Inductance and Capacitance

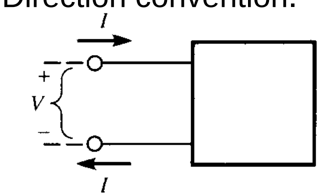
Lecture 6 review:
- Power Calculations
- Starting your car (model)

Today: (5.1, 5.2)
- Ideal inductors and capacitors
- Energy storage
- Practical capacitors and inductors

Power Calculations

Why?
- To find out how much power is being delivered to (from) some device, e.g. loud speaker.
- Alternative, it may be undesirable to delivered power to some part of the circuit, because of the heat it generates.
- Basic idea is addressed in the first lecture.

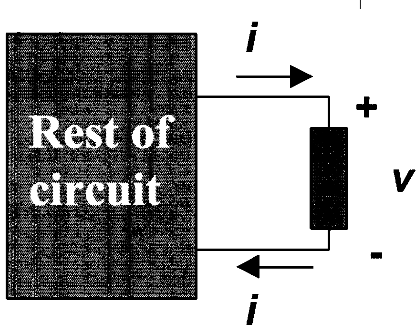
How?
Use the Associated Reference Direction convention.



Associated Reference Directions

- It is convenient to define the current through a circuit element as positive when entering the terminal associated with the + reference for voltage

For positive current and positive voltage, positive charge "falls down" a potential "drop" in moving through the circuit element: it absorbs power.



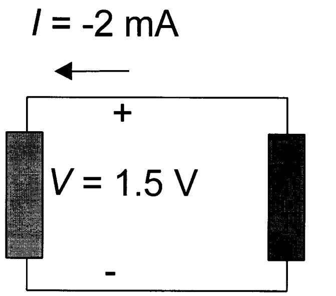
Power Definitions

- P = VI > 0 corresponds to the element absorbing power
  - How can a circuit element absorb power?
- By converting electrical energy into heat (resistors in toasters), light (light bulbs), acoustic energy (speakers); by storing energy (charging a battery)
- Negative power - releasing power to the rest of the circuit.

Conservation of Power

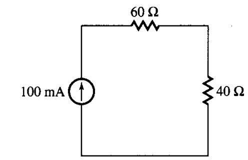
- Sum of the power absorbed by all circuit element must be zero.
- Concept: circuit elements are used to model all modes of energy conversion (heat, sound, batteries, voltage generators, etc.)
- Simple example:

Power released (VI <0) by the element on the left equals to the power absorbed by the element on the right.



Example of Power Flowing into Current Source

What is the power flow into the current source in the circuit on the right?



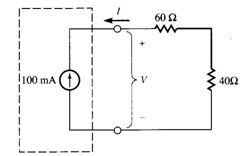
(a)

Put an imaginary box enclosing the current source and apply the associated reference direction (ARD) to the current source.

(1) Assume V convention on the right, ARD dictates that current has to go into + side of the terminals.

By KVL: V = 100mA×(60 Ω +40Ω) = 10V

By KCL: I = -100mA

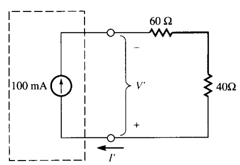


(b)

The power entering the box (in this case, current source) is:

Power = VI = 10V × -100mA = -1 W ∴1W is leaving the box

Example of Power Flowing into Current Source (cont)



(c)

(2) Assume V' convention on the right, ARD dictates that current has to go into + side of the terminals.

By KVL: V' = 100mA×(60 Ω +40Ω) = -10V

By KCL: I' = 100mA

The power entering the box (in this case, current source) is:

Power = V'I' = -10V × 100mA = -1 W ∴1W is leaving the box

Conclusions: V convention does not affect the results. Both conventions leads to the same conclusion that 1W is leaving the box or current source.

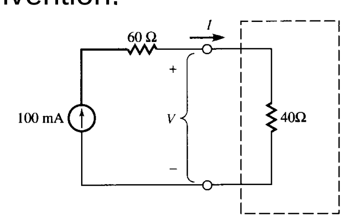
Example of Power Flowing into resistor

Find the power entering the 40 Ω resistor as shown in the circuit.

By associate reference direction convention. the V and I are defined.

