## Lecture 20

## LECT 19 Review

Gate delay in logic circuits

Today: Two Topics

1) Simple controlled switch model of logic
2) Capacitor charging and energy per logic cycle

Review of Energy in Switching
Efficiency
Dynamic Power

## How can we build inverters, Nand gates, etc. ?

We need some sort of controlled switch: that is a device in which a switch opens or closes in response to an input voltage (a control voltage). If we have a controlled switch it is an easy matter to build inverters, Nand gates, etc.

For example an electromagnetic relay has a coil producing a magnetic field causing some contacts to "snap shut" when a voltage is applied to the coil.

Lets imagine a simple controlled switch, but include in it some resistance (all real devices have non-zero resistance).

## Controlled Switch Model



The basic idea: We need a switch which is controlled by an input voltage. For example: Input $\mathrm{V}=0$ means the switch is open, whereas an input voltage of 2 V means that the switch is closed
(We will call this a "Type N controlled switch")

## Controlled Switch Model



> Type N controlled switch" means switch is closed if input is high. $\left(\mathrm{V}_{\mathrm{G}}>\mathrm{V}_{\mathrm{S}}\right)$

Type P controlled switch" means switch is closed if input is low. $\left(\mathrm{V}_{\mathrm{G}}<\mathrm{V}_{\mathrm{S}}\right)$

Now lets combine these switches to make an inverter.

## Controlled Switch Model of Inverter



So if $V_{\text {IN }}$ is $2 V$ then $S_{N}$ is closed and $S_{P}$ is open. Hence $V_{\text {OUT }}$ is zero.
But if $\mathrm{V}_{\text {IN }}$ is 0 V then $\mathrm{S}_{\mathrm{P}}$ is closed and $\mathrm{S}_{\mathrm{N}}$ is open. Hence $\mathrm{V}_{\text {Out }}$ is 2 V .

## Controlled Switch Model of Inverter



IF $V_{I N}$ is $2 V$ then $S_{N}$ is closed and $\mathrm{S}_{\mathrm{p}}$ is open. Hence $\mathrm{V}_{\text {out }}$ is zero (but driven through resistance $R_{N}$ ).


But if $\mathrm{V}_{\mathrm{IN}}$ is 0 V then $\mathrm{S}_{\mathrm{P}}$ is closed and $S_{N}$ is open. Hence $V_{\text {out }}$ is 2 V (but driven through resistance $\mathrm{R}_{\mathrm{P}}$ ).

## Controlled Switch Model of Inverter



IF there is a capacitance at the output node (there always is) then $\mathrm{V}_{\text {out }}$ responds to a change in $\mathrm{V}_{\mathrm{IN}}$ with our usual exponential form.


## Controlled Switch Model of Inverter

We will expand on this model in coming weeks.
The controlled switches will of course be MOS transistors.
The resistance will be the effective output resistance of the MOS devices.

The capacitance will be the input capacitance of the MOS devices.

But now lets briefly review the energy used in charging and discharging capacitances so we can start to estimate chip power.

## ENERGY AND POWER IN CHARGING/DISCHARGING CAPACITORS - A REVIEW

CASE 1-


Power out of "battery"
Power into C
$\mathrm{P}_{\mathrm{C}}=\mathrm{i}(\mathrm{t}) \mathrm{V}_{\mathrm{C}}(\mathrm{t})$
Energy out of "battery"

> Energy into C

$$
\begin{aligned}
\mathrm{E}=\int_{0}^{\infty} \mathrm{iV}_{\mathrm{DD}} \mathrm{dt} & =\mathrm{QV}_{\mathrm{DD}} & \mathrm{E}_{\mathrm{C}} & =\int_{\mathrm{iV}}^{\mathrm{C}} \mathrm{dt} \\
& =\mathrm{CV}_{\mathrm{DD}}^{2} & & =\frac{1}{2} \mathrm{CV}_{\mathrm{DD}}{ }^{2}
\end{aligned}
$$

Capacitor initially uncharged ( $\mathrm{Q}=\mathrm{CV}_{\mathrm{DD}}$ at end)

Switch moves @ t=0

Power into $R$
$\mathrm{P}_{\mathrm{R}}=[\mathrm{i}(\mathrm{t})]^{2} \mathrm{R}$
Energy into R (heat)
This must be difference of $E$ and $E_{C}$, i.e. $\frac{1}{2} C V_{D D}{ }^{2}$

## ENERGY AND POWER IN CHARGING



Energy out of "battery"

$$
=\mathrm{CV}_{\mathrm{DD}}^{2}
$$

Energy into C
$=\frac{1}{2} \mathrm{CV}_{\mathrm{DD}}{ }^{2}$

Capacitor initially uncharged ( $\mathrm{Q}=\mathrm{CV}_{\mathrm{DD}}$ at end)

Switch moves @ t=0

Energy into R (heat)

$$
\frac{1}{2} \mathrm{CV}_{\mathrm{DD}}{ }^{2}
$$

In charging a capacitor from a fixed voltage source $\mathcal{V}_{\mathcal{D D}}$ falf the energy from the source is delivered to the capacitor, and half is lost to the charging resistance, independent of the value of $R$.

