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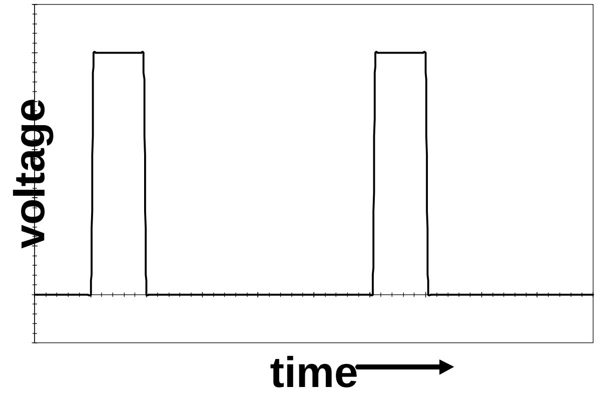
Review of charging and discharging in RC Circuits
(an enlightened approach)

- *Before* we continue with formal circuit analysis - lets review RC circuits
- Rationale: Every node in a circuit has capacitance to ground, like it or not, and it's the charging of these capacitances that limits real circuit performance (speed)

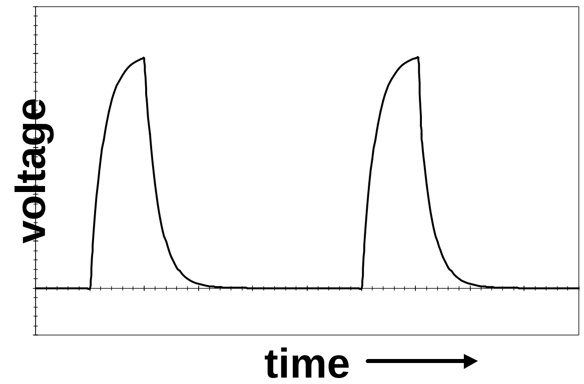
Relevance to digital circuits:

We communicate with pulses

We send beautiful pulses out



But we receive lousy-looking pulses and must restore them



RC charging effects are responsible …. So lets review them.

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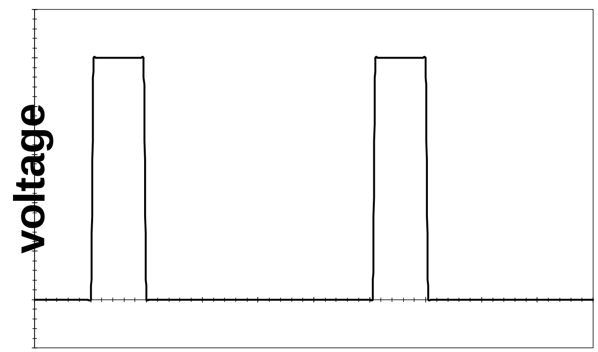
Simplification for time behavior of RC Circuits

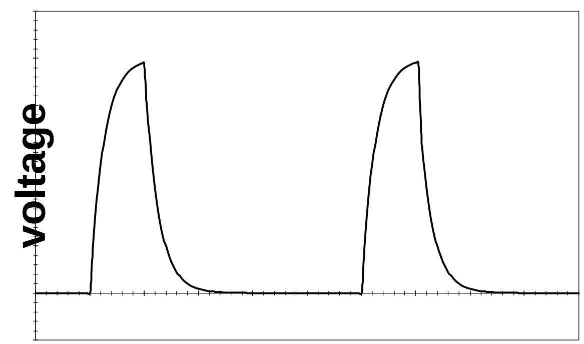
- Before any input change occurs we have a dc circuit problem (that is we can use dc circuit analysis to relate the output to the input).
- Long after the input change occurs things "settle down" …. Nothing is changing …. So again we have a dc circuit problem.

We call the time period during which the output changes the *transient*

We can predict a lot about the transient behavior from the pre- and post-transient dc solutions



time



time

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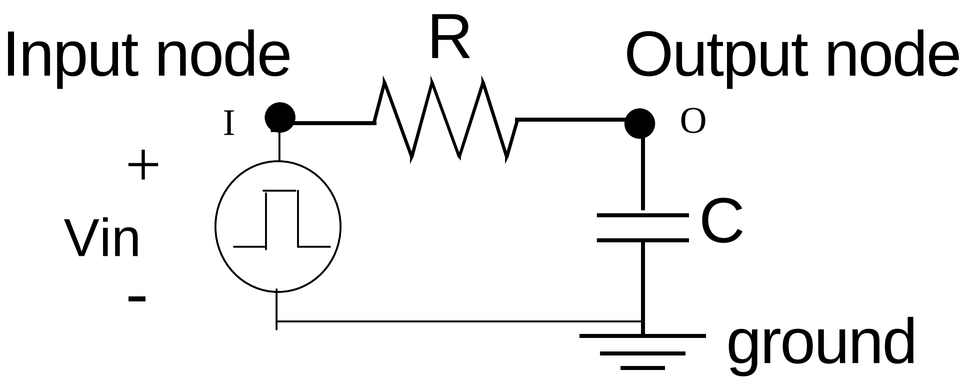
What environment do pulses face?

