

Solving 1st order Ordinary Differential Equations (ODEs):

0. Put ODE in Standard Form
- I. Find homogenous solution $y_H(t)$
- II. Find particular solution $y_P(t)$
- III. Form complete solution $y(t) = y_H(t) + y_P(t)$
- IV. Use initial conditions to find unknown coefficients

0. Standard Form

Desired form for a 1st order ODE is:

$$\frac{dy(t)}{dt} = \alpha y(t) + f(t)$$

Or more compactly:

$$y'(t) = \alpha y(t) + f(t)$$

Here $y(t)$ is a “state variable”, in the sense that y is a variable tracking the state of some physical process in the universe, like capacitor voltage or inductor current or the velocity of a bowl of petunias that has suddenly appeared over the surface of a distant planet or anything at all.

The differential equation tells us literally how $y(t)$ changes in time (because the left hand side says “the change in $y(t)$ with respect to time).

The $\alpha y(t)$ term tells us how the system’s state evolves as a function of its current state. For the example of the velocity of a bowl of petunias, this would capture air resistance, which increases with the speed of the bowl. In general α can vary as a function of time (this would be akin to taking into account a varying gravitational field, or in circuits, a varying capacitance, resistance, or inductance), but we won’t typically be doing this.

The $f(t)$ term tells us how the system’s state evolves independently of its current state. Again to the bowl, this would represent the influence of gravity, which exerts its cruel forces regardless of whatever else is going on.

If there is just one source in a circuit, $f(t)$ is probably just going to be that source or some scaled version of it. If there are many sources, then $f(t)$ will be some mix of all the sources that comes from the node voltage equations that you write in order to get your ODE.

I will call our equation from this point on the **SFODE**, a word I have just made up.

- I. Find the homogeneous solution $y_H(t)$
 - A. Set $f(t) = 0$

- B. Replace the state variable $y(t)$ with 1
Replace the state variable $y'(t)$ with s
- C. Solve for s (trivial)
- D. The homogeneous solution is just $y_H(t) = k_1 e^{st}$

The point of doing all this is to find the function which explicitly tells us the system's state if we just let the system do what it wants, independent of outside forces.

In circuits, we call the homogenous solution the "natural response", because it is what the circuit wants to do naturally.

II. Find the Particular Solution

- A. Make up some function $y_p(t)$, which hopefully satisfies our **original SFODE** [not the $f(t)=0$ version]. A good guess for $y_p(t)$ is:
 - a. $y_p(t) = Af(t) + Bf'(t) + Cf''(t) + \dots$

This doesn't always work but this isn't an ODE class, so I will give hints any time you have a strange $f(t)$. We'll almost always use $f(t)=\text{constant}$ or $f(t) = V \cos(\omega t)$ anyway

- B. Plug your guess into the original SFODE, and pick constants A, B, C, and so forth so that the particular solution $y_p(t)$ works.
- C. Verify your answer from part B when you get done finding constants, and make sure it all works out [optional]

We call $f(t)$ the forced response, because it tells us how the circuit reacts to meddling outside forces, intent on ruining the fragile balance of an otherwise stable circuit situation.

III. Form the complete solution $y(t)$

- A. $y(t) = y_H(t) + y_p(t)$

There's nothing really to this step. Done.

IV. Use initial condition $y(t_{initial})$ to find the unknown coefficient k_1 from the homogenous solution.

- A. Write out

$$y(t_{initial}) = y_H(t_{initial}) + y_p(t_{initial})$$

- B. Solve for k_1

Usually $t_{initial}$ is zero, but it isn't always.

Often these initial conditions are not explicitly given to you, and you have to think about what they should be. Finding a capacitor voltage for a circuit that has had no source for a long time, well it's probably zero. We'll go over more examples in class.

Obviously this is a long, seemingly arbitrary procedure, and after you do this a few times, you'll realize that the answer for a simple RC (resistor+capacitor) or RL (resistor+inductor) circuit is pretty much always the same. This is the so called "intuitive method" given in the book.

For a detailed example, see the webcast or the handwritten lecture notes from Lecture #8. Hope this helps!