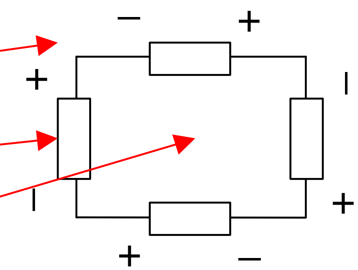
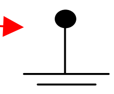
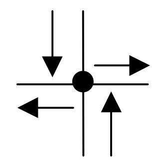
CIRCUIT ELEMENTS AND CIRCUIT ANALYSIS



Lecture 5 review:

- Terminology: Nodes and branches
- Introduce the implicit reference (common) node – defines node voltages
- Introduce fundamental circuit laws: Kirchhoff's Current and Voltage Laws





Today:

- Formal nodal analysis
- Resistors in series and parallel and the voltage divider

Rationale:

- Method applies to circuits of any size, dc or transient analysis
- Approach is algorithmic; Same as Spice or other circuit analysis programs

FORMAL CIRCUIT ANALYSIS

Systematic approaches to writing down KCL and KVL: Section 2.3 of Text - In particular use of KCL gives NODAL ANALYSIS

Mathematical foundation is rigorous: EECS 104

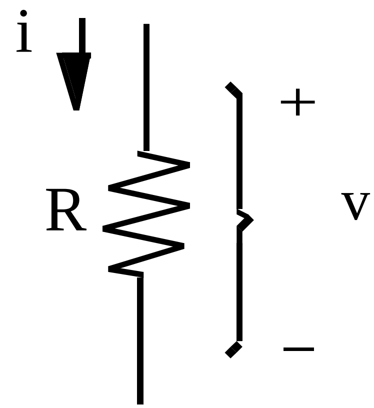
Nodal Analysis: Node voltages are the unknowns

Mesh Analysis: Branch currents are the unknowns

Use one **or** the other for circuit analysis

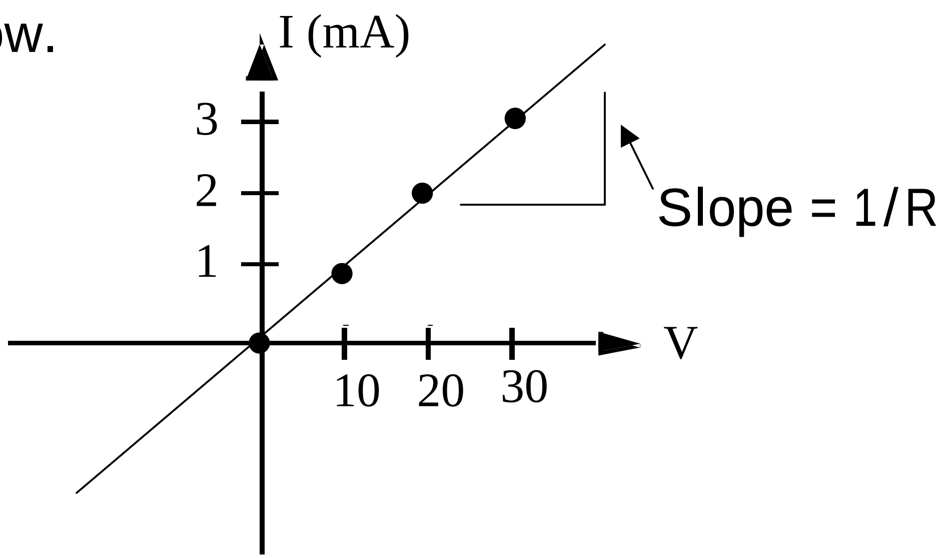
We will do only nodal analysis – (because voltages make more convenient variables than currents)

REVIEW OF IV CHARACTERISTICS OF A RESISTOR



If we use associated current and voltage (i.e., i is defined as into + terminal), then v = iR (Ohm's law)

Question: What is the I-V characteristic for a 10KΩ resistor? Draw on axes below.