- Every wire in a circuit has resistance.
- Every junction (called *nodes*) has capacitance to ground and other nodes.
- The active circuit elements (transistors) add additional resistance in series with the wires, and additional capacitance in parallel with the node capacitance.



A pulse originating at **node I** will arrive delayed and distorted at **node O** because it takes time to charge C through R

If we focus on the circuit which distorts the pulses produced by Vin, it consists simply of R and C. (Vin is just the time-varying source which produces the input pulse.)

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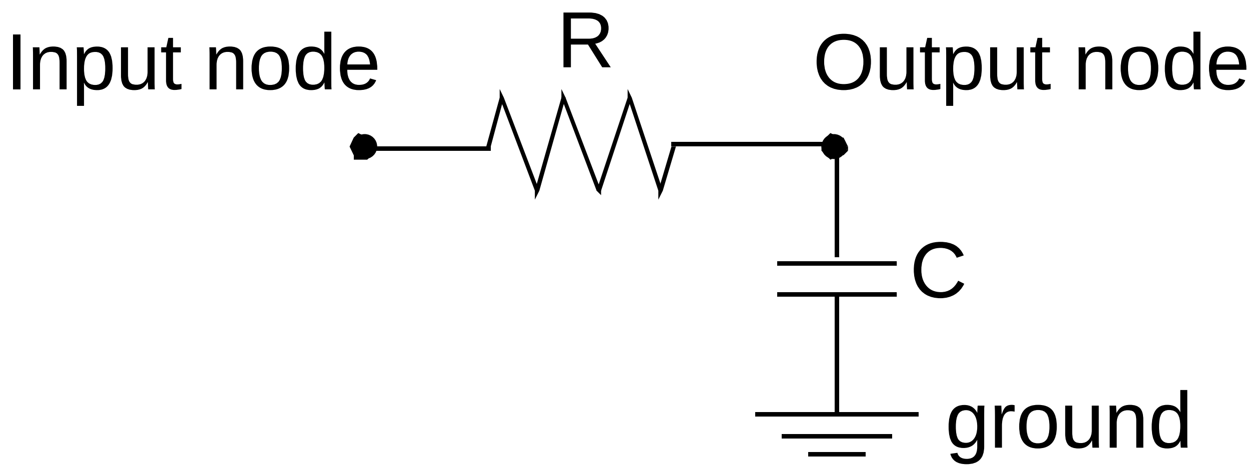
The RC Circuit to Study

(All single-capacitor circuits reduce to this one)



- R represents total resistance (wire plus whatever drives the input node)
- C represents the total capacitance from node to the outside world (from devices, nearby wires, ground etc)

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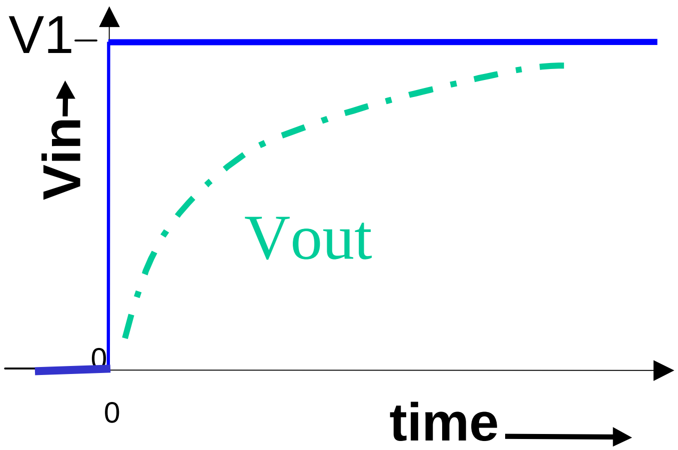
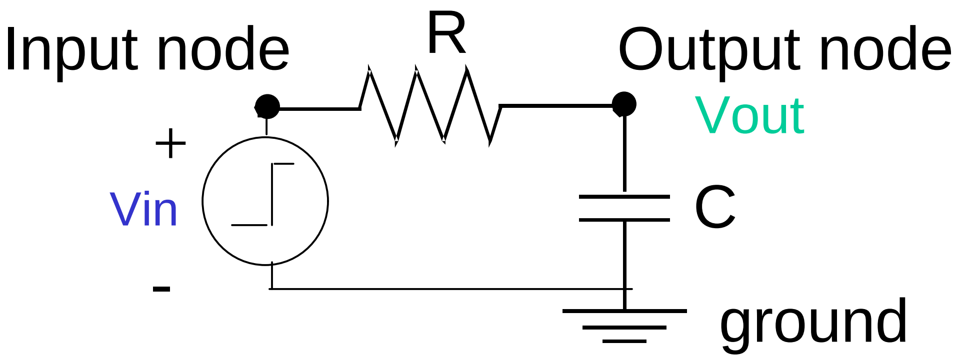
RC RESPONSE

Case 1 – Rising voltage. Capacitor uncharged: Apply + voltage step





- Vin “jumps” at t=0, but Vout cannot “jump” like Vin. Why not?

