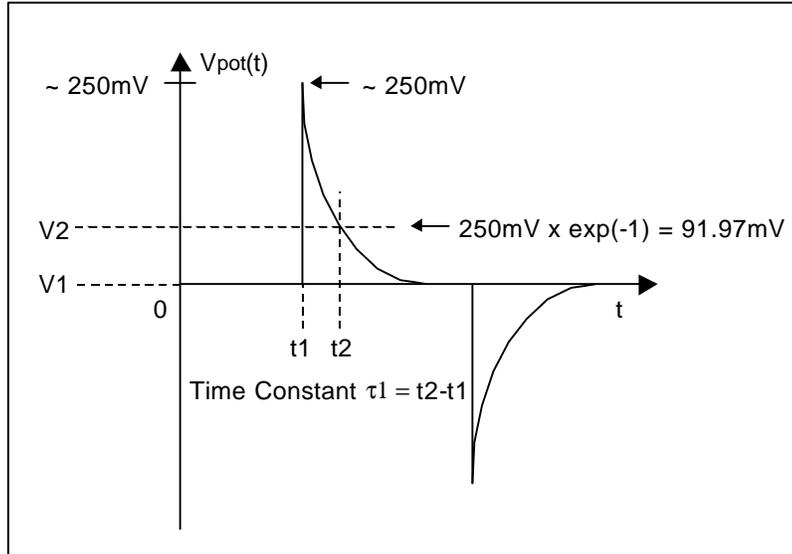
- (b) Set the amplitude of the square wave to 3 Volts peak-to-peak, with 0 Volts DC offset. Adjust the frequency of the square wave to  $\sim 50$  Hz. Make sure that you are using the waveform from the OUTPUT terminal of the function generator, not the SYNC terminal.
- (c) Note that the voltage drop across the potentiometer  $V_{\text{pot}}(t)$  is proportional to  $V_{\text{IN}} - V_{\text{C}}(t)$ .



**Figure 6:** Voltage waveforms in a series RC circuit

From the voltage across the pot shown on the oscilloscope, you can obtain the current going through the black box, since the current through the pot ( $= V_{\text{pot}} / R_{\text{pot}}$ ) is equal to the current going through the black box by Kirchhoff's Current Law.

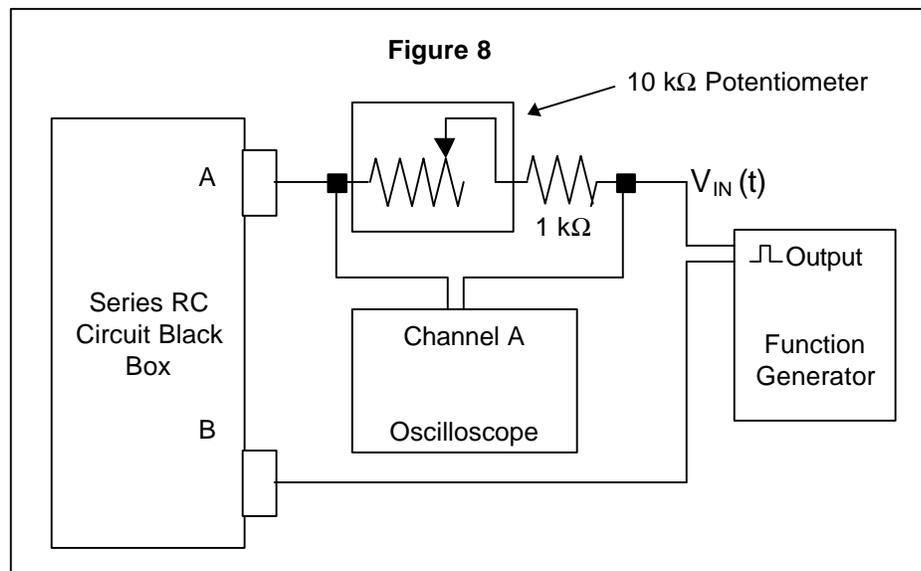
- (d) Reduce the pot's resistance until the amplitude of the voltage across the pot is at most one tenth of the input voltage amplitude. ( $\sim 250$  mV is good.) This will correspond to the pot resistance of approximately  $0.2 \text{ k}\Omega$ . You can almost treat the pot as a short circuit at this value, since it is much smaller than the resistor inside the black box. From the voltage across the pot waveform on the oscilloscope, determine the black box's RC time constant. (Use the "CURSOR" function of the oscilloscope.) Refer to Figure 7 if you are not sure how to do this.



