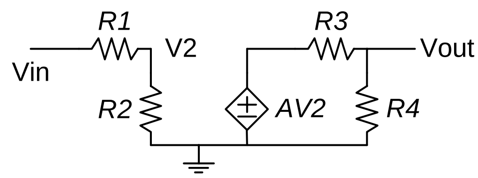
EE42 - Problem Set #7 Solution

7.1.a Voltage divider formed by R1 and R2: $V2 = Vin \cdot \frac{R2}{R1 + R2}$

Voltage gain of VCVS: $A \cdot V2 = Vin \cdot \frac{A \cdot R2}{R1 + R2}$

Voltage divider by R3 and R4: $Vout = A \cdot V2 \cdot \frac{R4}{R3 + R4}$

So, $$Vout = Vin \cdot \frac{A \cdot R2}{R1 + R2} \cdot \frac{R4}{R3 + R4} \quad (1)$$



7.1

7.1.b Let Vout = Vout_DC + ΔVout, Vin = VIn_DC + ΔVin in Eq. (1),

$$Vout\_DC + \Delta Vout = (Vin\_DC + \Delta Vin) \cdot \frac{A \cdot R2}{R1 + R2} \cdot \frac{R4}{R3 + R4}$$

Sort terms of DC and Δ, we get,

$$Vout\_DC = Vin\_DC \cdot \frac{A \cdot R2}{R1 + R2} \cdot \frac{R4}{R3 + R4}$$

$$\Delta Vout = \Delta Vin \cdot \frac{A \cdot R2}{R1 + R2} \cdot \frac{R4}{R3 + R4} \quad (2)$$

Derive the small-signal gain from (2):

$$Aac = \frac{\Delta Vout}{\Delta Vin} = A \cdot \frac{R2}{R1 + R2} \cdot \frac{R4}{R3 + R4}$$

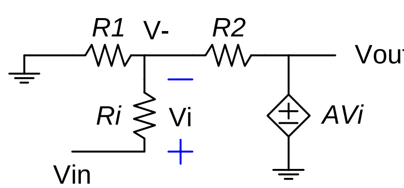
7.2.a KCL at node V- : $$\frac{V_- - V_{out}}{R2} + \frac{V_-}{R1} = 0 \quad (3)$$

We also know: $$V_{out} = A \cdot V_i = A \cdot (V_{in} - V_-) \quad (4)$$

Given $$\frac{V_- - V_{out}}{R2} + \frac{V_-}{R1} = 0$$

$$V_{out} = A \cdot (V_{in} - V_-)$$

$$\text{Find}(V_{out}, V_-) \rightarrow \begin{bmatrix} A \cdot V_{in} \cdot \frac{(R1 + R2)}{(R1 + R1 \cdot A + R2)} \\ R1 \cdot A \cdot \frac{V_{in}}{(R1 + R1 \cdot A + R2)} \end{bmatrix}$$



7.2

So, $$V_{out} = V_{in} \cdot \frac{R1 + R2}{R1 + \frac{R1 + R2}{A}} = 10.999395 \qquad (5)$$

Note: 1) In (5), term contains A in the denominator is the error term due to finite gain of VCVS.
2) If A = ∞, then Vout reduces to the form assuming op amp is ideal.

7.2.b A is large, so the term contains A in the denominator is small compared to R1. Apply Taylor expansion to (5), we have:

$$V_{out} = V_{in} \cdot \frac{R1 + R2}{R1 + \frac{R1 + R2}{A}} = V_{in} \cdot \left(\frac{R1 + R2}{R1}\right) \cdot \left(1 - \frac{R1 + R2}{A \cdot R1}\right)$$

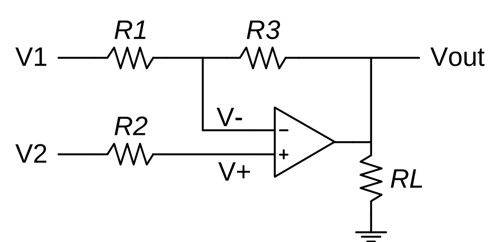
The fraction of deviation from ideal gain is: $$\frac{\Delta V_{out}}{V_{out}(ideal)} = -\frac{R1 + R2}{A \cdot R1} = \frac{-1}{A \cdot \frac{R1}{R1 + R2}} = -5.5 \cdot 10^{-5} \qquad (6)$$

Note: The term in the denominator of (6) is called "loop gain", i.e., the gain derived when going around the feedback loop of the op amp, including the resistive network forming a divider.