☞ Because an instantaneous change in a capacitor voltage would require instantaneous increase in energy stored ($1/2CV^2$), that is, infinite power. (Mathematically, V must be differentiable: $I=CdV/dt$)

$$\boxed{\text{V does not “jump” at t=0 , i.e. } V(t=0^+) = V(t=0^-)}$$

> Therefore the dc solution before the transient tells us the capacitor voltage at the beginning of the transient.

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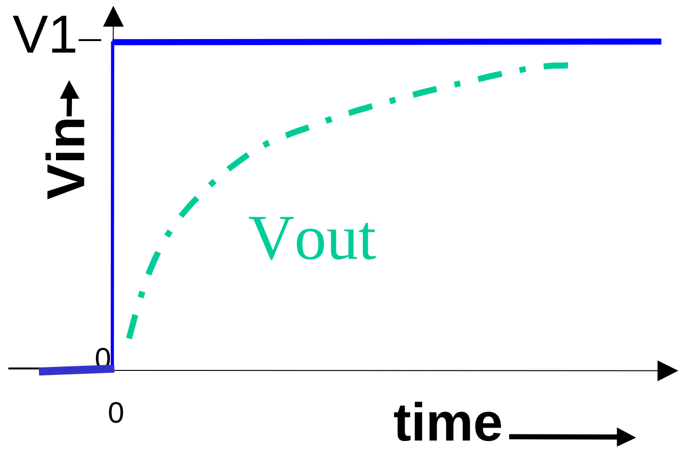
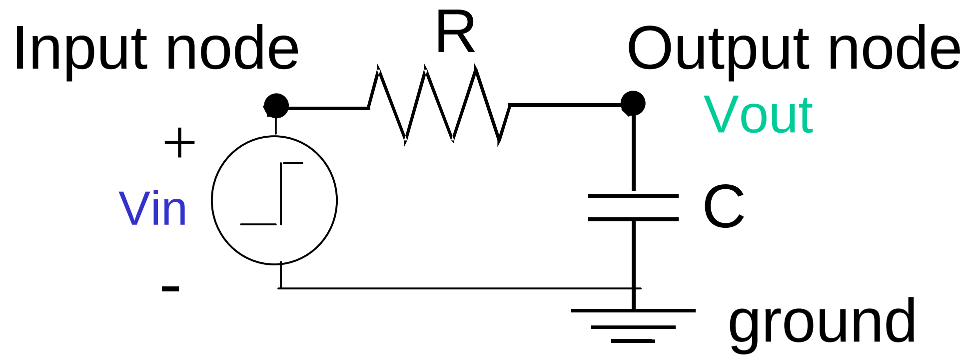
RC RESPONSE

Case 1 – Capacitor uncharged: Apply voltage step





- Vout approaches its final value asymptotically (It never quite gets to V1, but it gets arbitrarily close). Why?

After the transient is over (nothing changing anymore) it means d(V)/dt = 0 ; that is all currents must be zero. From Ohm's law, the voltage across R must be zero, i.e. Vin = Vout.

☞ That is, Vout → V1 as t → ∞. (Asymptotic behavior)

Again the dc solution (after the transient) tells us (the asymptotic limit of) the capacitor voltage during the transient.

