

## Charging and discharging in RC Circuits (continued)

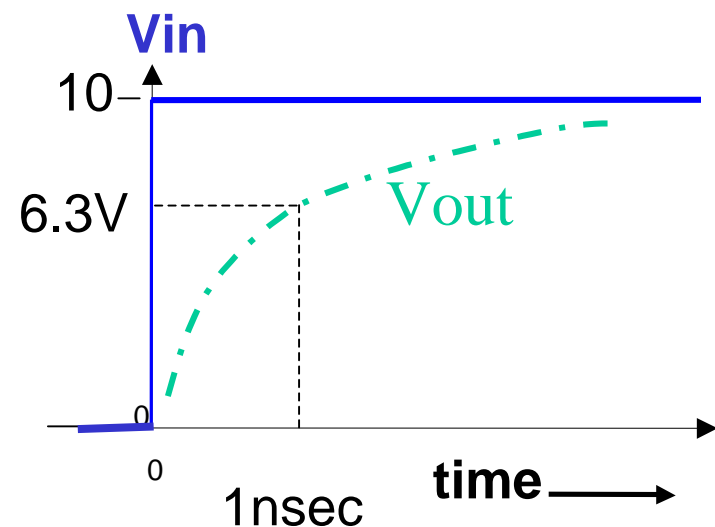
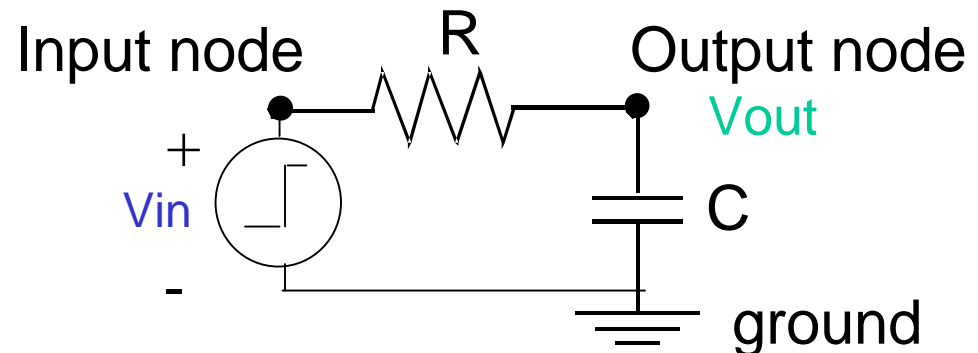
### Last Time:

We learned that simple the simple RC circuit with a step input has a universal exponential solution of the form:

$$V_{\text{out}} = A + Be^{-t/RC}$$

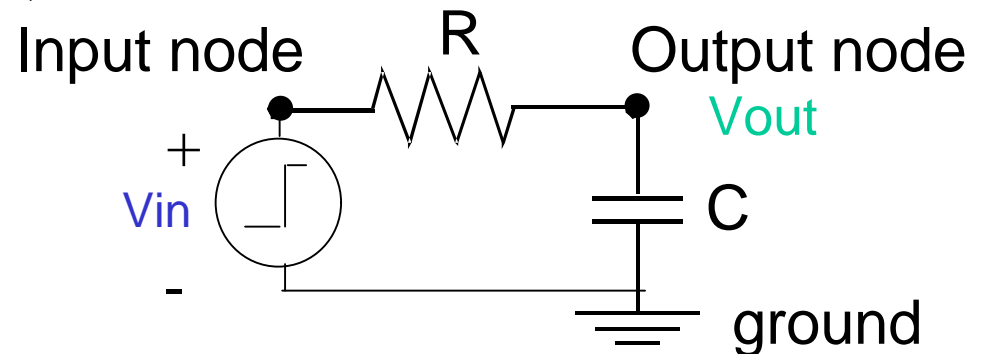
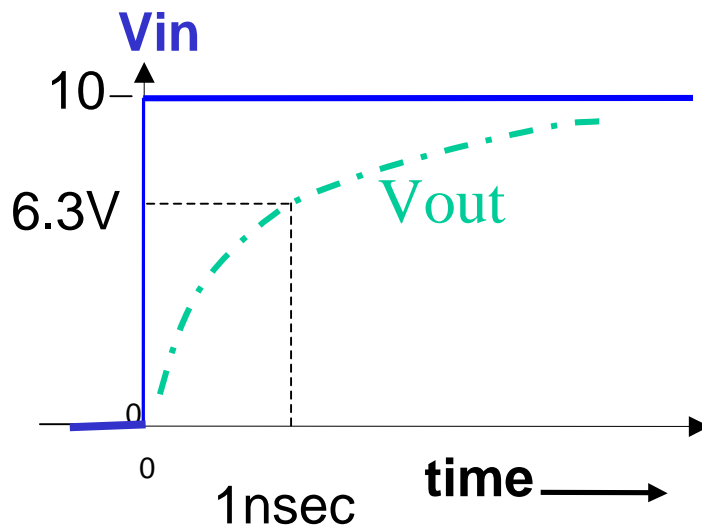
Example:  $R = 1\text{K}$ ,  $C = 1\text{pF}$ ,  $V_{\text{in}}$  steps from zero to 10V at  $t=0$ :