By inspection I = 100mA

$$V = I \times 40\Omega = 4V$$



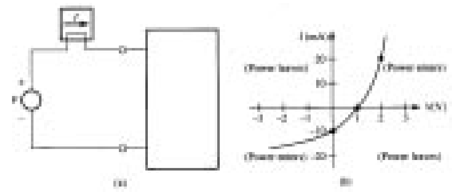
P = I × V = 100mA × 4V = 0.4W

∴0.4W is entering the box (or 40Ω resistor).

(Using the similar strategy, it is found that 0.6W is entering the 60Ω resistor.)

Experimental Measurement of Power using a Volt Source and a Ammeter

The box in the right contains an unknown circuit, experimentally it is found that the I-V diagram as shown.
(how? Set the voltage, and measure the current by the ammeter)



V (set)	I (measured)	P = V I (compute)
-3V	-18mA	+54mW (power entering)
-2V	-16mA	+32mW
-1V	-14mA	+14mW
0	-10mA	0 (no power transfer)
1V	-2mA	-2mW (power leaving)
2V	+20mA	+40mW
3V	+400mA	+1.2W

∴ In the first and the third quadrant of the I-V curve, power is entering.
In the second and the fourth quadrant, power is leaving.

Special cases

If a voltage exists across a resistor R,

Power dissipated in the resistor = VI = V (V/R) =V^2/R
(Ohm's law applies in this case)

If a current flow, I, through a resistor R,

Power dissipated in the resistor = VI = (IR) I =I^2R
(Ohm's law also applies in this case)

Instantaneous Power and Average Power

When V and I are function of time, we write as v(t) and i(t), then instantaneous power entering the box is:

$P(t) = v(t)i(t)$ (instantaneous power, power as function of t)

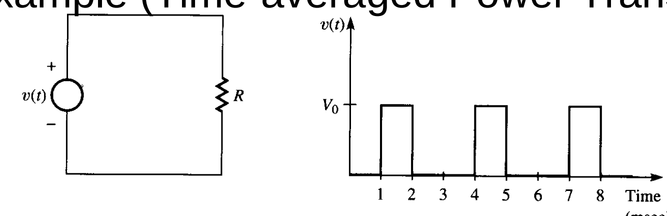
This is particularly useful to compute the maximum power received or delivered at any instant.

$P_{AV} = \frac{1}{T}\int_0^T v(t)i(t)dt$ (time-averaged power)

This is particularly useful for computing the trend, the average power received or delivered.

Does DC current have instantaneous or time-average power?

Example (Time-averaged Power Transferred)



Find the time-averaged power transferred into resistor R, if the time varying voltage source *v(t)* is as shown on the right.

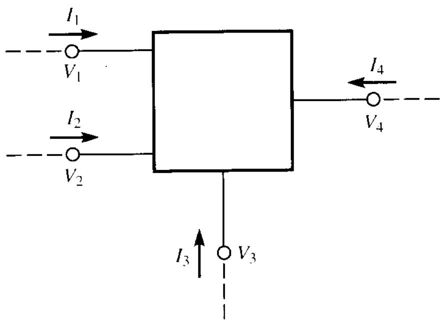
Observation: (1) v(t) is a square wave (max at V_0, min at 0)
i(t) is a ______wave (max at , min at)
Instantaneous power = v(t)i(t) = or
(2) Period from 0ms to 3ms, or 1ms to 4ms, or 2ms to 5ms.

$$P_{AV} = \frac{1}{0.003-0}\int_0^{0.003} v(t)i(t)dt$$

$$P_{AV} = \frac{1}{0.003}\left(\int_0^{0.001} 0dt + \int_{0.001}^{0.002}\frac{V_0^2}{R}dt + \int_{0.002}^{0.003} 0dt\right) = \frac{1}{0.003}\frac{V_0^2}{R}(0.001) = \frac{V_0^2}{3R}$$

Multi-terminal Elements

We dealt with circuit element with 2 terminals in the preceding sections.



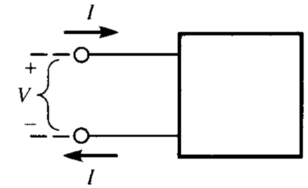
Here we have 4 terminals

$P = V_1I_1+V_2I_2+V_3I_3+V_4I_4$

Notice the reference direction of I is toward the terminals, the terminals are labeled V_1, V_2, V_3, V_4.