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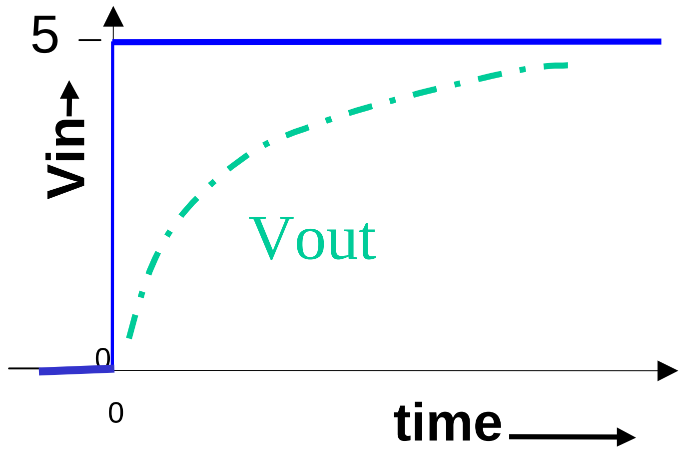
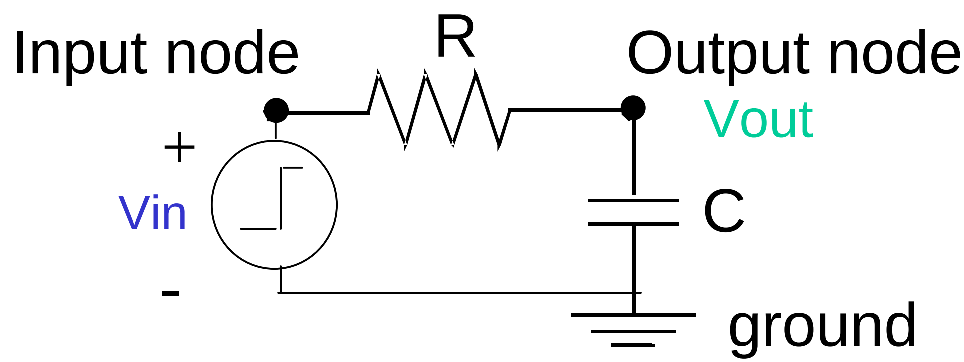
RC RESPONSE

Example – Capacitor uncharged: Apply voltage step of 5 V





- Clearly Vout starts out at 0V (at t = 0^+) and approaches 5V.
- We know this because of the pre-transient dc solution (V=0) and post-transient dc solution (V=5V).

So we know a lot about Vout during the transient - namely its initial value, its final value , *and we know the general shape* .

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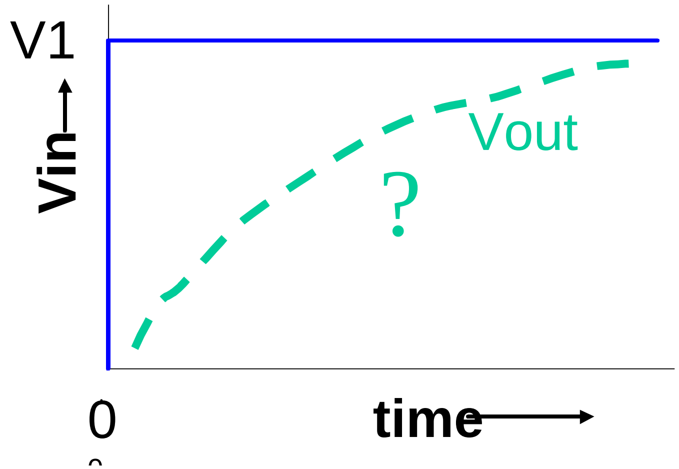
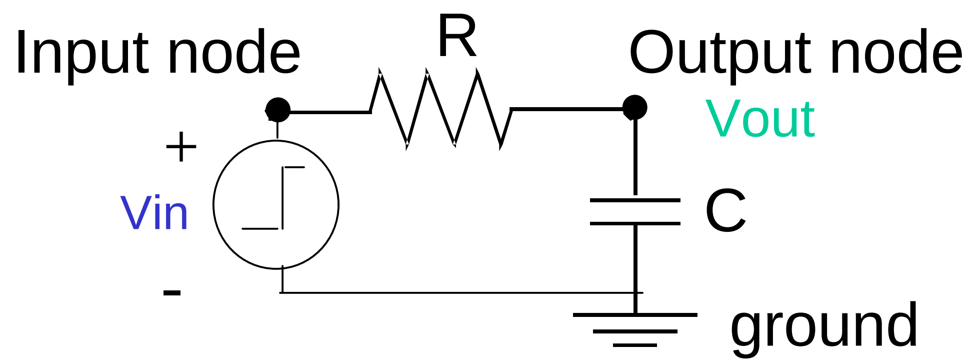
RC RESPONSE: Case 1 (cont.)