- Initial value of  $V_{\text{out}}$  is 0
- Final value of  $V_{\text{out}}$  is 10V
- Time constant is  $10^{-9}$  sec
- $V_{\text{out}}$  reaches  $0.63 \times 10$  in  $10^{-9}$  sec



## Charging and discharging in RC Circuits

For this example:  $R = 1\text{K}$ ,  $C = 1\text{pF}$ ,  
 $V_{in}$  steps from zero to 10V at  $t=0$ :



Note that we found this graph without even using the equation

$$V_{out} = A + Be^{-t/RC} \quad (\text{That is we did not try to evaluate } A \text{ and } B).$$

We simply used the dc solution for  $t < 0$  and the dc solution for  $t \gg 0$  to get the limits and we used the time constant to get the horizontal scale. We only need the equation to remind us the solution is an exponential. So this will be the basis of our **easy method**.

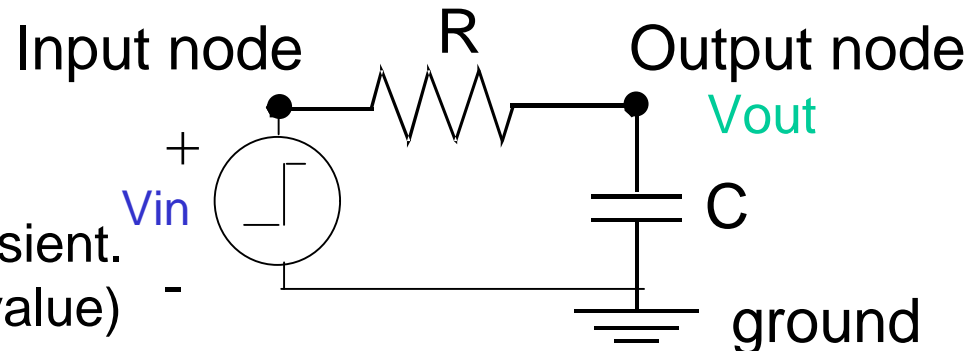
## Charging and discharging in RC Circuits (The official EE42 Easy Method)

**Method of solving for any node voltage in a single capacitor circuit.**

1) Simplify the circuit so it looks like one resistor, a source, and a capacitor (it will take another two weeks to learn all the tricks to do this.) But then the circuit looks like this:

2) The time constant of the transient is  $\tau = RC$ .

3) Solve the dc problem for the capacitor voltage before the transient. This is the starting value (initial value) for the transient voltage.



4) Solve the dc problem for the capacitor voltage after the transient is over. This is the asymptotic value.

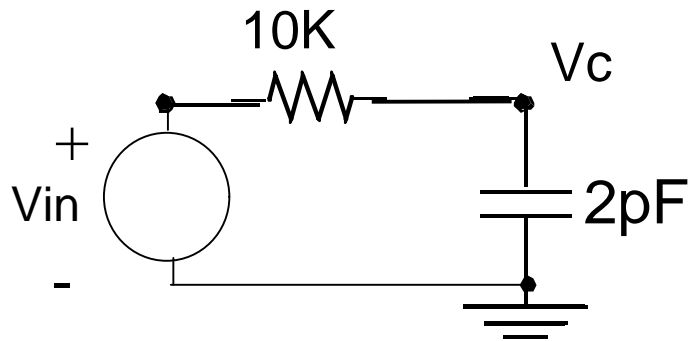
5) Sketch the Transient. It is 63% complete after one time constant.

