P1: Basics - Things you now know that you didn't know you knew (25 pts)

a) Birds routinely land and relax on power lines which carry tens of thousands of volts of electricity. Explain why these birds do not get electrocuted. (6 pts)

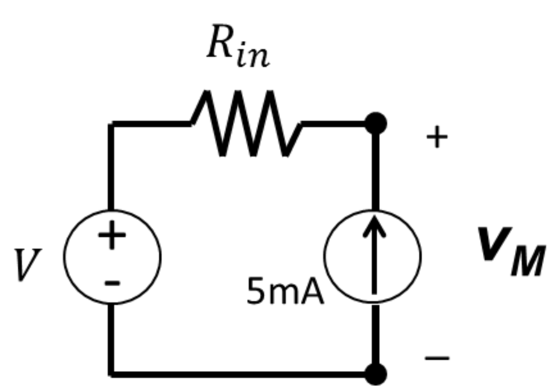
The resistance of the air is too large for current to flow from the line to the bird to the ground. Alternately, the resistance is too large for current to divert from the wire, into the bird, and then back into the wire. It is not because of wire insulation. Birds could land on uninsulated wires just fine. 6 pts for right answer. 1 pt for something vaguely correct.

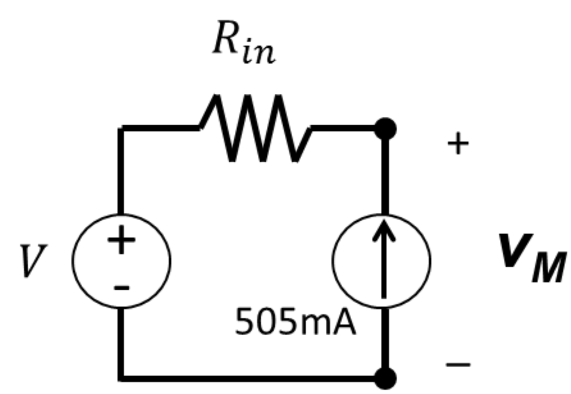
b) Suppose your car battery is dead, and you need to charge it using another car's battery. You have a pair of cables which you can use to connect the terminals of the batteries. In order to charge your car battery, should you connect the same terminals (positive of one battery to the positive of the other, and the same with the negative) or the opposite terminals (positive of one battery to the negative of the other battery)? Why did you choose this configuration?

First note that real batteries have internal resistance, so the universe will not explode if you directly connect two car batteries. Sorry guys.

Next, if the opposite terminals are connected, then P=VI and common sense tells us that both batteries will be supplying power, and the internal resistances will be consuming power, so clearly no power is being delivered by either battery. If the same terminals are connected, then we can see that if one battery is of a higher voltage than the other, it will provide power to the other battery. Key to this realization is to know that battery voltages drop as the battery is depleted. This was not discussed in class explicitly, but was hopefully clear.
6 pts for right answer. 2 pts if you try to say there is no current if two batteries are connected + to + because voltage sources are perfectly balanced.

c) A standard procedure for testing the internal resistance of a battery is the "dual pulse" test. We first attach an ideal 5 mA current source between the terminals of the battery, so that current flows in the usual direction (positive to negative), and measure the voltage across the battery terminals. We then remove the 5 mA source, and attach an ideal 505 mA current source instead, and again measure the battery terminals.





i. Suppose that we find a 1.485V voltage with the 5 mA source, and a 1.385 with the 505 mA source, what is the internal resistance?

Name: ____________________

There was a bug in the above diagram, current should have been flowing from + to – as it says in the text!

$\frac{V-1.485}{R_{in}} = 5mA$ $\frac{(V-1.385)}{R_{in}} = 505mA$

Easy way to solve these is to subtract 2nd from first, giving $\frac{-0.1}{R_{in}} = -500mA$, or $R_{in} = 0.2\Omega$. **If you did this the other way around and got $R_{in} = -0.2\ \Omega$, then you are owed a point.** 3 pts for right answer. 2 pts if sign error or algebra mistake.

ii. Assuming the battery is perfectly linear (i.e. accurately modeled by a Thevenin equivalent), is it possible to find the voltage provided by the battery with no load attached **using the data above**? If so, what is it? If not, why not?

Plug R_{in} in to one of our equations above, e.g. $\frac{V-1.485}{0.2} = 5mA$, giving $V - 1.485 = 1mV$, or finally $V = 1.486$. If you did it with the current source the other way, you should still have gotten 1.486V.