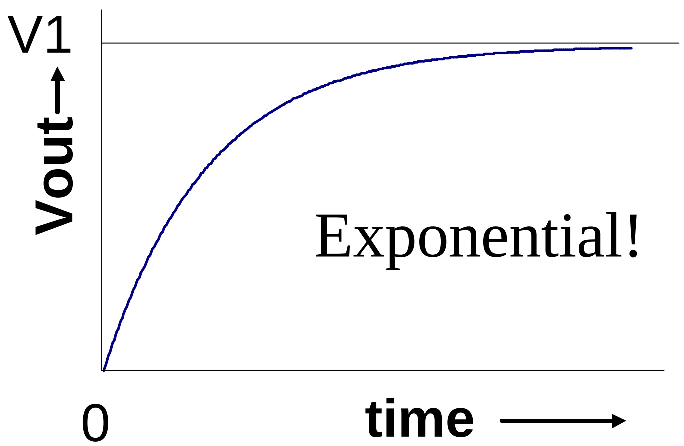


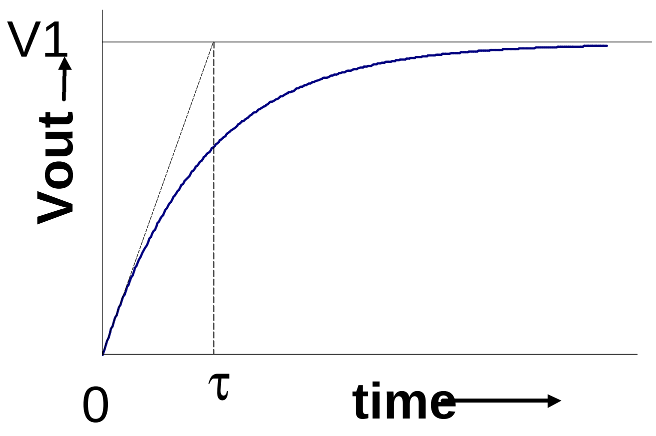


Exact form of Vout?

Equation for Vout: Do you remember general form?

$$V_{out} = V_1(1-e^{-t/\tau})$$





Review of simple exponentials.

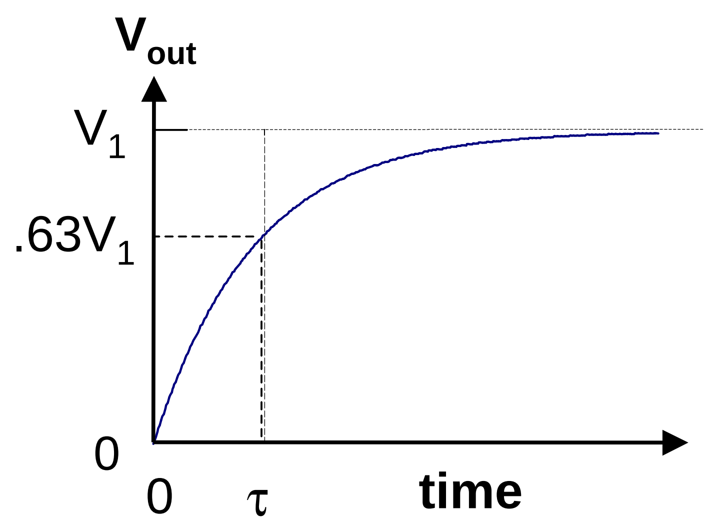
Rising Exponential from Zero

$$V_{out} = V_1(1-e^{-t/\tau})$$

at $t = 0$, $V_{out} = 0$, and

at $t \rightarrow \infty$, $V_{out} \rightarrow V_1$ also

at $t = \tau$, $V_{out} = 0.63\ V_1$



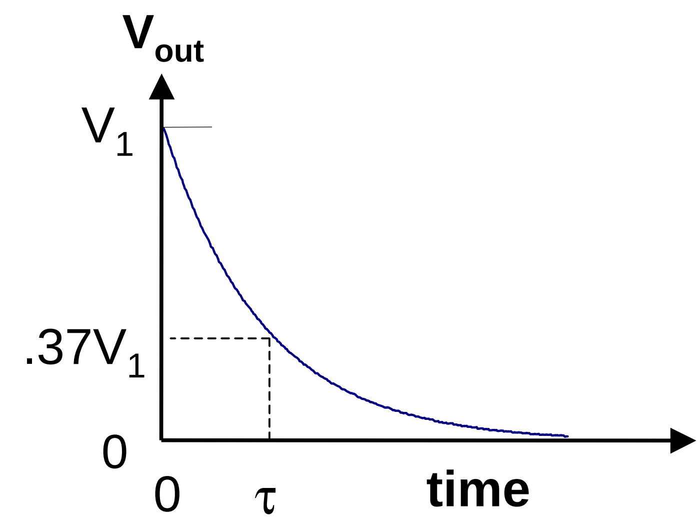
Falling Exponential to Zero

$$V_{out} = V_1 e^{-t/\tau}$$

at $t = 0$, $V_{out} = V_1$, and

at $t \rightarrow \infty$, $V_{out} \rightarrow 0$, also

at $t = \tau$, $V_{out} = 0.37\ V_1$



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Further Review of simple exponentials.