6) Write the equation by inspection.

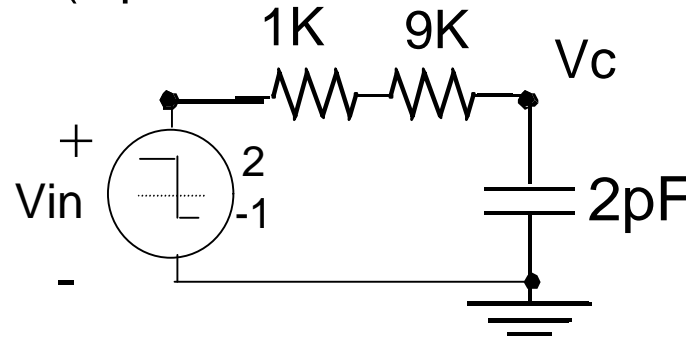
## Charging and discharging in RC Circuits (Example 1 of the EE42 Easy Method)

Find  $V_c(t)$  for the following circuit: (input switches from 2V to -1V at  $t = 0$ )

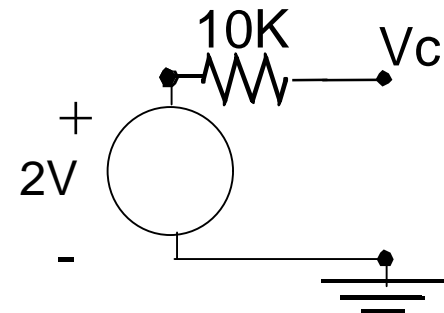
1) Simplify the circuit :



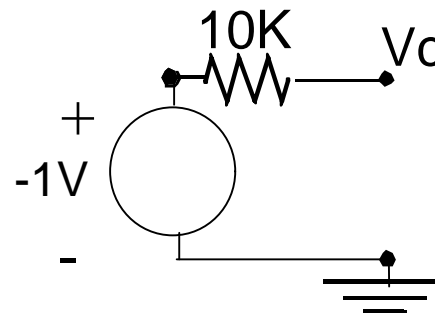
2) The time constant of the transient is  $\tau = RC = 20\text{nsec}$



3) Before the transient  
 $V_{in} = 2V$  so  $V_c = 2V$



4) After the transient is over  $V_{in} = -1V$  so  $V_c = -1V$ . This is the asymptotic value.

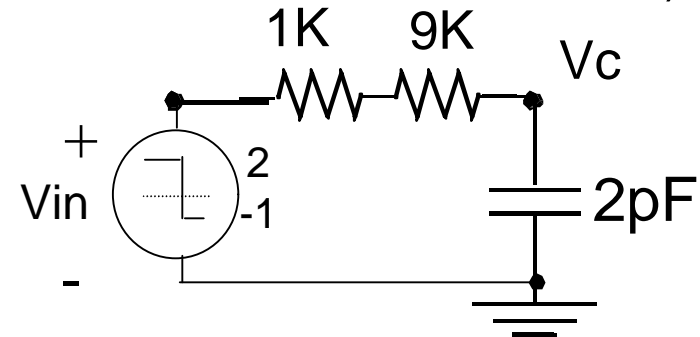


## Charging and discharging in RC Circuits (Example 1 of the EE42 Easy Method)

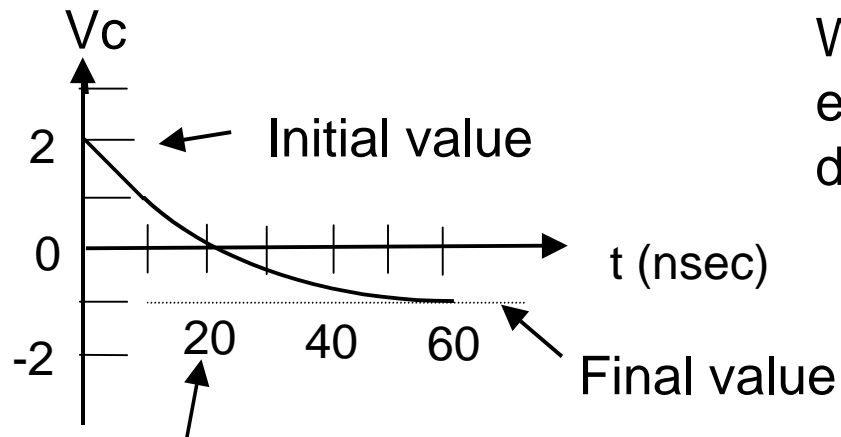
Find  $V_c(t)$  for the following circuit: (input switches from 2V to -1V at  $t = 0$ )

We have : Initial value of  $V_c$  is 2V,  
final value is -1V and  $\tau = 20\text{nsec}$

5) Sketch  $V_c(t)$  :



What is the equation for an exponential beginning at 2V, decaying to -1V, with  $\tau = 20\text{nsec}$ ?