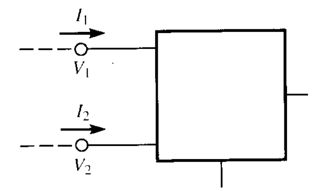
This can be generalized into any N terminals devices.

Example



Show that the 2 terminals circuit and formula is a special case to our 4 terminals circuit formula.

Is P=VI a special case of
$P= V_1I_1+V_2I_2+V_3I_3+V_4I_4$?
Generalize into a 2 terminals circuit:
$P= V_1I_1+V_2I_2$

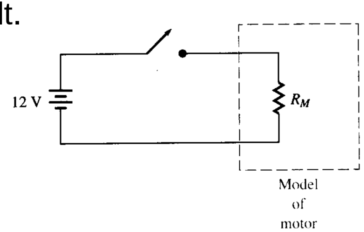


Comparing the 2 circuits,
$I=I_1$, $I=-1_2$
$P= V_1I_1+V_2I_2$ becomes $P= V_1I-V_2I=(V_1-V_2)I$
But $V=V_1-V_2$ (comparing the 2 circuits)

P=VI which is our original Power formula.
∴ P=VI is a special case of $P= V_1I_1+V_2I_2+V_3I_3+V_4I_4$ general formula.

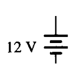
Starting a Car

A starting motor of a car typical of a range of 20-40 Amp and 12 volt.



If we take 30Amp and 12 volt. The motor can be modeled as a equivalent resistance with value $R_m=12/30 = 0.4\Omega$ as shown in the diagram on the right.

Can we replace the bulky car battery by eight 1.5v AA size batteries in series? Why not? They are both 12V.

The battery model 

needs to be replaced by a more accurate electrical model.

Battery model

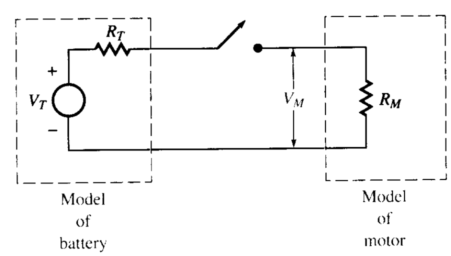
We can model each battery as an ideal 12V voltage source. But this would not help us to understand the difference between eight 1.5V batteries in series and a 12V car battery.

Next, we can model the battery (2 terminal voltage source) as a Thévenin equivalent circuit as shown below.

Both batteries have an identical V_T (open circuit voltage), so what is the difference between the two?

By voltage divider formula:

$$V_M = V_T \frac{R_M}{R_T + R_M}$$



Since V_T and R_M are the same for both batteries, the difference is in R_T.

Battery model (cont)

The typical car battery has a Thévenin resistance of 0.05 Ω.

$$V_M = 12\frac{0.4}{0.05+0.4} = 10.67V \qquad I = \frac{12}{0.05+0.4} = 26.67A$$

The typical AA size battery has a Thévenin resistance of 20 Ω.

$$V_M = 12\frac{0.4}{20+0.4} = 0.235V. \qquad I = \frac{12}{20+0.4} = 0.588A$$

That is not sufficient voltage and current to start a car.
This is no surprising because you don't expect AA size batteries to be sufficient to start a car!
Typically, the resistance (R_T) is a function of the cross section of the device (in this case, battery). The larger the cross section, the smaller the resistance.

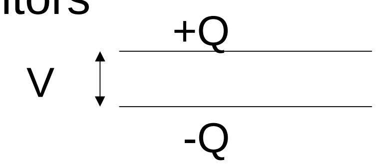
Points To Remember:

- Power flow can be calculated from the expressions **P = VI** for two-terminal circuit elements and **P = Σ** $V_n I_n$ for multi-terminal circuit element. However, it is essential that the signs of the various voltages and currents be stated correctly.
- If voltage and current vary, the quantity *v(t)i(t)* is known as the *instantaneous power*. The *time-averaged power* is the average over time of the instantaneous power.

Duality

- \+ -
- black, white
- Voltage, Current
- Capacitors, inductors.