Rising Exponential from Zero

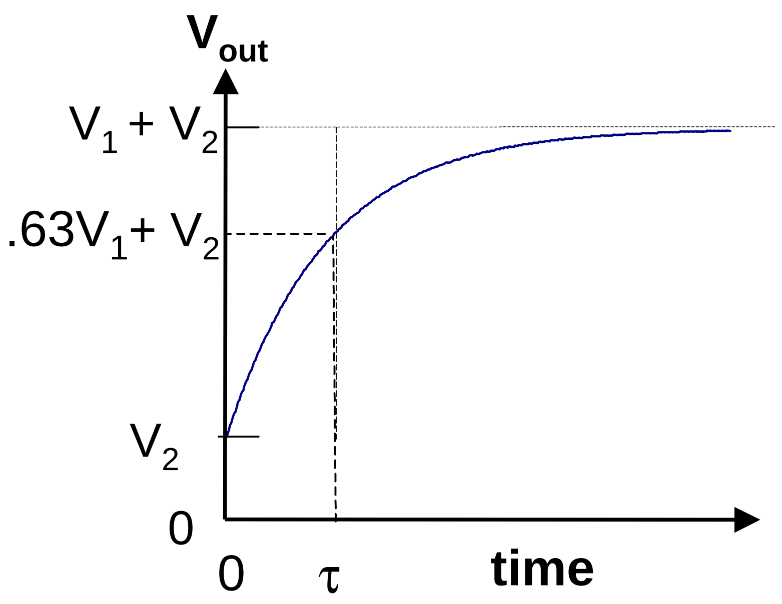
$$V_{out} = V_1(1-e^{-t/\tau})$$

Falling Exponential to Zero

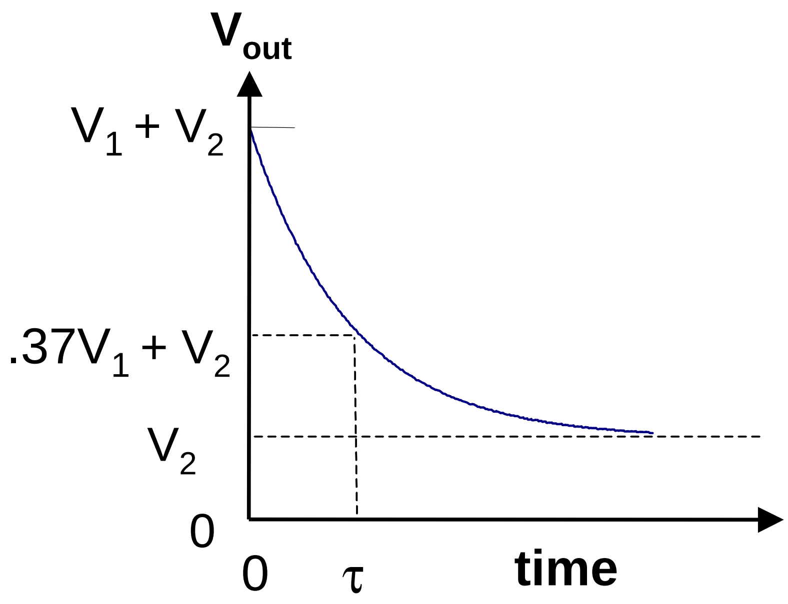
$$V_{out} = V_1 e^{-t/\tau}$$

We can add a constant (positive or negative)

$$V_{out} = V_1(1-e^{-t/\tau}) + V_2$$



$$V_{out} = V_1 e^{-t/\tau} + V_2$$



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Further Review of simple exponentials.