3 pts for right answer. 2 pts if sign error or algebra mistake

d) An EE40 student has constructed a purely resistive circuit which drains the connected battery too quickly. He decides to add another battery in parallel to fix the problem (assume internal resistance of the battery is not important).

a. Will adding another battery in parallel actually reduce the power provided by each battery?
For this problem, imagine replacing the resistive circuit in the middle with an equivalent resistor. In this case, each battery will provide only half the current that a single battery would provide (draw it if you're confused), and thus will use only half the power. 3 pts

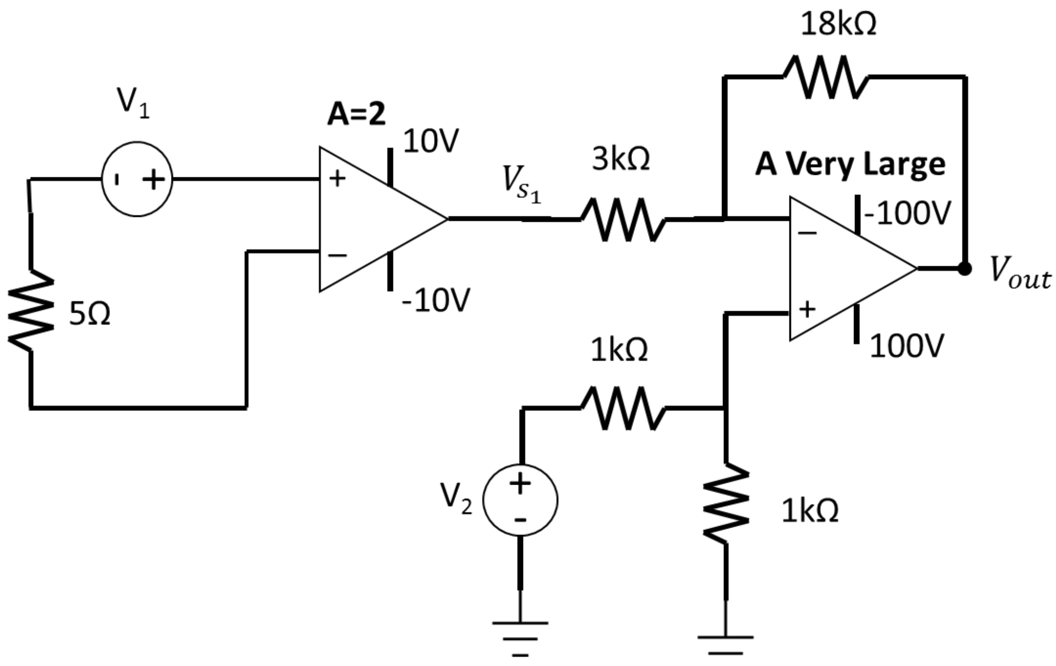
b. Will the load still receive the same total amount of current?
Yes, because the voltage across the resistive circuit is still the same and so Ohm's Law tells us the current will be the same. 3 pts

e) An EE40 student takes a random black box device (the contents of the box are unknown) and connects it to a 5V source. He finds that there is a 1A current and claims that the black box must contain only resistors, and that these resistors have a 5Ω equivalent resistance. Is the student right or wrong? If he is right, why? If he is wrong, give a specific counterexample of something else that might be in the box.

No, for example, there could just be a 1A current source. 6 pts

Name: ______________________

P3: Op-Amptimus Prime **(25 pts)** **[still needs grade breakdown but I am falling asleeeeep, so here's the answers just for now]**



a) Explain why it is safe to use the summing point constraint on the op-amp on the right.

We have negative feedback and a very large A. +5 pts

b) Find V_{s_1} and V_{out} as a function of $V_1$and V_2. Assume that the input signals are small enough that the op-amps do not saturate. If you're not sure about V_{s_1}, you can also give V_{out}in terms of V_{s_1}.

$V_{s_1}$is by definition of op-amps just $A(V_{left}^{+} - V_{left}^{-})$=2$V_1$ (+5 pts) (-2 for not getting V_1 in there)

In the portion on the right, we have what looks mostly like an inverting amplifier, except that the + side is not grounded. We can see by inspection that $V^{+} = \frac{V_2}{2}$. Thus, we know that $V^{-} = \frac{V_2}{2}$.

Then we have:

$$\frac{\frac{V_2}{2} - 2V_1}{3000} + \frac{\frac{V_2}{2} - V_{out}}{18000} = 0$$

Solving, we just get $V_{out} = \frac{7}{2}V_2 - 12V_1$ +10 pts

c) If $V_2 = 1V$, and $V_1 > 0$, how large must V_1be before one of the op-amps saturates? Which op-amp will saturate first? (If you didn't get part b, you may assume for this sub-problem that $V_{out} = V_1 + V_2$ [this is not the right answer to part b]) (+5 pts)

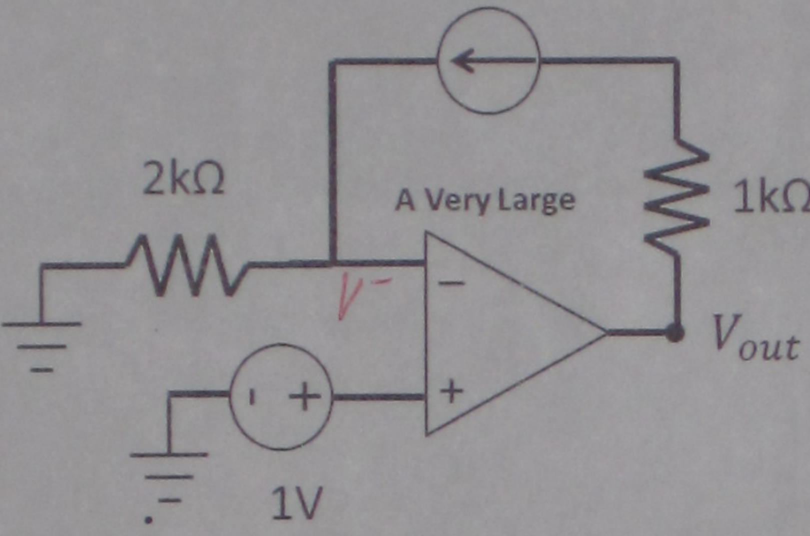
Left saturates at 5V. On the right we have $\frac{7}{2}V_2 - 12V_1$=$\frac{7}{2} - 12V_2$. So obviously left op-amp will saturate first.

