## Solution Problem Set \# 3 (by Farinaz Koushanfar)

3.1 Capacitance.
a) Charge accumulation rate x time $=\mathrm{Q}(\mathrm{t})=\left(9 \mathrm{x} 10^{5} \mathrm{~m} / \mathrm{hr}\right)(1 / 3600 \mathrm{~s} / \mathrm{hr})\left(10^{17} \mathrm{e} / \mathrm{m}\right)\left(-1.6 \times 10^{-}\right.$ $\left.{ }^{19} \mathrm{C} / \mathrm{e}\right) \mathrm{t}=4 \mathrm{t} \mathrm{C}$.
b) $\mathrm{V}(\mathrm{t})=\left(1 / \mathrm{C}_{\text {PLANE }}\right) \mathrm{Q}(\mathrm{t})=10^{+3} \times 4 \mathrm{t}=4.0 \times 10^{3} \mathrm{t} \mathrm{V}$.
c) $100 \mathrm{KV} /\left(4.0 \times 10^{3} \mathrm{~V} / \mathrm{s}\right)=25 \mathrm{~s}$
d) $(1 / 2) \mathrm{CV}^{2}=(1 / 2)\left(10^{-3}\right)\left(10^{5}\right)^{2}=5.0 \mathrm{MJ}$

### 3.2 Capacitance and voltage versus time.

a) From 0 to $2 \mu \mathrm{~s}, \mathrm{v}(\mathrm{t})=\mathrm{V}(\mathrm{t}=0)+(1 / \mathrm{C}) \mathrm{i}(\mathrm{t}) \mathrm{t}=0+\left(1 / 10^{-12}\right)\left(1 \times 10^{-6}\right) \mathrm{t}=0+1 \mathrm{t}(\mathrm{V})$

From 2 to $4 \mu \mathrm{~s}, \mathrm{v}(\mathrm{t})=\mathrm{V}(\mathrm{t}=2)+(1 / \mathrm{C}) \mathrm{i}(\mathrm{t})(\mathrm{t}-2 \mathrm{us})=2+\left(1 / 10^{-12}\right)\left(2 \times 10^{-6}\right)(\mathrm{t}-2 \mu \mathrm{~s})$
$=2+2(\mathrm{t}-2 \mu \mathrm{~s})(\mathrm{V})$
From 4 to $7 \mu \mathrm{~s}, \mathrm{v}(\mathrm{t})=\mathrm{V}(\mathrm{t}=4)+(1 / \mathrm{C}) \mathrm{i}(\mathrm{t})(\mathrm{t}-4 \mu \mathrm{~s})=6+\left(1 / 10^{-12}\right)\left(-4 \times 10^{-6}\right)(\mathrm{t}-2 \mu \mathrm{~s})=6-4(\mathrm{t}-$ $4 \mu \mathrm{~s}$ ) (V)
If $v(t)=0 \Rightarrow 6-4(t-4 \mu s)=0 \Rightarrow 22-4 t=0 \Rightarrow t=5.5 \mu s$
b) $\mathrm{E}=\stackrel{\mathrm{v}=\mathrm{V}_{\text {Final }} \mathrm{Cv} \mathrm{Cv}=\frac{1}{2} \mathrm{CV}_{\text {Final }} 2-\frac{1}{2} \mathrm{CV}_{\text {Initial }} 22=(1 / 2) \times 10^{-12} \times(4-0)=2 \mathrm{pJ}, \mathrm{V}_{\text {Initial }}}{ }$

d) $E=\frac{1}{2} \mathrm{C}\left(\mathrm{V}_{\text {Final }}-V_{\text {Initial }}\right)^{2}=(1 / 2)\left(10^{-12}\right)(6-2)^{2}=8 \mathrm{pJ}$ which is NOT correct!

Since $\left(\mathrm{V}_{\text {Final }}-\mathrm{V}_{\text {Initial }}\right)^{2}$ is not the solution to the integral

$$
\mathrm{E}=\int_{0}^{\infty} \mathrm{i}(\mathrm{t}(. \mathrm{v}(\mathrm{t}) \cdot \mathrm{dt}
$$

3.3 Equivalent Circuits.
a) For $\mathrm{V}_{\text {OUT }}=1.2 \mathrm{~V}$, we know from last weeks HW that this is the open circuit voltage and so $\mathrm{I}_{\mathrm{OUT}}=0$. This means that there is really no external element and the circuit to the left of $V_{\text {OUT }}$ can