Rising Exponential

$$V_{out} = V_1(1-e^{-t/\tau}) + V_2$$

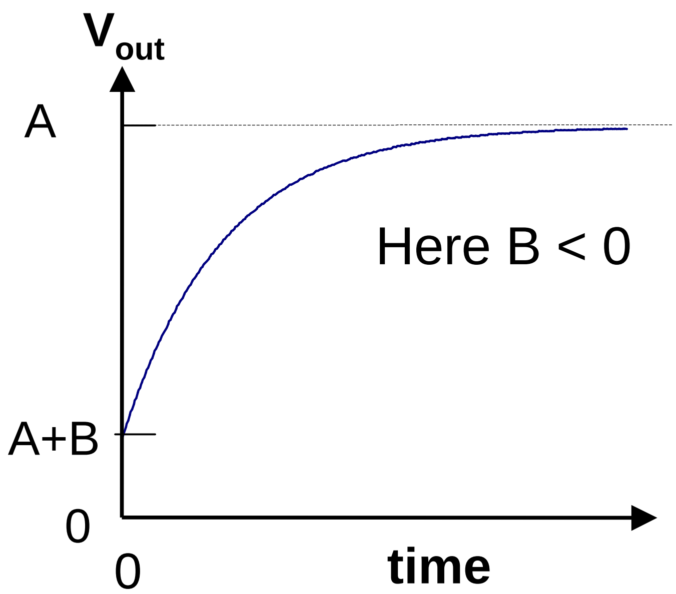
Falling Exponential

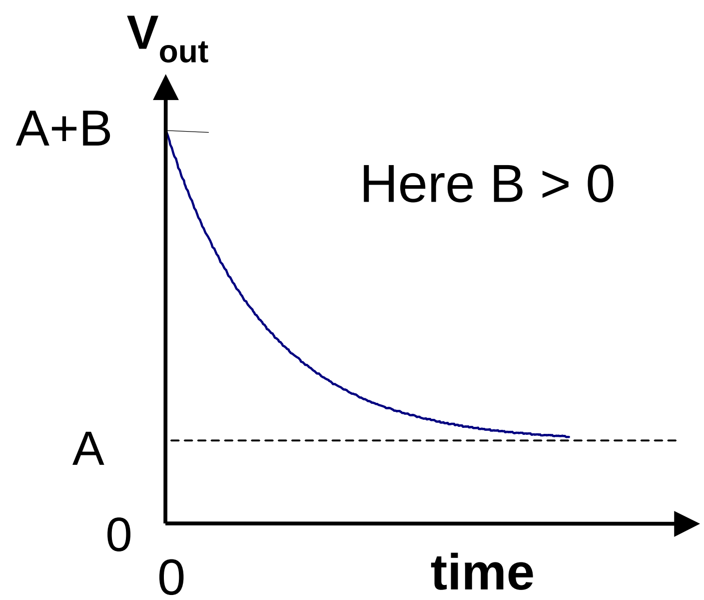
$$V_{out} = V_1 e^{-t/\tau} + V_2$$

Both equations can be written in one simple form: $\boxed{V_{out} = A + Be^{-t/\tau}}$

Initial value (t=0) : $V_{out} = A + B$. Final value ($t>>\tau$): $V_{out} = A$

Thus: if $B < 0$, rising exponential; if $B > 0$, falling exponential





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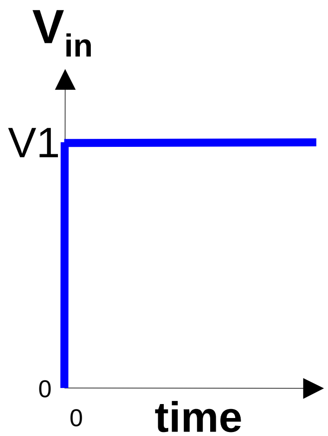
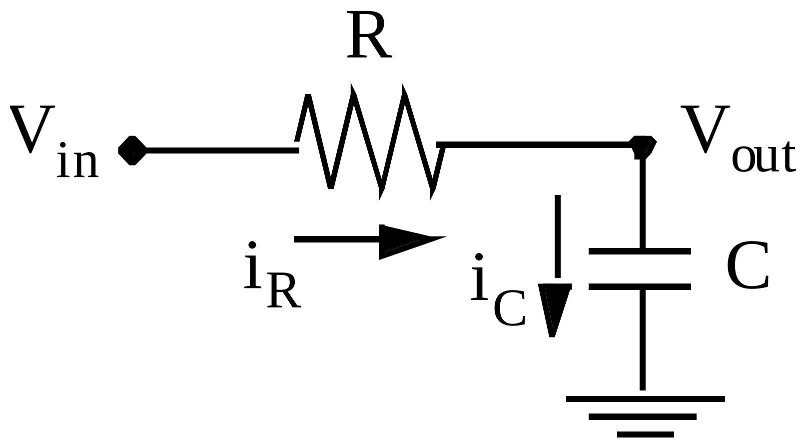
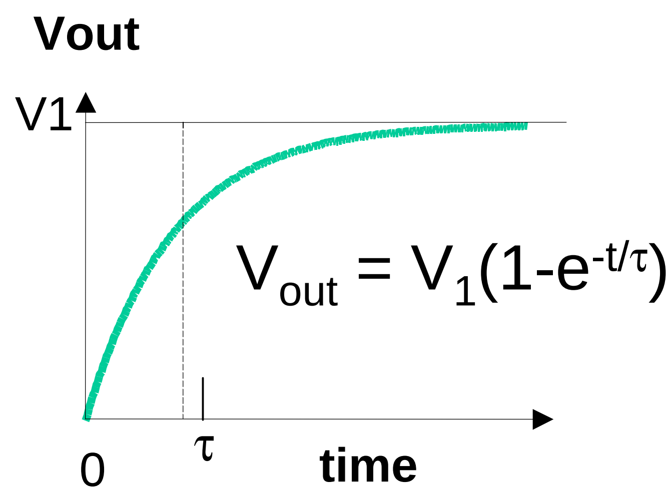
RC RESPONSE: Case 1 (Rising exponential)







- How is τ related to R and C ?
  - If C is bigger, it takes longer ($\tau\uparrow$).
  - If R is bigger, it takes longer ($\tau\uparrow$).
  - ✍ Thus, τ is proportional to RC.

☞ In fact, τ = RC !