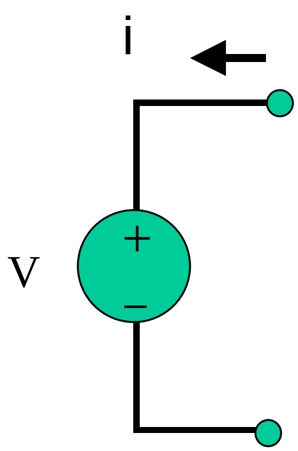
☞Answer: $V = 0 \Rightarrow I = 0$

$V = IR \Rightarrow I = 1$ mA when V = 10V

etc, etc

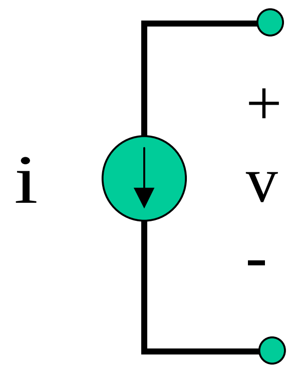
REVIEW OF CURRENT-VOLTAGE CHARACTERISTICS OF VOLTAGE & CURRENT SOURCES

Describe a two-terminal circuit element by plotting current vs. voltage



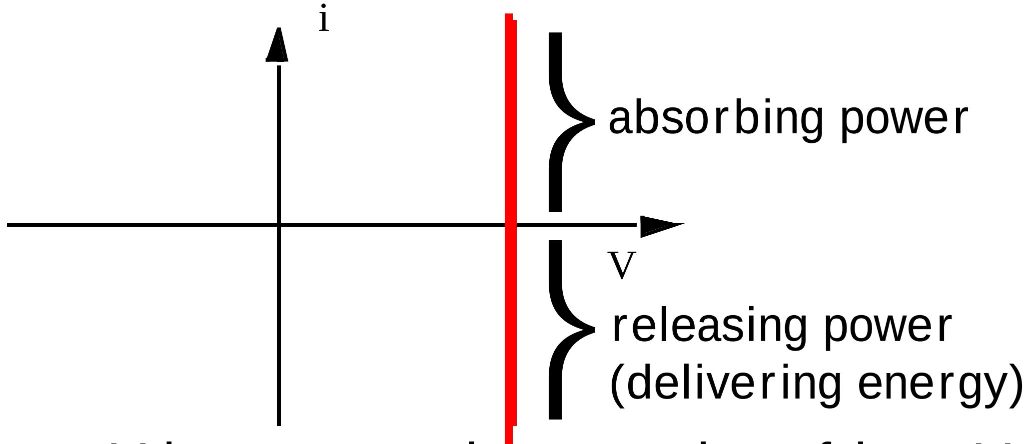
Ideal voltage source

If we use associated signs so i goes into + terminal

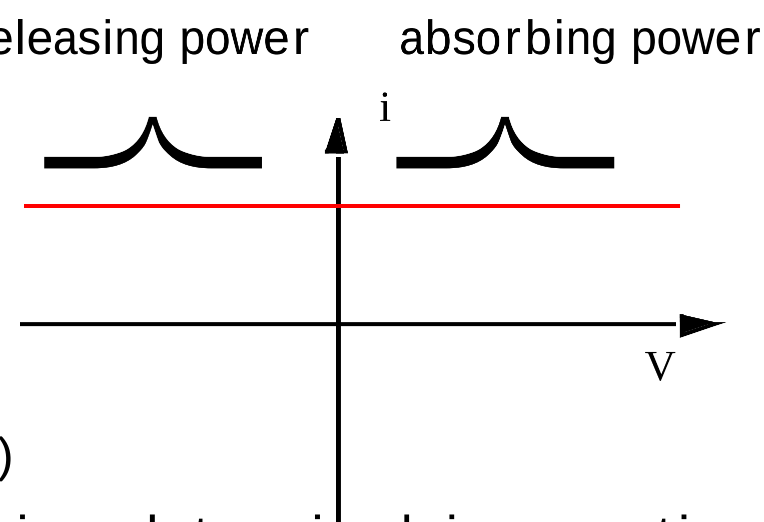


Ideal current source

If we assume associated signs so i flows into + terminal



V is constant, irrespective of i



V is undetermined, irrespective of i

Now lets move on to learning about a technique for analyzing circuits of arbitrary complexity (i.e. lots of nodes and branches).

Our preferred techinque is called Nodal Analysis and is simply the repeated application of Kirchoff's current law.

  W. G. Oldham

FORMAL CIRCUIT ANALYSIS USING KCL: NODAL ANALYSIS

(Memorize these steps and apply them rigorously!)

1 Choose a Reference Node

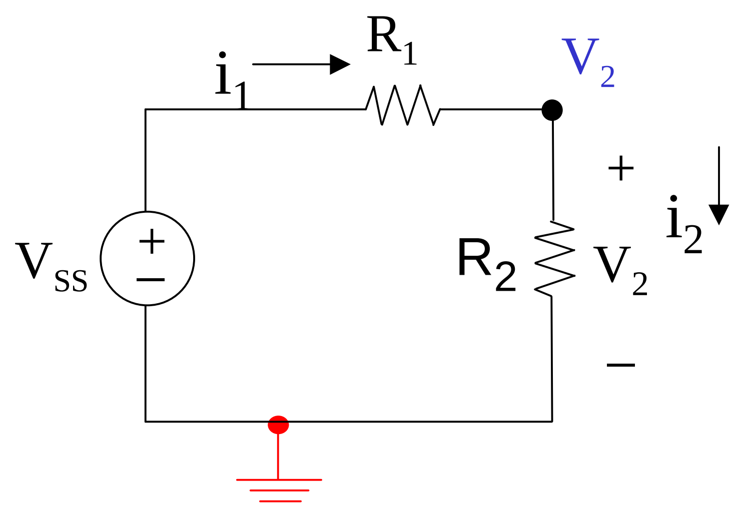
2 Define unknown node voltages (those not fixed by voltage sources)