## ENERGY AND POWER IN CHARGING/DISCHARGING CAPACITORS



Capacitor initially charged $\left(\mathrm{Q}=\mathrm{CV}_{\mathrm{DD}}\right)$ and discharges.

Switch moves @ t=0

Power out of battery
$=0$
Energy out of battery
$=0$
Power in/out of $R$
$=0$

Power out of C
$\mathrm{P}_{\mathrm{C}}=\mathrm{i}(\mathrm{t}) \mathrm{V}_{\mathrm{C}}(\mathrm{t})$
Energy out of $C$

$$
\begin{aligned}
\mathrm{E}_{\mathrm{C}} & =\int_{\mathrm{i} \mathrm{~V}_{\mathrm{C}} \mathrm{dt}} \\
& 0 \\
& =\frac{1}{2} \mathrm{CV}_{\mathrm{DD}}{ }^{2}
\end{aligned}
$$

Power into $R_{D}$ $\mathrm{P}_{\mathrm{R}}=[\mathrm{i}(\mathrm{t})]^{2} \mathrm{R}$

Energy into $\mathrm{R}_{\mathrm{D}}$ (heat)
This must be energy
initially in C , i.e.
$\frac{1}{2} \mathrm{CV}_{\mathrm{DD}}{ }^{2}$

## ENERGY IN DISCHARGING CAPACITORS



Capacitor initially charged ( $\mathrm{Q}=\mathrm{CV}_{\mathrm{DD}}$ ) and discharges.

Switch moves @ t=0

Energy out of $C \quad$ Energy into $R_{D}$ (heat)

$$
=\frac{1}{2} \mathrm{CV}_{\mathrm{DD}}^{2} \quad \frac{1}{2} \mathrm{CV}_{\mathrm{DD}}^{2}
$$

When a capacitor is discharged into a resistor the energy originally stored in the capacitor $\left(1 / 2 \mathcal{C V} \mathcal{D D}^{2}\right)$ is dissipated as feat in the resistor

## POWER DISSIPATION in DIGITAL CIRCUITS

Each node transition (i.e. charging or discharging) results in a loss of $(1 / 2)(\mathrm{C})\left(\mathrm{V}_{\mathrm{DD}}\right)^{2}$ How many transitions occur per second? Well if the node is pulsed up then down at a frequency $f$ (like a clock frequency) then we have $2 f$ dissipation events.

A system of $N$ nodes being pulsed at a frequency $f$ to a signal voltage $\mathrm{V}_{\mathrm{DD}}$ will dissipate energy equal to ( N ) $(2 \mathrm{f})\left(1 / 2 \mathrm{CV}_{\mathrm{DD}}\right)^{2}$ each second

Therefore the average power dissipation is $(\mathrm{N})(\mathrm{f})\left(\mathrm{CV}_{\mathrm{DD}}\right)^{2}$

## LOGIC POWER DISSIPATION

Power $=($ Number of gates $) \times($ Energy per cycle) $\times$ (frequency $)$

$$
\mathrm{P}=(\mathrm{N})\left(\mathrm{CV} \mathrm{~V}_{\mathrm{DD}}\right)^{2}(\mathrm{f})
$$

$\mathrm{N}=10^{7} ; \mathrm{V}_{\mathrm{DD}}=2 \mathrm{~V}$; node capacitance $=10 \mathrm{fF} ; \mathrm{f}=10^{9} \mathrm{~s}^{-1}(1 \mathrm{GHz})$
$\mathrm{P}=400 \mathrm{~W}$ ! -- a toaster!

Pretty high but realistic
What to do? ( N increases, f increases, hmm )

1) Lower $V_{D D}$
2) Turn off the clock to the inactive nodes

Clever architecture and design!
Lets define $\alpha$ as the fraction of nodes that are clocked (active). Then we have a new formula for power.

## LOGIC POWER DISSIPATION with power mitigation

Power $=($ Energy per transition) $\times$ (Number of gates) $\times$ (frequency) $\times$ fraction of gates that are active ( $\alpha$ ).

$$
\mathrm{P}=\alpha \mathrm{NfCV} \mathrm{DD}^{2}
$$

In the last 5 years $\mathrm{V}_{\mathrm{DD}}$ has been lowered from 5 V to about 1.5 V . It cannot go very much lower. But with clever design, we can make $\alpha$ as low as 1 or $10 \%$. That is we do not clock those parts of the chip where there is no computation being made at the moment.

Thus the 400W example becomes 4 to 40W, a manageable range ( 4 W with heat sink, 40 W with heat sink plus fan on the chip).