**Figure 7:** How to measure the time constant  $\tau_1$  on the oscilloscope

**Question 5:** What is the time constant  $\tau_1$ ?

Now connect a  $1\text{k}\Omega$  resistor in series with the pot, as shown in Figure 8.



- (e) Increase the pot's resistance until the time constant of the voltage waveform doubles in value. You will have to repeat the process of increasing the pot's resistance and measuring the time constant until  $\tau = 2\tau_1$ . Once the time constant has doubled, disconnect the pot and the  $1\text{k}\Omega$  resistor from the circuit and measure their series resistance using an ohmmeter.

**Question 6:** What is the resistance of the series combination (pot +  $1\text{k}\Omega$  resistor)?

You may wonder what the reason is for doubling the time constant. Let  $R_{\text{pot}}$  = resistance of pot,  $R_{1k}$  = resistance of  $1k\Omega$  resistor,  $R_{\text{box}}$  = series resistor in the black box, and  $C_{\text{box}}$  = series capacitor in the black box. When the pot is at a very small value, the entire circuit (RC series circuit in the black box and the pot together) has a RC time constant of roughly  $R_{\text{box}}C_{\text{box}}$ . When the pot is adjusted such that  $R_{\text{pot}} + R_{1k} = R_{\text{box}}$ , the time constant becomes  $(R_{\text{pot}} + R_{1k} + R_{\text{box}}) C_{\text{box}} = 2 R_{\text{box}} C_{\text{box}}$ ; therefore, doubling the time constant requires setting the pot +  $1k\Omega$  resistor series combination to a resistance of  $R_{\text{box}}$ .

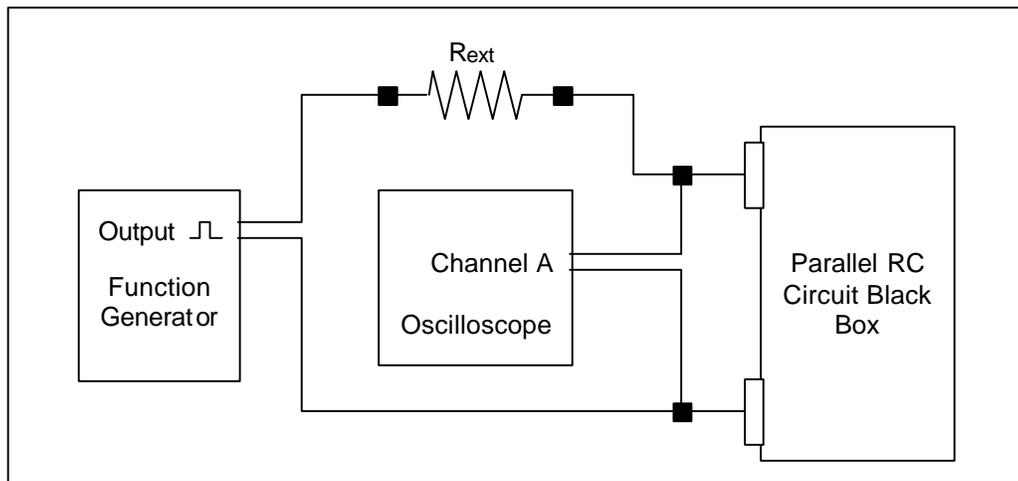
**Question 7:** Can you find the resistance and capacitance  $R_{\text{box}}$  and  $C_{\text{box}}$ ? Draw a possible RC circuit for the black box. Ask your TA for the resistance and capacitance of the resistor and the capacitor inside the black box. Are they in good agreement with the values you have obtained experimentally? Explain if there is(are) any significant difference(s).

## 2) Parallel RC Circuit Black Box

(a) Measure the resistance of the circuit inside the black box using the ohmmeter.

**Question 8:** What is the value of the resistor inside the black box?

(b) Select a resistor of a value comparable to the resistor inside the black box and construct the circuit in Figure 9. Measure the time constant of the circuit with  $R_{\text{ext}}$ .



**Figure 9:** Setup for finding  $R$  and  $C$  of an unknown parallel RC circuit

**Question 9:** What is the time constant of the circuit with  $R_{\text{ext}}$ ? What is the value of the capacitor inside the black box? Ask your TA for the values of the resistor and the capacitor inside the black box. Are they in good agreement with the values you have obtained experimentally? Explain if there are any significant differences.