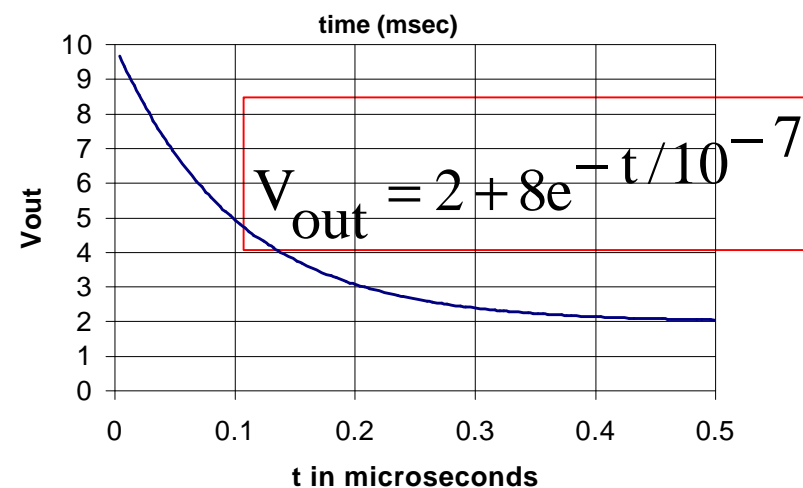
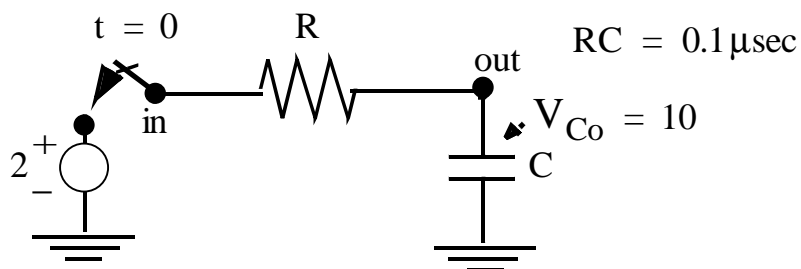
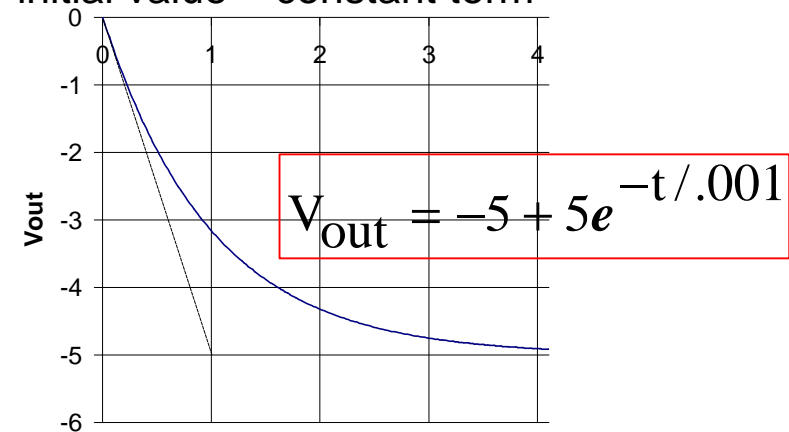
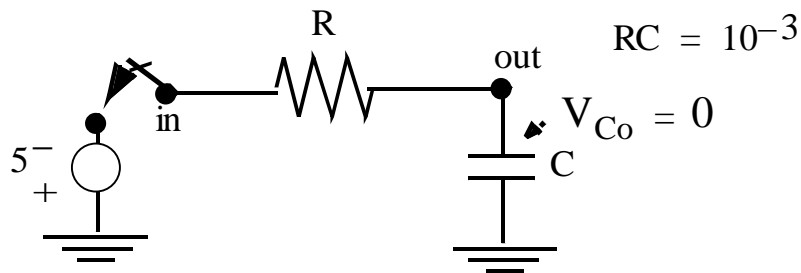