Capacitors



- Q = CV

  where Q=charge

  V=voltage difference between 2 plates

  C= capacitance

take derivative with respect to t on both sides

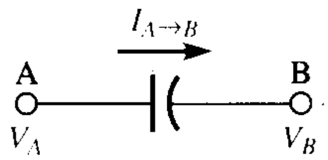
- dQ/dt = C dV/dt = i
- i=CdV/dt (remember I-V diagram)
- current = constant X time derivative of voltage
- Ohm's law tells us about the relationship between

V and I for a resistor. This equation describes the relationship between i and V for a capacitor.

Capacitors (continue)

- The I-V relationship for a capacitor is:

$$I_{A\to B} = C\frac{d}{dt}(V_{AB})$$



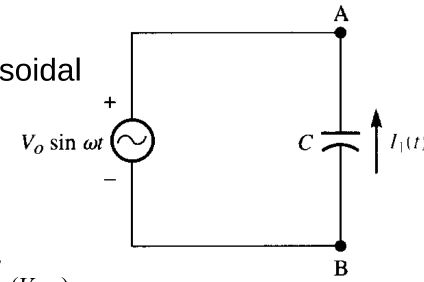
Where C is the capacitance in Farad or F, mF, μF, nF, pF

Notice the current depends on the derivative. If the derivative is zero, then there is no current. The derivative is zero when the voltage remains constant and does not change with time. An example would be: dc circuit.

∴No current goes through a capacitor in a dc circuit.

Capacitor example

- Find the current $I_1(t)$ that passes through the capacitor as shown. The voltage source is a sinusoid $V_0\sin\omega t$, where V_0 and ω are given constants and t is time.



Since the voltage source is sinusoidal (change with time), the current across the capacitor is nonzero.

From circuit $V_A - V_B = V_0 \sin\omega t$

From previous page $I_{A\to B} = C\frac{d}{dt}(V_{AB})$

$$I_{A\to B} = C\frac{d}{dt}(V_0 \sin\omega t)$$

$$I_{A\to B} = CV_0\omega\cos\omega t$$

But i is from B to A direction:

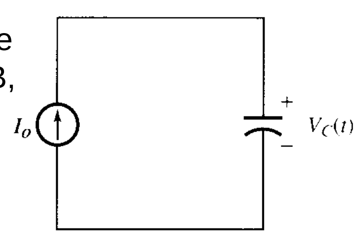
$$I_{B\to A} = I_1 = -CV_0\omega\cos\omega t$$

Capacitor example

- Find the $v_c(t)$ across the capacitor as shown. The current I_0 through the current source is constant.

Apply the I-V equation for capacitor from the previous page (when the current direction A->B, then Voltage is V_{AB})

$$I_{A\to B} = C\frac{d}{dt}(V_{AB})$$



In the circuit on the right, current I_0 is entering cap.

$$I_0 = C\frac{d}{dt}(V_c) \Rightarrow \frac{I_0}{C}dt = dV_c \Rightarrow V_c = \int dV_c = \frac{I_0}{C}\int dt = \frac{I_0 t}{C} + K$$

∴ The voltage is increasing proportional to time, cap. Is charged.

Capacitors in series and parallel

- We studied that when R_1 and R_2 are in series, the R_{eq} is equal to R_1+R_2, the R_{eq} for resistor in parallel is: $R_1R_2/(R_1+R_2)$
- What about capacitors?

If 2 capacitors are in parallel, the voltage across both would be the same (=V)

$$I = C_1\frac{dV}{dt} + C_2\frac{dV}{dt} = (C_1 + C_2)\frac{dV}{dt} = C_{eq}\frac{dV}{dt} \qquad C_{eq} = C_1 + C_2$$

If 2 capacitors are in series, the current across both would be the same (=I)

$$I = C_1\frac{dV_1}{dt} + C_2\frac{dV_2}{dt}$$

Capacitors in series

If 2 capacitors are in series, the current across both would be the same (=I) $I = C_1 \frac{dV_1}{dt} = C_2 \frac{dV_2}{dt}$ (1)

In series, $V = V_1 + V_2 \Rightarrow \frac{dV}{dt} = \frac{dV_1}{dt} + \frac{dV_2}{dt}$ (2)