$\mathrm{I}_{\mathrm{SS}}=1 \mathrm{~mA}, \mathrm{R}_{1}=1 \mathrm{k} \Omega, \mathrm{R}_{2}=2 \mathrm{k} \Omega, \mathrm{R}_{3}=3 \mathrm{k} \Omega$
Fig. P3.3 be analyzed by itself. $\mathrm{I}_{\mathrm{SS}} \times \mathrm{R} 2 \| \mathrm{R} 3=1.2 \mathrm{~V}$ as expected.
For $\mathrm{V}_{\text {Out }}=0.6 \mathrm{~V}$ the current through R 1 plus through R 2 is:
$(0.6 \mathrm{~V} / 2 \mathrm{k} \Omega)+(0.6 \mathrm{~V} / 3 \mathrm{k} \Omega)=0.3 \mathrm{~mA}+0.2 \mathrm{~mA}=0.5 \mathrm{~mA} \Rightarrow \mathrm{I}_{\text {OUT }}=1 \mathrm{~mA}-0.5 \mathrm{~mA}=0.5 \mathrm{~mA}$.
For $\mathrm{V}_{\text {Out }}=0 \mathrm{~V}$ there is no voltage drop on R 2 or $\mathrm{R} 3 \Rightarrow$ there is no current through them.
This greatly simplifies the circuit to the left by removing R1 and R2 and $\mathrm{I}_{\mathrm{OUT}}=1.0 \mathrm{~mA}$.
b) The short circuit case $\mathrm{V}_{\text {out }}=0 \mathrm{~V}$ was easiest followed by the open circuit case $\mathrm{V}_{\text {out }}=1.2 \mathrm{~V}$. The solution technique for $\mathrm{V}_{\text {OUT }}=0.6 \mathrm{~V}$ is completely general and could be used for any output voltage.
c) $\mathrm{I}_{\text {OUT }}=1 \mathrm{~mA}-\mathrm{V}_{\text {OUT }} /(\mathrm{R} 2 \| \mathrm{R} 3)=\left(1-\mathrm{V}_{\text {OUT }} /(1.2 \mathrm{~K} \Omega)\right)$
lout (A)

d) Slope $=10^{-3} / 1.2(\mathrm{~A} / \mathrm{V})$. The inverse of the slope gives $1.2 \mathrm{k} \Omega$ and the voltage is 1.2 V .
e) Show that $\mathrm{R}_{\mathrm{T}}=-\mathrm{V}_{\mathrm{OC}} / \mathrm{I}_{\mathrm{SC}}=-(1.2 \mathrm{~V} /-1 \mathrm{~mA})=1.2 \mathrm{k} \Omega$.
f) This value is $R_{2} \| R_{3}$. Turning the current source to zero makes $I_{S S}=0$ and there is no current path through $\mathrm{I}_{\mathrm{Ss}}$. Thus only $\mathrm{R}_{2}$ and $\mathrm{R}_{3}$ are in the circuit in this case.

### 3.4 Equivalent Circuits and Ideal Supplies.

a) $\mathrm{V}_{\mathrm{OC}}=\mathrm{V}_{\mathrm{AA}}=1 \mathrm{~V}$ as there is no current through $\mathrm{R}_{2} . \quad \mathrm{V}_{\mathrm{AA}}=1 \mathrm{~V}, \mathrm{I}_{\mathrm{SS}}=1 \mathrm{~mA}, \mathrm{R}_{1}=1 \mathrm{k} \Omega, \mathrm{R}_{2}=2 \mathrm{k} \Omega$
b) In the book $\mathrm{I}_{\mathrm{SC}}$ flows into the output. $\mathrm{I}_{\mathrm{SC}}=\left(-\mathrm{V}_{\mathrm{AA}} / \mathrm{R}_{2}\right)=(-1 \mathrm{~V} / 2 \mathrm{k} \Omega)=-0.5 \mathrm{~mA}$.
c) $\mathrm{I}_{\mathrm{N}}=-\mathrm{I}_{\mathrm{SC}}=0.5 \mathrm{~mA} . \mathrm{R}_{\mathrm{N}}=\mathrm{R}_{\mathrm{TH}}=-\left(\mathrm{V}_{\mathrm{OC}} / \mathrm{I}_{\mathrm{SC}}\right)=-(1 \mathrm{~V} /(-0.5 \mathrm{~mA}))=2 \mathrm{k} \Omega$.
d) $\mathrm{V}_{\mathrm{SS}}=\mathrm{I}_{\mathrm{SS}} \times \mathrm{R}_{1}+\mathrm{V}_{\mathrm{AA}}=1 \mathrm{~mA} \times 1 \mathrm{k} \Omega+1 \mathrm{~V}=2 \mathrm{~V}$.
e) $\mathrm{I}_{\mathrm{AA}}=\mathrm{I}_{\mathrm{SS}}-$ Current through $\mathrm{R}_{2}=1 \mathrm{~mA}-\mathrm{V}_{\mathrm{AA}} / \mathrm{R}_{2}$ $=1 \mathrm{~mA}-1 \mathrm{~V} / 2 \mathrm{k} \Omega=0.5 \mathrm{~mA}$.
3.5 Nonlinear Load and Power. Use a graphical method to find the solution for the following

Fig. P3. 4 loads when attached to $\mathrm{V}_{\text {Out }}$ in Fig. P3.3. For a) The answer is around 0.9 V and 0.225 mA with 0.20 mW into the load. For b) it is about $0.7 \mathrm{~V}, 0.4 \mathrm{~mA}$ and 0.28 mW . For c ) it is at about 1.35 V , 0.2 mA and 0.3 mW .