3 Write KCL at each unknown node, expressing current in terms of the node voltages (using the constitutive relationships of branch elements*)

4 Solve the set of equations (N equations for N unknown node voltages)

* with inductors or floating voltages we will use a modified Step 3

NODAL ANALYSIS USING KCL
– The Voltage Divider –



1 Choose reference node

2 Define unknown node voltages

3 Write KCL at unknown nodes

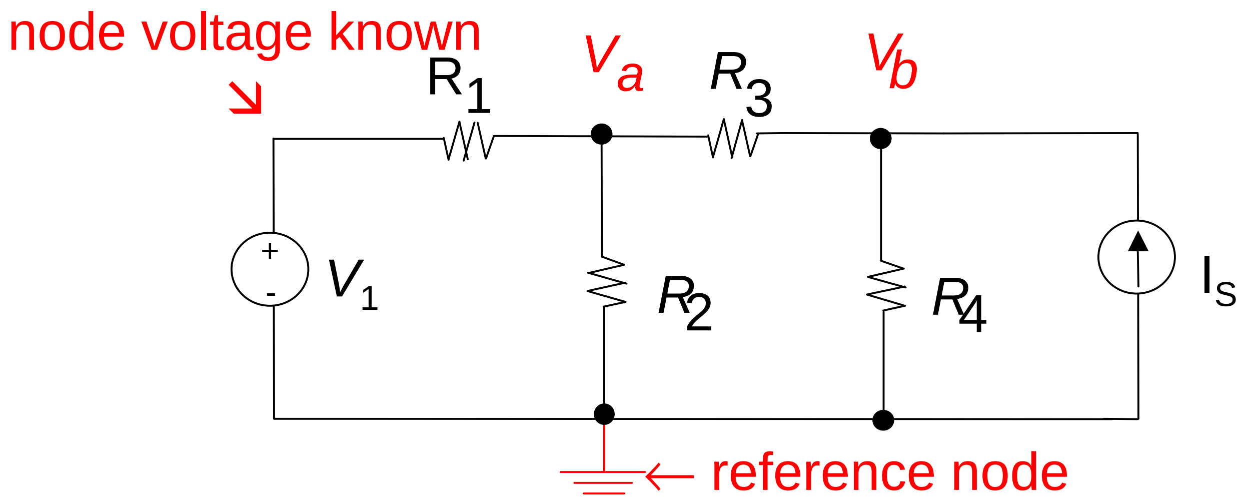
$$\boxed{\frac{V_{SS}-V_2}{R_1}=\frac{V_2-0}{R_2}}$$

4 Solve:

$$V_2 = V_{SS}\cdot\frac{R_2}{R_1+R_2}$$

EXAMPLE OF NODE ANALYSIS

Choose a reference node and define the node voltages (except reference node and the one set by the voltage source);



Apply KCL:

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_b}{R_3} + \frac{V_a}{R_2} = 0$$

$$\frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} - I_S = 0$$

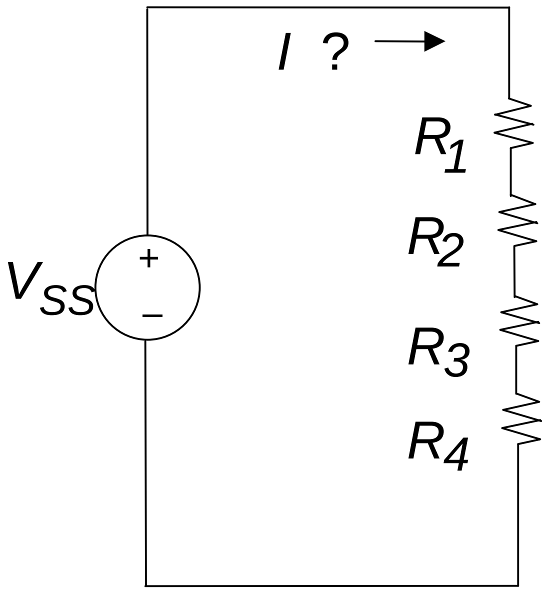
You can solve for V_a, V_b.

What if we used different ref node?

RESISTORS IN SERIES

(Here its more convenient to use KVL than node analysis)

Circuit with several resistors in series – Can we find an equivalent resistance?