Name: Answer Key

P2: Are these trick questions? Yes. All of them. **(30 pts)**

All of these problems might not quite be what they seem at first. There's a really easy way to do each of these that doesn't involve pages of algebra.

a) Find V^- in the circuit below [remember the summing point constraint doesn't always work. Maybe it does in this case, maybe it doesn't. You'll have to think about it carefully.]



Notice that a 1mA current is forced to flow into the V^- node. Since no current goes into the op amp, all of that current appears across the 2kΩ resistor.

$V^- = (1mA)(2k\Omega) = \boxed{2V}$.

+6 pts.

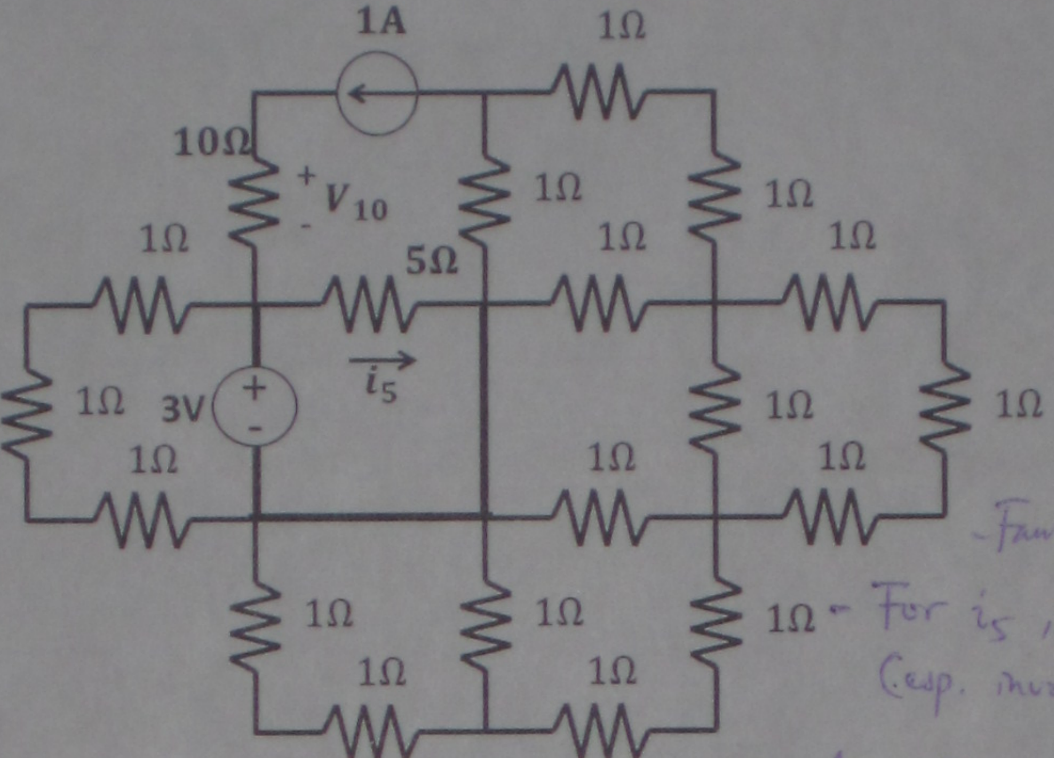
Assuming $V^- = V^+$ -4

Assuming no voltage drop across 1mA source -4

Assuming V_{out} = some value (not given) -4

Name: Answer Key

b) Find V_{10}, the voltage across the 10Ω resistor, and I_5, the current through the 5Ω resistor



- Faulty superposition -2
- For i_5, faulty KCL statements (esp. involving 3V source) -2
- Assuming no current flows -2

