EE42 - Problem Set #5 Solution

5.1.a KCL for Vb, $\frac{Vaa - Vb}{R1} + \frac{Vc - Vb}{R3} = 0$ (1)

KCL for Vc, $\frac{Vaa - Vc}{R2} + \frac{Vb - Vc}{R3} - \frac{Vc}{R4} = 0$ (2)

5.1.b Let R1+R3 = R5 = 4kΩ, redraw the circuit

KCL for Vc, $\frac{Vaa - Vc}{R5} + \frac{Vaa - Vc}{R2} - \frac{Vc}{R4} = 0$ (3)

5.1.c Plug (1) into (2), we get

$$\frac{Vaa - Vc}{R5} + \frac{Vaa - Vc}{R2} - \frac{Vc}{R4} = 0$$ same as (3)

5.1.d $R1 := 1 \quad R2 := 2 \quad R3 := 3 \quad R4 := 4 \quad Vaa := 3$

$R5 := R1 + R3$

Given $\frac{Vaa - Vc}{R5} + \frac{Vaa - Vc}{R2} - \frac{Vc}{R4} = 0 \qquad \text{Find}(Vc) \rightarrow \frac{9}{4}$

Vc = 9/4 (V)

5.1.e

Given $\frac{Vaa - Vb}{R1} + \frac{Vc - Vb}{R3} = 0$

$\frac{Vaa - Vc}{R2} + \frac{Vb - Vc}{R3} - \frac{Vc}{R4} = 0 \qquad \text{Find}(Vb, Vc) \rightarrow \begin{pmatrix} \frac{45}{16} \\ \frac{9}{4} \end{pmatrix}$

Vb = 45/16 (V)

5.2.a Treat nodes Va and Vb as supernode

$$\frac{Vaa - Va}{R1} - \frac{Va}{R2} - \frac{Vb}{R3} = 0$$

constraint $Va - Vb = Vbb$

5.2.b $Vaa := 2 \quad Vbb := 1 \quad R1 := 1 \quad R2 := 2 \quad R3 := 3$

Given $\frac{Vaa - Va}{R1} - \frac{Va}{R2} - \frac{Vb}{R3} = 0$

$Va - Vb = Vbb \qquad \text{Find}(Va, Vb) \rightarrow \begin{pmatrix} \frac{14}{11} \\ \frac{3}{11} \end{pmatrix}$

5.2.b (cont'd) Va = 14/11 (V)

5.2.c Treat nodes Va and Vb as supernode

$$Iss - \frac{Va}{R2} - \frac{Vb}{R3} = 0$$

constraint $Va - Vb = Vbb$

5.2.d $Vaa := 2 \quad Vbb := 1 \quad R1 := 1 \quad R2 := 2 \quad R3 := 3 \quad Iss := 1$

Given

$$Iss - \frac{Va}{R2} - \frac{Vb}{R3} = 0$$

$$Va - Vb = Vbb$$

$$\text{Find}(Va, Vb) \rightarrow \begin{pmatrix} \frac{8}{5} \\ \frac{3}{5} \end{pmatrix}$$

Vb = 3/5 (V)

5.3.a KCL for Va, $\frac{Vaa - Va}{R1} + \frac{Vb - Va}{R5} - \frac{Va}{R2} = 0$

KCL for Vb, $\frac{Vaa - Vb}{R3} + \frac{Va - Vb}{R5} - \frac{Vb}{R4} = 0$

5.3.b-c $Vaa := 3 \quad R1 := 1 \quad R2 := 2 \quad R3 := 2 \quad R4 := 4 \quad R5 := 5$

Given

$$\frac{Vaa - Va}{R1} + \frac{Vb - Va}{R5} - \frac{Va}{R2} = 0$$

$$\frac{Vaa - Vb}{R3} + \frac{Va - Vb}{R5} - \frac{Vb}{R4} = 0$$

$$\text{Find}(Va, Vb) \rightarrow \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

Va = Vb = 2 (V)

5.3.d Remove R5 from the Wheatstone bridge, now we get two individual branches of the original circuit, each one forms a voltage divider. We can calculate voltages at node Va and Vb as follows:

$$Va = \frac{R2}{R1 + R2} \cdot Vaa = \frac{1}{\frac{R1}{R2} + 1} \cdot Vaa \qquad Vb = \frac{R4}{R3 + R4} \cdot Vaa = \frac{1}{\frac{R3}{R4} + 1} \cdot Vaa$$

So, when $\frac{R1}{R2} = \frac{R3}{R4}$ holds, $Va = Vb$ also holds.

When all these hold, Va and Vb are equal-potential, i.e., there will be no current following through R5 if it is put back into the circuit.

5.4.a KCL for Vb $\frac{V1 - Vb}{R1} + I2 = 0$

$V1 := 2 \quad R1 := 1 \quad I2 := 1$

Given $\frac{V1 - Vb}{R1} + I2 = 0$ $\quad$ $Find(Vb) \rightarrow 3$

Vb = 3 (V)

5.4.b Short V1 and open I2, the circuit simplifies into R1 in series with R2, thus the Thevenin resistance looking into port Vout is:

$$Rth(Vout) = R1 + R2 = 3(k\Omega)$$

5.5 $Vc(t = 0) = \frac{R2}{R1 + R2} \cdot Vs = 2(V)$

$Vc(t = \infty) = 6(V)$

In general, $Vc(t) = A + Be^{\frac{-t}{\tau}}$ for t >= 0

evaluate Vc(t) for t=0, we get $A + B = 2$

evaluate Vc(t) for t=infinity, we get $A = 6$ so $B = -4$

By inspection, we also know $\tau = R1 \cdot C = 4(ns)$

So, we have $Vc(t) = 6 - 4e^{\frac{-t}{4ns}}$ for t >= 0