7.3.a Assuming ideal op amp, 1) V+ = V-; 2) no current into + and - terminals, so V- = V+ = V2 holds.

KCL at node V-, we get: $$\frac{V2 - V1}{R1} + \frac{V2 - Vout}{R3} = 0$$

Solve, we have: $$Vout = V2 \cdot \left(1 + \frac{R3}{R1}\right) - V1 \cdot \frac{R3}{R1}$$

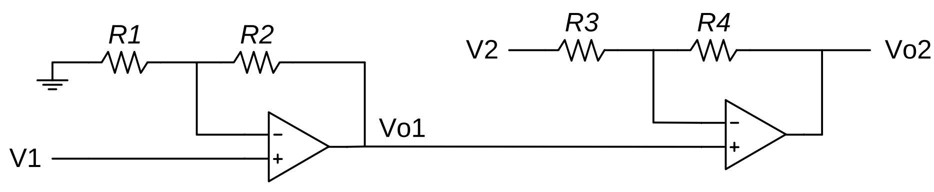


7.3

7.3.b Op amp is ideal, no current flowing through R2, so V+ = V2 holds independent to the value of R2.

7.3.c VCVS is a voltage source, so RL at output in parallel to the VCVS will not affect the its voltage.

7.4.a



$$Vo1 = V1 \cdot \left(1 + \frac{R2}{R1}\right)$$

7.4.b $$Vo2 = Vo1 \cdot \left(1 + \frac{R4}{R3}\right) - V2 \cdot \frac{R4}{R3} = V1 \cdot \left(1 + \frac{R2}{R1}\right) \cdot \left(1 + \frac{R4}{R3}\right) - V2 \cdot \frac{R4}{R3}$$

7.5.a KCL at node V2: $$\frac{Vin - V2}{R1} + I_B - \frac{V2}{R2} = 0$$

KCL at node Vout: $$\frac{Vout}{R4} + \frac{Vout - Vcc}{R3} + gm \cdot V2 = 0$$

Given $$\frac{Vin - V2}{R1} + I_B - \frac{V2}{R2} = 0$$

$$\frac{Vout}{R4} + \frac{Vout - Vcc}{R3} + gm \cdot V2 = 0$$

$$\text{Find}(Vout, V2) \rightarrow \begin{bmatrix} -R4 \cdot \dfrac{\left(-Vcc \cdot R1 - Vcc \cdot R2 + gm \cdot R2 \cdot R3 \cdot Vin + gm \cdot R2 \cdot R3 \cdot R1 \cdot I_B\right)}{(R3 \cdot R1 + R3 \cdot R2 + R4 \cdot R1 + R4 \cdot R2)} \\ R2 \cdot \dfrac{\left(Vin + R1 \cdot I_B\right)}{(R1 + R2)} \end{bmatrix}$$

So, $$Vout = \frac{-gm \cdot R2 \cdot R3 \cdot R4 \cdot \left(Vin + I_B \cdot R1\right)}{(R1 + R2) \cdot (R3 + R4)} + \frac{Vcc \cdot R4}{R3 + R4} \qquad (7)$$

Note: If superposition is applied to the analysis, it is very easy to eyeball the solution in (7) consisting of 3 terms:
1) Vin term, due to the cascade of divider R1 & R2, VCCS gain gm, output resistance R3 || R4;
2) IB term, silimar to Vin, but need to go through a Thevenin equivalent transform at Vin;
3) Vcc term, simply a divider formed by R3 and R4.

7.5.b Set IB = Vcc = 0, rewrite KCL's

KCL at node V2: $$\frac{Vin - V2}{R1} - \frac{V2}{R2} = 0$$

KCL at node Vout: $$\frac{Vout}{R4} + \frac{Vout}{R3} + gm \cdot V2 = 0$$

Solve, we have $$Vout = \frac{-gm \cdot R2 \cdot R3 \cdot R4 \cdot Vin}{(R1 + R2) \cdot (R3 + R4)} \qquad (8)$$

7.5.c If we set IB = Vcc = 0 in (7), we see we get exactly (8).