$V_{10} = (1A)(10\Omega) = \boxed{10\ V}$ +3 pts. All or nothing

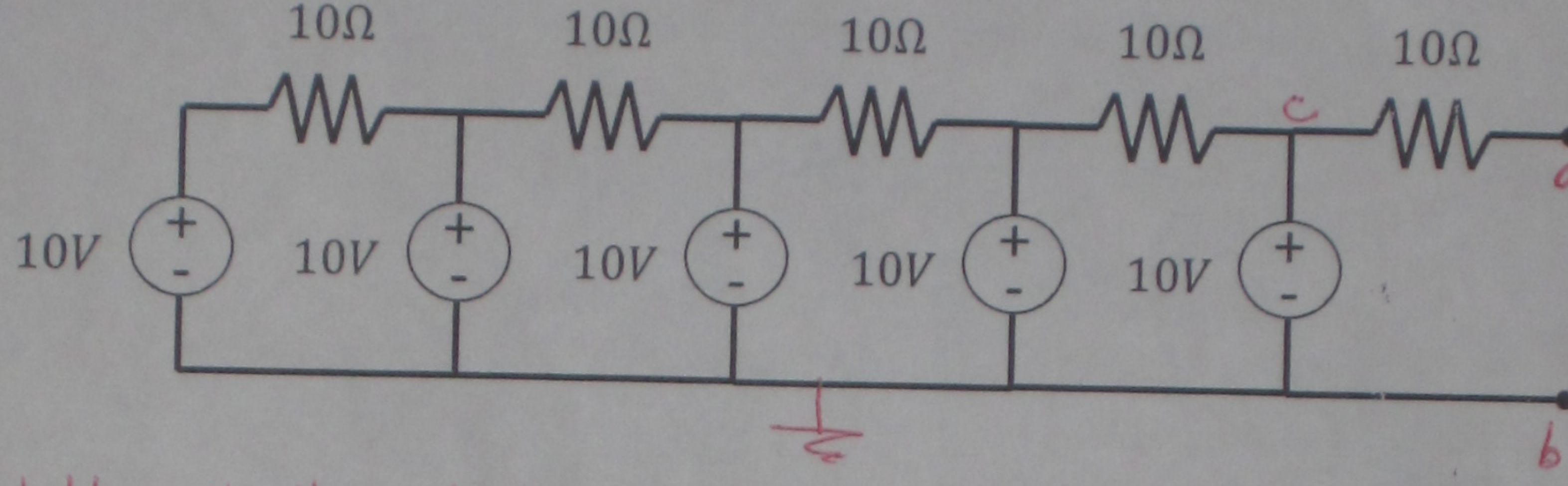
$i_5 = \frac{3V}{5\Omega} = \boxed{0.6\ A}$. +3pts.

For V_{10}, notice that the 1A source in the same branch forces a 10V drop across the resistor.

The 3V source defines a 3V drop across the 5Ω resistor. So the current i_5 cannot be affected by the rest of the circuit.

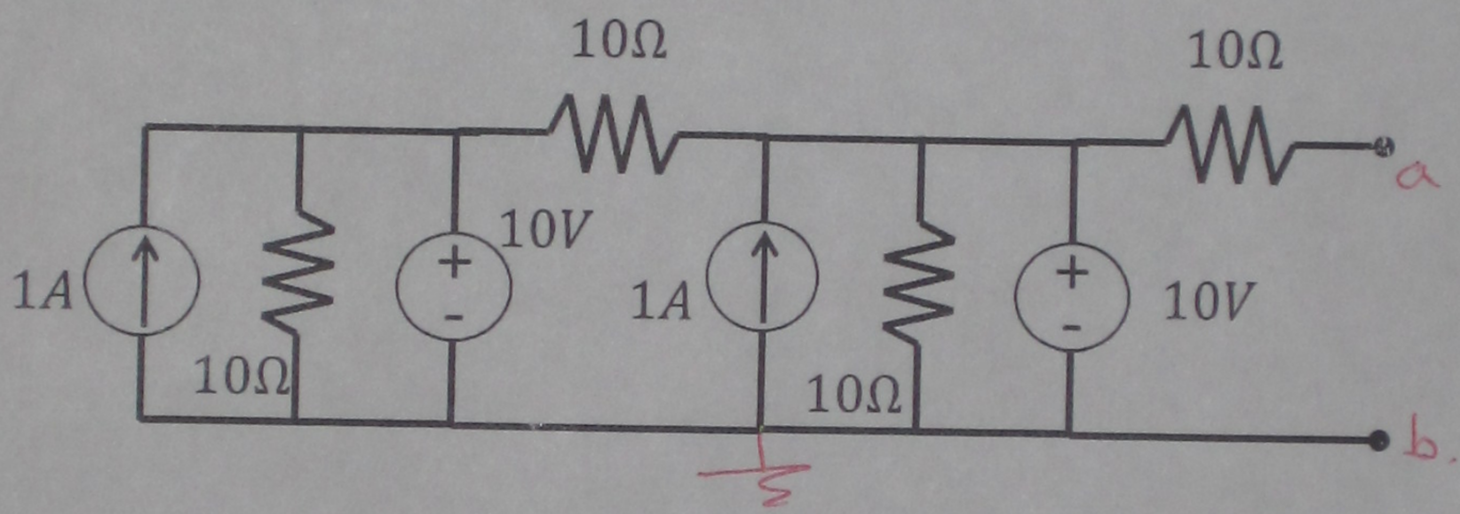
Name: Answer Key

c) Find the Thevenin equivalent of the circuit below at the two terminals on the far right:

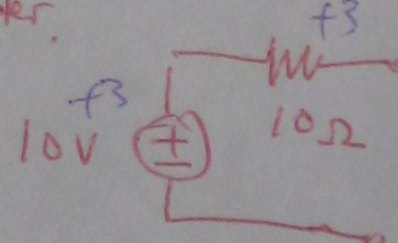


Let's call the node at b to be ground. Then $V_{th} = V_a$. Notice that $V_c = V_a = 10V$, as no current can flow through the rightmost resistor. Thus, $\boxed{V_{th} = 10V}$ +3. If we short the rightmost 10 V source to find R_{th}, we immediately see that the rest of the circuit doesn't matter. So $\boxed{R_{th} = 10\Omega}$ +3

d) Find the Thevenin equivalent of the circuit below at the two terminals on the far right:



The approach to this problem is identical to the one above. Anything to the left of the rightmost 10V source and the 10Ω resistor doesn't matter.



For both problems (partial credit)

Writing correct node-equations: +2

" incorrect " : +1

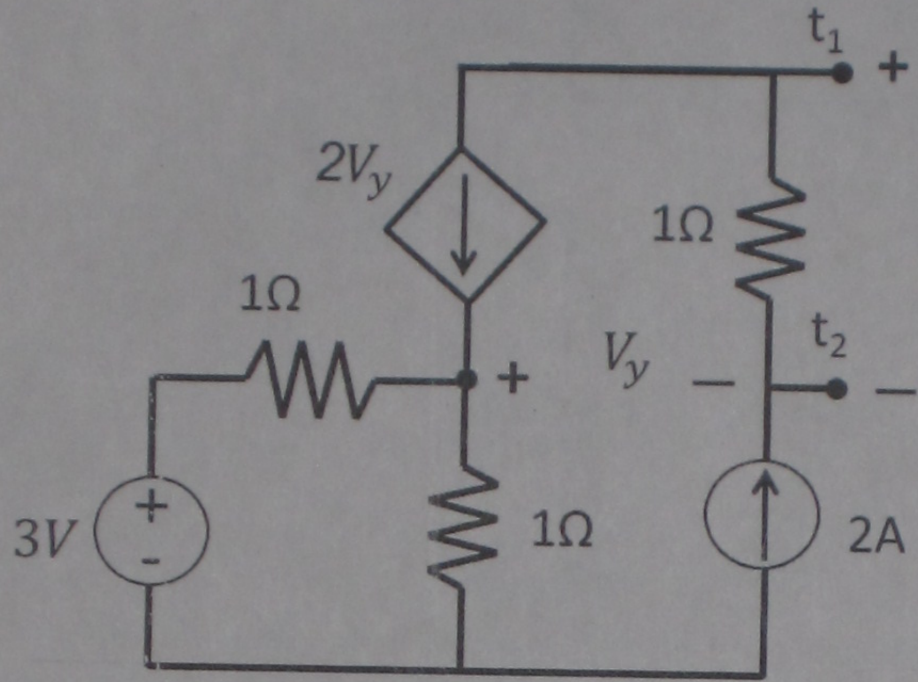
Zeroing out sources to find R_{th}: +2

Not recognizing shorts -1

Incorrect application of superposition -2

Name: Answer Key

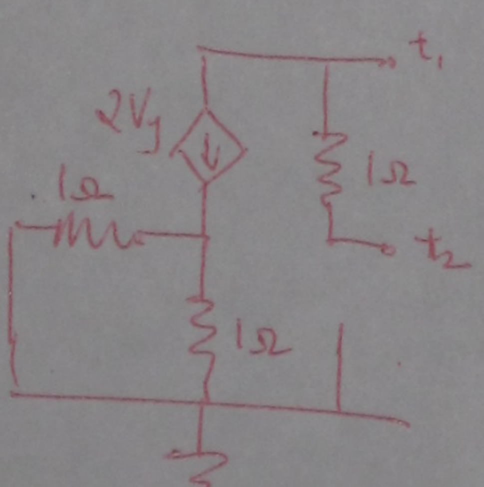
e) Find the Thevenin equivalent of the circuit below between terminals t_1 and t_2 [Think about the various algorithms we've used to find Thevenin equivalents. If you get stuck, complete the rest of these tricky problems and come back to this one and maybe some inspiration will reach you]:



Wrong sign of V_{th} -2
Wrong terminals (not t_1 & t_2)
Correct node-voltage eqs +2
Incorrect " " +1
Negative resistance -1

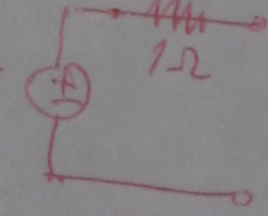
First we need to find V_{th} between t_1 & t_2. Notice that 2A flows across the resistor between t_1 & t_2 no matter what. By Ohm's Law, $t_2 - t_1 = (2A)(1\Omega) = 2V$

So $V_{th} = t_1 - t_2 = \boxed{-2V}$ +3 To find R_{th}, we ~~do~~ zero out all independent sources: Since t_1 & t_2 can only be connected via the 1Ω resistor, we conclude that