- KCL tells us same current flows through every resistor
- KVL tells us

$$I \cdot R_1 + I \cdot R_2 + I \cdot R_3 + I \cdot R_4 = V_{SS}$$

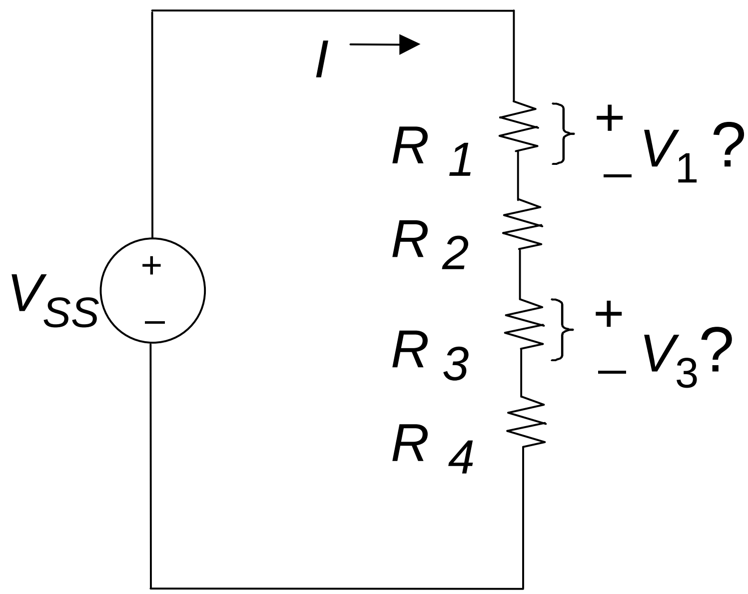
- Clearly,

$$I = V_{SS}/(R_1 + R_2 + R_3 + R_4)$$

☞ Thus, equivalent resistance of resistors in series is the simple sum

GENERALIZED VOLTAGE DIVIDER

Circuit with several resistors *in series*



- We know

$$I = V_{SS}/(R_1 + R_2 + R_3 + R_4)$$

- Thus,

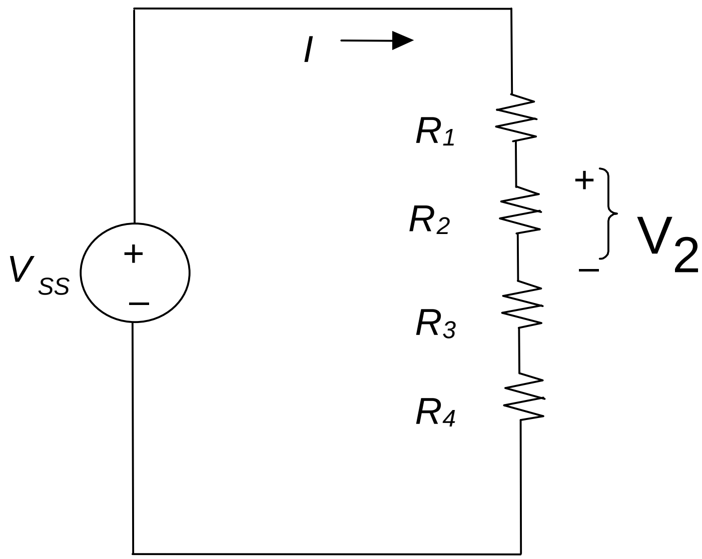
$$V_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

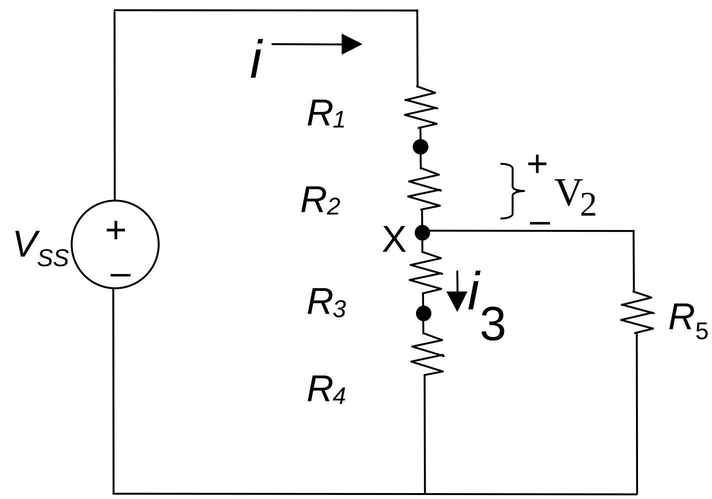
and

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

etc.. etc..

WHEN IS VOLTAGE DIVIDER FORMULA CORRECT?





$$V_2 = \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

Correct if nothing else connected to nodes

$$V_2 \neq \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

because R_5 removes condition of resistors in series – i.e. $i_3 \neq I$

What is V_2?

Answer: $$\frac{R_2}{R_1 + R_2 + R_5 \| (R_3 + R_4)} \cdot V_{SS}$$