## EE 42

## Homework \#5 Solutions

## Problem 1:

For there to be between 20 mA and 100 mA of current flowing, the diode must be forward biased. The diode will take 2 V of $\mathrm{V}_{\text {digital }}$, and the resistor will take the rest of $\mathrm{V}_{\text {digital }}$.

The current flowing through the circuit will thus be
$\mathrm{I}_{\text {diode }}=\left(\mathrm{V}_{\text {digital }}-2 \mathrm{~V}\right) / \mathrm{R}_{\text {limit }}$
$\mathrm{R}_{\text {limit }}$ must be small enough so that $\mathrm{I}_{\text {diode }}$ is at least 20 mA when
 $\mathrm{V}_{\text {digital }}$ is at its smallest, 4 V .
$(4 \mathrm{~V}-2 \mathrm{~V}) / \mathrm{R}_{\text {limit }} \geq 20 \mathrm{~mA}$
$R_{\text {limit }} \leq 100 \Omega$
$\mathrm{R}_{\text {limit }}$ must be big enough so that $\mathrm{I}_{\text {diode }}$ is less than 100 mA when $\mathrm{V}_{\text {digital }}$ is at its biggest, 5 V .
$(5 \mathrm{~V}-2 \mathrm{~V}) / \mathrm{R}_{\text {limit }} \leq 100 \mathrm{~mA}$
$\mathrm{R}_{\text {limit }} \geq 30 \Omega$

Problem 2:


So transistor must be in triode mode.
$I_{D}=-\left(1 \mathrm{~mA} / \mathrm{V}^{2}\right)\left(-2.5 \mathrm{~V}--1 \mathrm{~V}-\left(\mathrm{V}_{\mathrm{DS}} / 2\right)\right)\left(\mathrm{V}_{\mathrm{DS}}\right)$
It's still true that $\quad(4 \mathrm{k} \Omega) \mathrm{I}_{\mathrm{D}}+\mathrm{V}_{\mathrm{DS}}+5 \mathrm{~V}=0$ so substitute to get
$-\left(5 \mathrm{~V}+\mathrm{V}_{\mathrm{DS}}\right) / 4 \mathrm{k} \Omega=-\left(1 \mathrm{~mA} / \mathrm{V}^{2}\right)\left(-2.5 \mathrm{~V}--1 \mathrm{~V}-\left(\mathrm{V}_{\mathrm{DS}} / 2\right)\right)\left(\mathrm{V}_{\mathrm{DS}}\right)$
Solutions: $\mathrm{V}_{\mathrm{DS}}=\{-1 \mathrm{~V},-2.5 \mathrm{~V}\}$
$V_{D S}=-2.5 \mathrm{~V}$ not possible for triode mode (need $\mathrm{V}_{\mathrm{DS}}>\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{T H(\mathrm{p})}$ ) so $\mathrm{V}_{\mathrm{DS}}=-1 \mathrm{~V}$
$I_{D}=-\left(1 \mathrm{~mA} / \mathrm{V}^{2}\right)(-2.5 \mathrm{~V}--1 \mathrm{~V}-(-1 \mathrm{~V} / 2))(-1 \mathrm{~V})=-1 \mathrm{~mA}$
$\mathrm{V}_{\text {out }}=\mathrm{V}_{\mathrm{DS}}+5 \mathrm{~V}=4 \mathrm{~V}$
(Typically, the numbers do not work out this nicely!)
Problem 3:


Notice that no current goes into or out of the gate.
So the 4 k and 6 k transistors have the same current.
Voltage division or Ohm's law tells us that the 4 k resistor has 2 V and the 6 k resistor has 3 V as shown.

So $\mathrm{V}_{\mathrm{GS}}=3 \mathrm{~V}$ (over the 6 k resistor).
The transistor is not cutoff.
Guess saturation mode:
$\mathrm{I}_{\mathrm{D}}=\left(1 \mathrm{~mA} / \mathrm{V}^{2}\right)(1 / 2)(3 \mathrm{~V}-1 \mathrm{~V})^{2}(1+0 \mathrm{~V} \mathrm{VS}) \quad \mathrm{I}_{\mathrm{D}}=2 \mathrm{~mA}$
By KVL, $\quad(1 \mathrm{k} \Omega) \mathrm{I}_{\mathrm{D}}+\mathrm{V}_{\mathrm{DS}}-5 \mathrm{~V}=0$ so $\mathrm{V}_{\mathrm{DS}}=3 \mathrm{~V}$
For saturation, $\mathrm{V}_{\mathrm{DS}}>\mathrm{V}_{G S}-\mathrm{V}_{T H(n)} \quad$ Yes! Saturation was the right guess. $\quad \mathrm{V}_{\text {out }}=\mathrm{V}_{\mathrm{DS}}=3 \mathrm{~V}$
Problem 4:

$\mathrm{V}_{\mathrm{GS}}=1.5 \mathrm{~V} \quad$ Transistor is not cutoff. Guess saturation:
$\mathrm{I}_{\mathrm{D}}=\left(1 \mathrm{~mA} / \mathrm{V}^{2}\right)(1 / 2)(1.5 \mathrm{~V}-1 \mathrm{~V})^{2}\left(1+0 \mathrm{~V}_{\mathrm{DS}}\right) \quad \mathrm{I}_{\mathrm{D}}=125 \mu \mathrm{~A}$
Now we must find $\mathrm{V}_{\mathrm{DS}}$. The only way to do this is by KVL.
Guess that the diode is forward biased (small $\mathrm{V}_{\text {in }}$ means big $\mathrm{V}_{\text {out }}$ for our inverter). For large-signal model, this means diode has $\mathrm{V}_{\mathrm{F}}=2 \mathrm{~V}$ and some positive current. Now starting KVL at ground and going clockwise,
$-V_{D S}+5 k \Omega I_{x}+2 V=0 \quad$ Need to find $I_{x}$.
KVL again, starting at ground, going up diode branch and 1 k resistor branch, dropping back to ground over 5 V source:

$$
-2 \mathrm{~V}-5 \mathrm{~kW} \mathrm{I}_{\mathrm{x}}-1 \mathrm{k} \Omega\left(\mathrm{I}_{\mathrm{x}}+125 \mu \mathrm{~A}\right)+5 \mathrm{~V}=0
$$

$$
I_{x}=479 \mu \mathrm{~A}
$$

$\mathrm{V}_{\mathrm{DS}}=5 \mathrm{k} \Omega \mathrm{I}_{\mathrm{x}}+2 \mathrm{~V} \quad$ (by previous KVL ) $\quad \mathrm{V}_{\mathrm{DS}}=4.395 \mathrm{~V}$
For saturation, $\mathrm{V}_{\mathrm{DS}}>\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{TH}(\mathrm{n})}$ Yes! Saturation was the right guess. $\quad \mathrm{V}_{\text {out }}=\mathrm{V}_{\mathrm{DS}}=4.395 \mathrm{~V}$