37% of transient  
remaining at one  
time constant

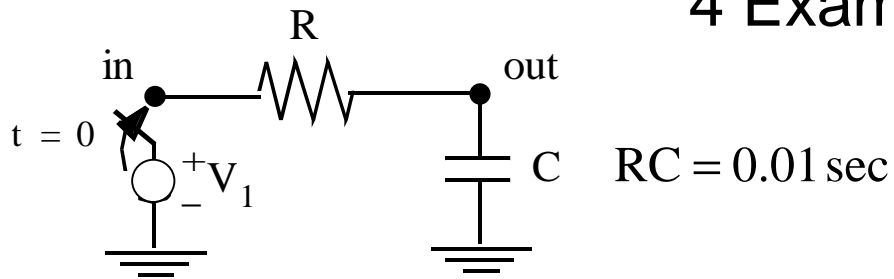
$$V_c(t) = -1 + 3e^{-t/\tau}$$

## OUR METHOD AVOIDS ALL MATH!

- ① Sketch waveform (starts at  $V_{Co}$ , ends asymptotically at  $V_1$ , initial slope intersects at  $t = RC$  or transient is 63% complete at  $t=RC$ )
- ② Write equation: **2a.** constant term  $A = \text{limit of } V \text{ as } t \rightarrow \infty$   
**2b.** pre-exponent  $B = \text{initial value} - \text{constant term}$