$R_{Th} = \boxed{1\Omega}$

+3



Name: Answer key

P4: Amplifier Design (15 pts)

You are tasked with designing an amplifier circuit with the following specifications:

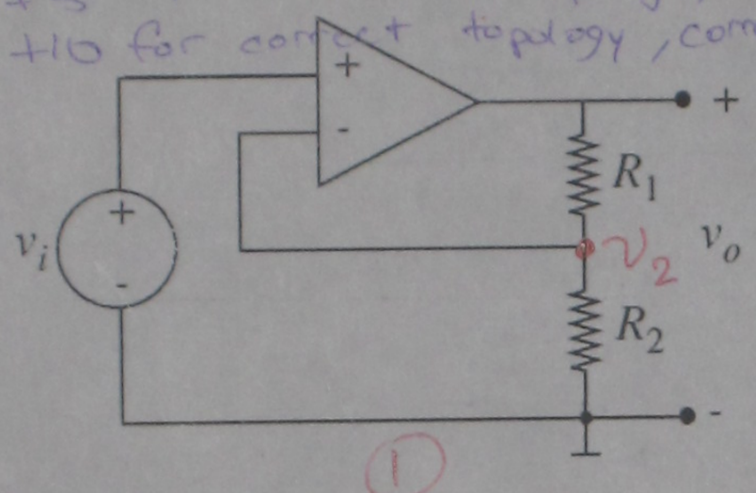
a) Generate an output gain V_o/V_i of approximately 1000 or approximately -1000 (either is fine)
b) Minimize the amount of power that the input source (V_i) must deliver to the amplifier

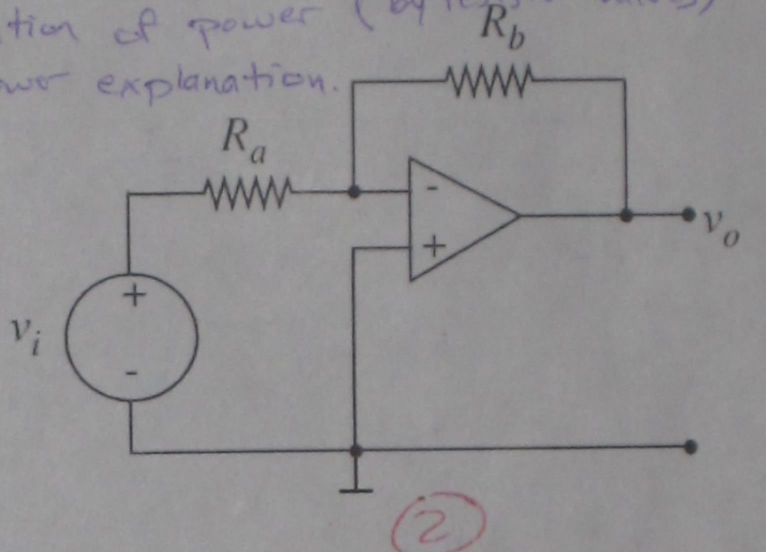
You may use any resistor values between 0.001Ω and 1,000,000Ω. You must use one of the two resistor configurations given below.

Choose one of the two circuits below, and choose resistor values so that you best meet the specification given above. If you don't know how to do one of these things, make an amplifier that meets the goal you understand how to meet.

Assume that we have op-amps which are **IDEAL**.

+2.5 for correct topology, no explanation of power (or incorrect explanation)
+5 for incorrect topology, explanation of power (by resistor values)
+10 for correct topology, correct power explanation.





Use the space below to pick a circuit and resistor values. Make sure to clearly indicate which circuit you have chosen, and draw a box around the resistor values you've selected.

Choose topology 1

No power delivered by the source since current into non-inverting input is 0A.

So $P = (v_i)(0A) = \boxed{0W}$

Have negative fb so apply summing

$v^- = v^+ = v_{in}, \quad v_2 = v^- = v_{in}$

By KCL @ node v_2,

$$\frac{v_o - v_2}{R_1} = \frac{v_2}{R_2}$$ no current into neg input.

$$v_o = R_1 (v_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = v_{in}\left(\frac{R_1 + R_2}{R_2}\right)$$

Give the equation which demonstrates that your circuit has the correct gain [approximately -1000 or 1000]. Also explain why your design minimizes power delivered by the input source.