From eq (1) $I = C_1 \frac{dV_1}{dt} = C_2 \frac{dV_2}{dt} \Rightarrow \frac{dV_2}{dt} = \frac{C_1}{C_2}\frac{dV_1}{dt}$ Substitute into eq (2)

$$\Rightarrow \frac{dV}{dt} = \frac{dV_1}{dt} + \frac{C_1}{C_2}\frac{dV_1}{dt} = \frac{dV_1}{dt}\left(\frac{C_2 + C_1}{C_2}\right) \quad (3)$$

From eq (1) $I = C_1 \frac{dV_1}{dt} \Rightarrow \frac{dV_1}{dt} = \frac{I}{C_1}$ Substitute into eq (3)

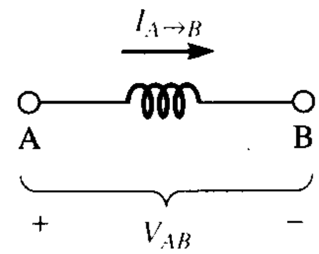
$$\Rightarrow \frac{dV}{dt} = \frac{I}{C_1}\left(\frac{C_2 + C_1}{C_2}\right) = I\left(\frac{C_2 + C_1}{C_1 C_2}\right) \Rightarrow I = C_{series}\frac{dV}{dt} \qquad \therefore C_{series} = \frac{C_1 C_2}{C_1 + C_2}$$

Series capacitors are similar to parallel resistors.

Inductors

- Ideal inductor is a 2-terminal device.

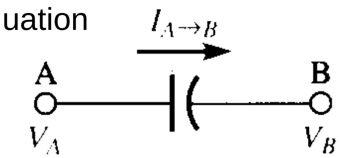
$$V_{AB} = L\frac{d}{dt}I_{A\to B}$$



- Where L is a constant called inductance with unit in Henry or H, mH, µH, nH.

Notice similarity with capacitance equation

$$I_{A\to B} = C\frac{d}{dt}(V_{AB})$$



Inductors (example)

- Assume there is no current going thru an inductor, at time =0, a time varying current i(t) is applied thru the inductor terminals. What is the voltage across the inductor terminals as a function of time.

Apply the inductor equation in the previous page:

$$v(t) = L\frac{d}{dt}i(t)$$

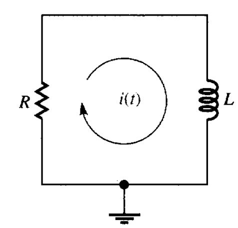
If a time varying voltage v(t) is applied across its terminals. What is the current thru the inductor as a function of time.

Again, apply the inductor equation in the previous page:

$$v(t) = L\frac{d}{dt}i(t) \Rightarrow di(t) = \frac{1}{L}v(t)dt \Rightarrow i(t) = \int di(t) = \int_0^t \frac{1}{L}v(t)dt$$

Inductance (example)

- Write a loop equations for the loop current i(t)



The voltage drop in the inductor is: $L\frac{d}{dt}i(t)$

The voltage drop in the resistor is: $i(t)R$

So, the loop equation (KVL) is: $L\frac{d}{dt}i(t) + i(t)R = 0$

What if you have the same circuit in series with a capacitor C?

Recall the I-V eq. for Capacitor: $i(t) = C\frac{d}{dt}v(t)$

$$i(t) = C\frac{d}{dt}v(t) \Rightarrow dv(t) = \frac{1}{C}i(t)dt \Rightarrow v(t) = \int \frac{1}{C}i(t)dt$$

The voltage drop in the capacitor is: $v(t) = \int \frac{1}{C}i(t)dt$

So, the new loop equation (KVL) is: $L\frac{d}{dt}i(t) + i(t)R + \int \frac{1}{C}i(t)dt = 0$

Parallel/Series Inductors

- Inductor in series is similar to resistor (sum):

$$L_{series} = L_1 + L_2$$

- Inductor in parallel is similar to resistor (product over sum):

$$L_{parallel} = \frac{L_1 L_2}{L_1 + L_2}$$

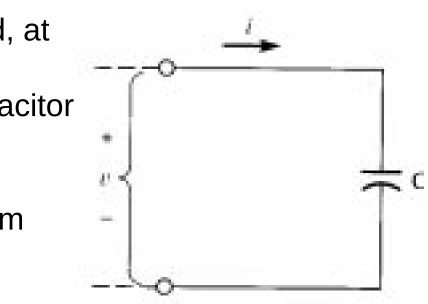
Energy Storage

Element		equation	energy+ or -?	energy eq.
• Resistor	R	V=IR	dissipate energy	V^2/R or I^2R
• Capacitor	C	I=CdV/dt	stored	???
• Inductor	L	V=LdI/dt	stored	???