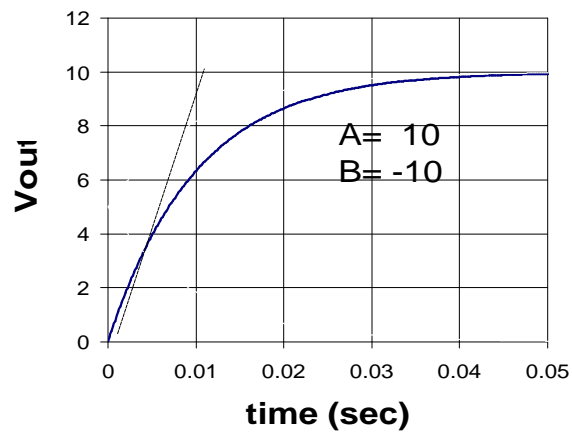
## 4 Examples



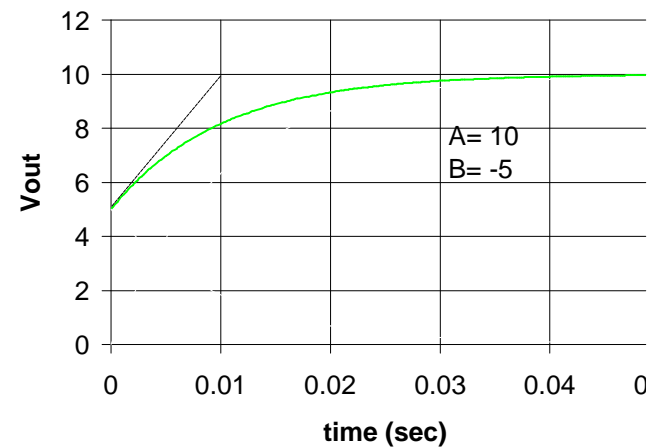
$$V_{\text{out}} = A + Be^{-t/RC}$$

$$V_{C0} = 0$$

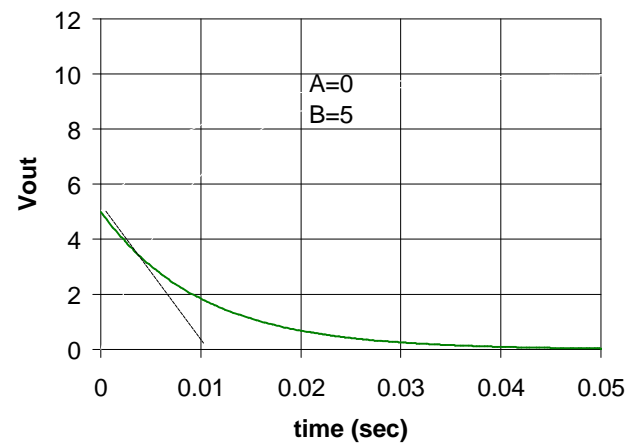
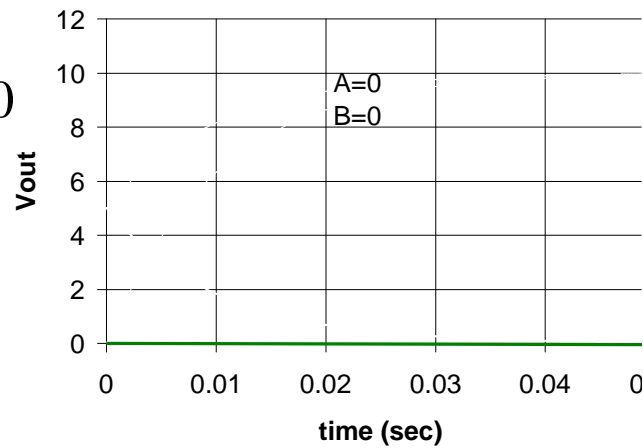
$$V_1 = 10$$



$$V_{C0} = 5 \text{ V}$$



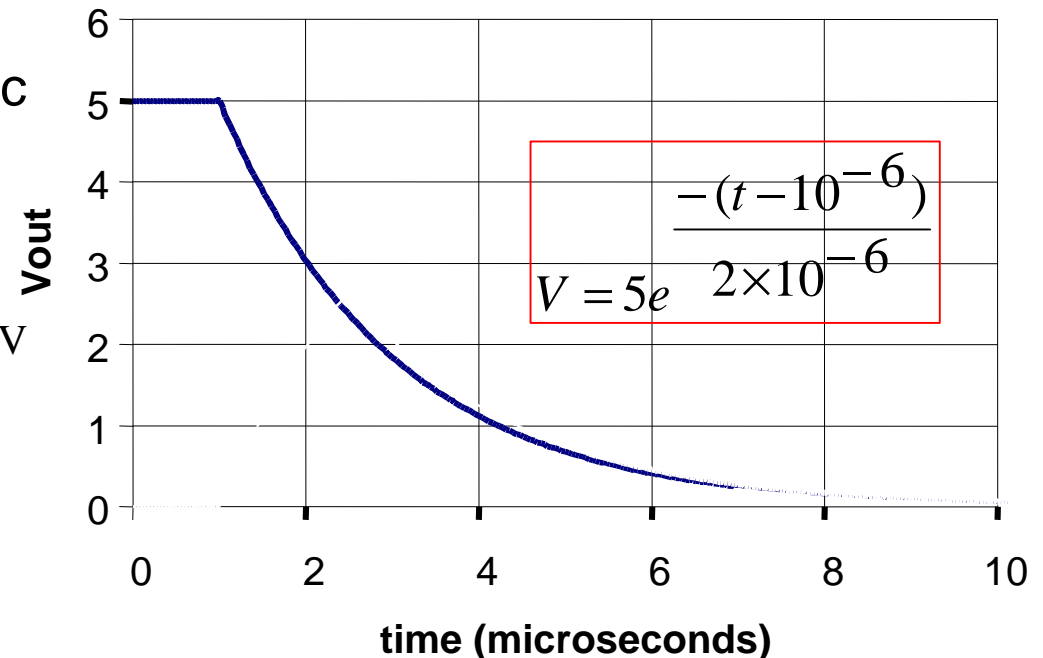
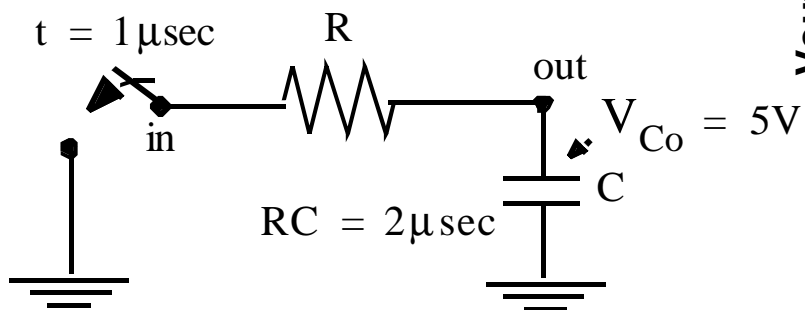
$$V_1 = 0$$