✍ Thus, $V_{out} = V_1(1\text{-}e^{-t/\tau})$

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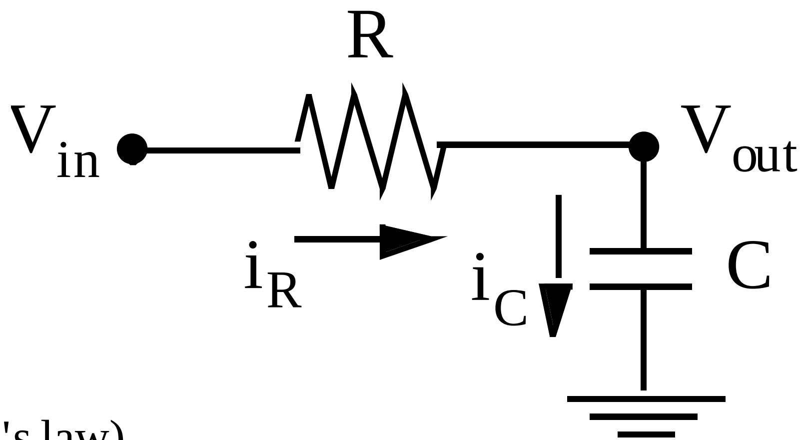
RC RESPONSE: Case 1 (cont.)

Proof that $V_{out} = V_1(1 - e^{-t/RC})$



$$i_R = \frac{V_{in} - V_{out}}{R} \quad \text{(Ohm's law)}$$

$$i_C = C\frac{dV_{out}}{dt} \quad \text{(capacitance law)}$$

But $i_R = i_C$!

$$\text{Thus,} \quad \frac{V_{in} - V_{out}}{R} = C\frac{dV_{out}}{dt}$$

or

$$\boxed{\frac{dV_{out}}{dt} = \frac{1}{RC}(V_{in} - V_{out})}$$

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RC RESPONSE Case 1 (cont.)

Proof that $V_{out} = V_1(1 - e^{-t/RC})$

We have: $\dfrac{dV_{out}}{dt} = \dfrac{1}{RC}(V_{in} - V_{out})$

But $V_{in} = V_1 = \text{constant}$

and $V_{out} = 0$ at $t = 0^+$

I claim that the solution to this first-order linear differential equation is:

$$V_{out} = V_1(1 - e^{-t/RC})$$

Proof by substitution:

$$\frac{dV_{out}}{dt} \overset{?}{=} \frac{1}{RC}(V_{in} - V_{out})$$

$$\downarrow$$

$$\cancel{-}\frac{V_1}{RC} e^{-t/RC} \overset{?}{=} \frac{1}{RC}(V_1 \cancel{-} V_1(1 \cancel{-} e^{-t/RC}))$$

clearly

$$\frac{V_1}{RC} e^{-t/RC} = \frac{V_1}{RC} e^{-t/RC}$$

and

$V_{out} = 0$ at $t = 0^+$ OK

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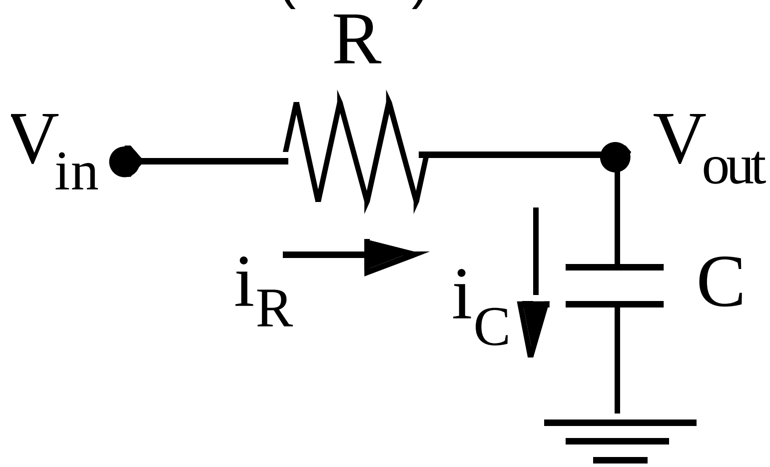
RC RESPONSE (cont.)



Generalization

Vin switches at t = 0; then for any time interval t > 0, in which Vin is a constant, Vout is ***always*** of the form:

$$\boxed{V_{out} = A + Be^{-t/\tau}}$$

We determine A and B from the initial voltage on C, and the value of Vin. Assume Vin "switches" at t=0 from Vco to V1:

First, at $t = 0 \quad V_C \equiv V_{Co}$ initial voltage

☞ Thus, $\boxed{A + B = V_{Co}}$

as $t \to \infty, V_C \to V_1$

☞ Thus, $\boxed{A = V_1} \Rightarrow \boxed{B = V_{Co} - V_1}$