$$\frac{v_o}{v_{in}} = \frac{R_1 + R_2}{R_2}$$

$$1000 = \frac{R_1 + R_2}{R_2}$$

$1000 R_2 - R_2 = R_1, \ \boxed{999 R_2 = R_1}$

Choose $\boxed{R_2 = 100\Omega}$

Then $\boxed{R_1 = 99.9 k\Omega}$

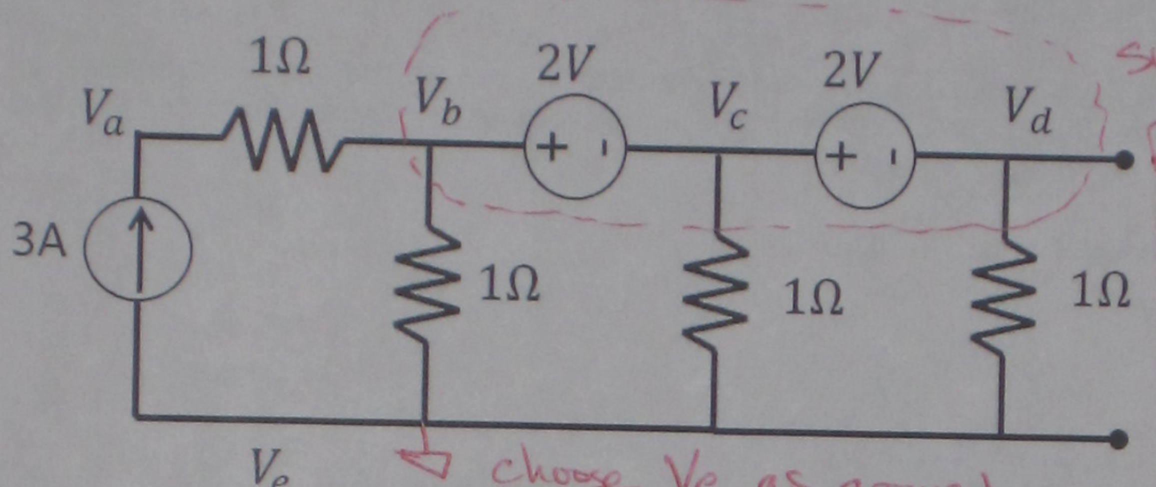
$[(999)(100\Omega)]$

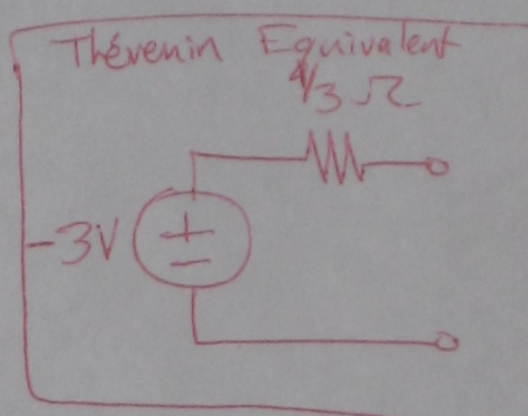
+5 for gain equation and good resistor values

Name: ____________________

P5: Thevenin Equivalents (35 pts)

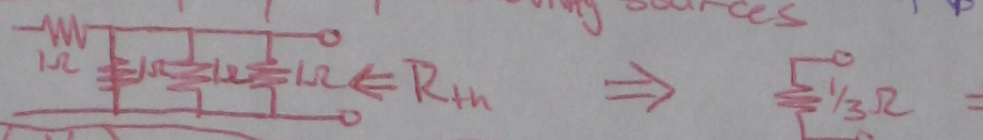
a) Find the Thevenin equivalent of the circuit between the terminals on the far right. Recall that you can always check your work by finding all three Thevenin variables (V_{th}, I_{sc}, R_{th}) and seeing that they agree. When showing your work, please use our voltage labels to make grading quicker and more fun for everyone (of course, you can still pick a ground!)





Rth simplify by removing sources

+7 for Rth, -3 for not removing sources



$\Rightarrow \boxed{R_{th} = \frac{1}{3}\Omega}$

Voc (=Vth)

$V_d = V_c - 2 \quad V_c = V_b - 2$

By KCL @ supernode

$3A = \frac{V_b}{1\Omega} + \frac{V_c}{1\Omega} + \frac{V_d}{1\Omega}$

$3 = (V_d+4) + (V_d+2) + V_d$

$3V_d = 3 - 6$

$\boxed{V_d = -3 = V_{oc} = V_{th}}$

Isc (=INorton)

$I_{sc} = -\frac{V_c}{1\Omega} - \frac{V_b}{1\Omega} + 3A, \quad V_b = V_c + 2$

$\boxed{I_{sc} = -2 - 4 + 3 = -3A}$

note $V_{th} = R_{th} * I_N$ holds!

+8 for Isc or Voc

b) Using your answer from part a, if you were to connect a 2/3Ω resistor across the terminals to the far right, how much power would be delivered to this resistor? (If you didn't get part A, you can assume V_{th}=3V and R_{th}=7/3Ω just for the purposes of this problem [these are not the right answers to part a])

+10

-3 for wrong resistance for current calc

With part A

$V_L = V_{th}\left(\frac{2/3}{1/3 + 2/3}\right) = \frac{2}{3}V_{th} = \boxed{2V}$