## COMPLICATION: Event Happens at $t \neq 0$

(Solution: Shift reference time to time of event)

Example: switch closes at  $1\mu\text{sec}$



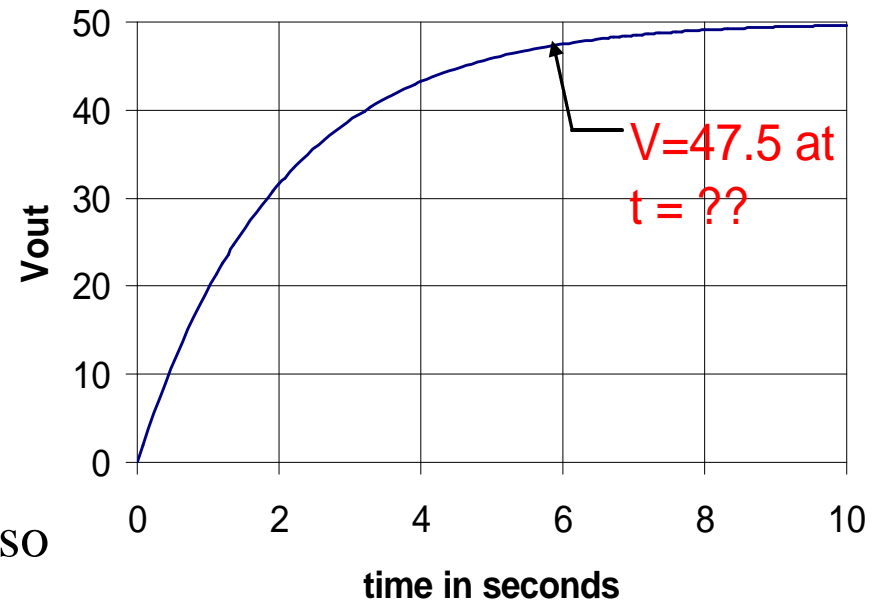
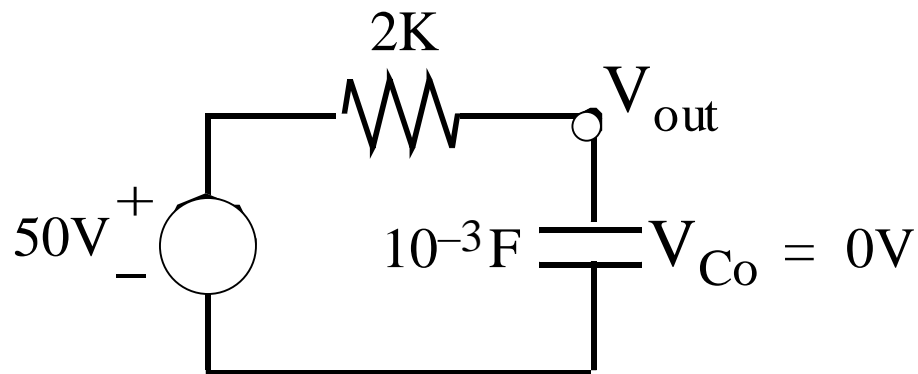
We shift the time axis here by one microsecond, i.e. imagine a new time coordinate  $t^* = t - 1\mu\text{sec}$  so that in the new time domain, the event happens at  $t^* = 0$  and our standard solution applies. Of course we replace  $t^*$  by  $t - 1\mu\text{sec}$  in the equations and plots. Thus instead of  $t^* = 0$  we have  $t = 1\mu\text{sec}$ , etc.



## FINAL EXAMPLE

Your photo flash charges a  $1000\mu\text{F}$  capacitor from a  $50\text{V}$  source through a  $2\text{K}$  resistor. If the capacitor is initially uncharged, how long must you wait for it to reach 95% charged ( $47.5\text{ V}$ )?

**Solution:**  $RC = 2\text{K} \times 10^{-3} = 2\text{ sec}$



By inspection:  $V_o = 50 - 50e^{-t/2}$ , so

$$47.5 = 50(1 - e^{-\frac{t_x}{2}}) \Rightarrow e^{-\frac{t_x}{2}} = (1 - \frac{47.5}{50}) \Rightarrow \boxed{t_x = 6\text{ sec}}$$