Assume the capacitor is uncharged, at t=0, a voltage v(t) is applied. The instantaneous power enter the capacitor is: p(t)=v(t)i(t)

The energy enter the capacitor (from time=0 to t)is:

$$E = \int_0^t p(t)dt = \int_0^t v(t)i(t)dt$$



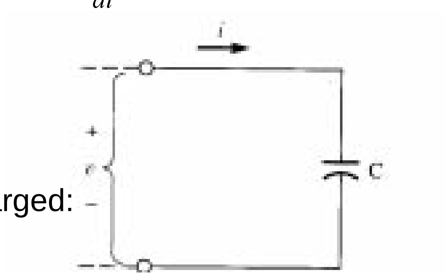
Energy Storage (continue)

$$E = \int_0^t p(t)dt = \int_0^t v(t)i(t)dt$$

Recall the I-V eq. for Capacitor: $i(t) = C\frac{d}{dt}v(t)$

$$E = \int_0^t vidt = \int_0^t vC\frac{dv}{dt}dt = \int_0^t \frac{1}{2}Cd(v^2)$$

$$E = \frac{1}{2}C[v(t)]^2 - \frac{1}{2}C[v(0)]^2$$



If the Capacitor is initially uncharged:

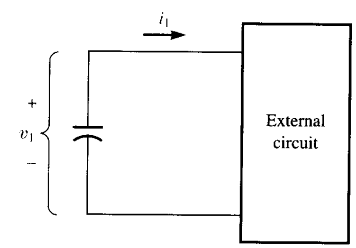
$$E = \frac{1}{2}C[v(t)]^2$$

∴Energy store in a capacitor is: 1/2 CV^2

Energy Storage (continue)

- Once energy is stored in capacitor, is there way we can regain the energy?

Consider the circuit on the right. Suppose the capacitor is initially charged to voltage V, is to discharge to an external circuit.



The energy recovered from the capacitor (entered the external circuit) after an infinite length of time:

$$E = \int_0^\infty p(t)dt = \int_0^\infty v_1(t)i_1(t)dt$$

$$E = \int_0^\infty v(t)\left[-C\frac{dv}{dt}\right]dt = -C\int_0^\infty v(t)dv = -C\int_0^\infty d(\frac{v^2}{2})$$

Minus sign is due to the reference direction

$$E = -\frac{1}{2}C\left[v(\infty)^2 - v(0)^2\right]$$

The voltage is fully discharged (V=0) when t=∞

$$E = \frac{1}{2}C\left[v(0)^2\right]$$

Energy storage for inductors

$$E = \int_0^t v(t)i(t)dt = \int_0^t L\frac{di}{dt}idt = \int_0^t \frac{1}{2}Ld(i^2)$$

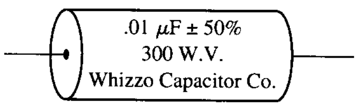
$$E = \frac{1}{2}L[i(t)]^2 - \frac{1}{2}L[i(0)]^2$$

$$E = \frac{1}{2}LI^2$$

Where I is the final current at time t, assumed the current through the inductor is zero.

Practical Capacitors and inductors

Practical capacitor = ideal capacitor in series with a resistor



(a)


(b)

The resistor part dissipates energy, thus, practical capacitor can never retain energy definitely, e.g. every DRAM cell need to be refreshed periodically to retain its value.

Capacitors use below 1GHz: mica, ceramic, and tantalum (see Figure a). Capacitors are specified by their capacitance value, maximum voltage applied across terminals, their tolerance.

Practical Capacitors and inductors (cont)

- a practical inductor can be replaced by an ideal inductor in series with a resistor and then in parallel with a capacitor

Again, practical inductor can dissipate energy because of the present of the resistor.