$I_L = \frac{V_{th}}{(2/3 + 1/3)} = \frac{-3}{1} = \boxed{3A}$

$\boxed{P_L = V_L * I_L = 6W}$

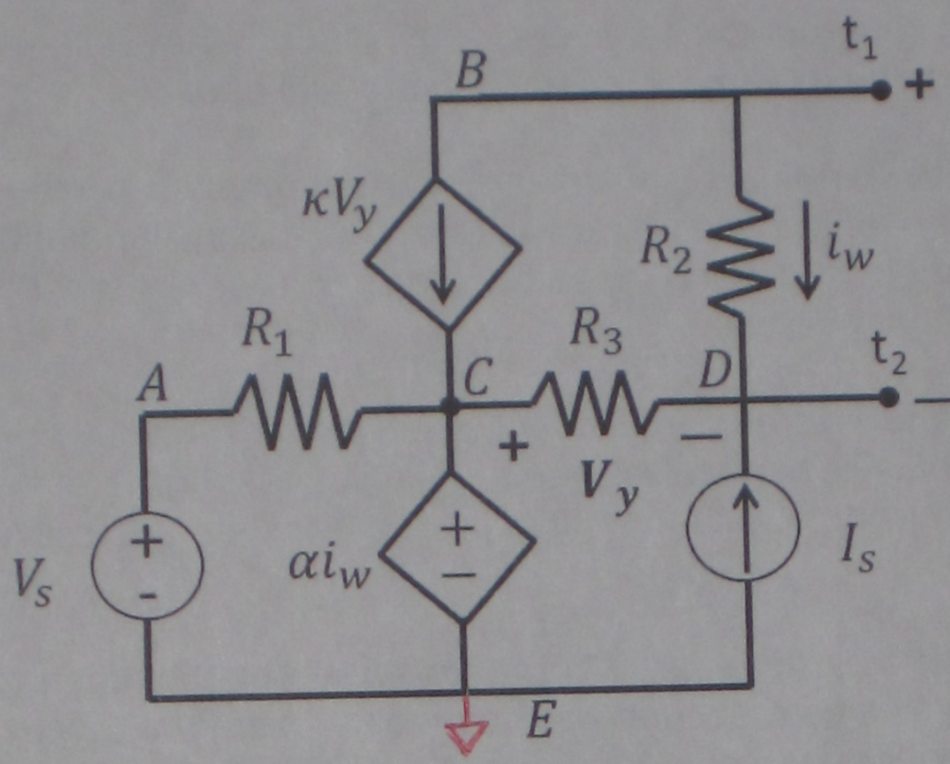
Using $V_{th} = 3V$, $R_{th} = \frac{7}{3}\Omega$

$V_L = 3\left(\frac{2/3}{2/3 + 7/3}\right) = \cancel{3}\left(\frac{2/3}{\cancel{3}}\right) = \frac{2}{3}V$

$I_L = \frac{(3)}{(2/3 + 7/3)} = \frac{3}{3} = 1A \Rightarrow \boxed{P_L = \left(\frac{2}{3}\right)(1) = \frac{2}{3}W}$

Name: ______________________

c) Write node equations (**DO NOT SOLVE!!**), which will allow you to find the open circuit voltage (voltage between terminals t_1 and t_2) of the circuit below:



Make sure you have as many equations as unknowns. $V_s, I_s, R_1, R_2, R_3, \kappa, \alpha$ are all known.

(1) $V_y = V_C - V_D$

(2) $i_w = \dfrac{V_B - V_D}{R_2}$

+2 per equation

−0.5 for sign error

−1 for voltage/current switching

(3) $\dfrac{V_B - V_D}{R_2} = kV_y$

(4) $\dfrac{V_C - V_D}{R_3} + \dfrac{V_B - V_D}{R_2} + I_S = 0$

(5) $V_C = \alpha i_w$

unknowns: V_B, V_C, V_D, V_y, i_w

5 unknowns
5 eqns ✓

P6: The hard problem that I promised **(25 pts)**
[Do this one last! It's much harder than the others]

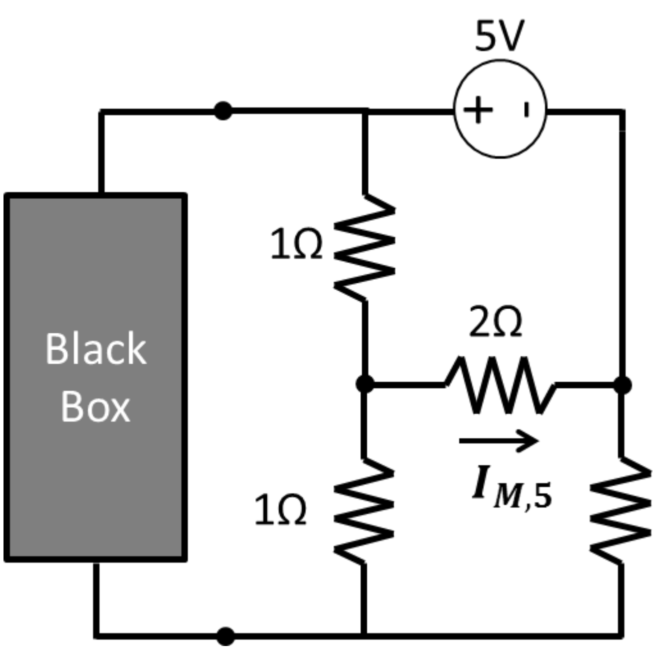
It is a grim post-apocalyptic future, but the survivors (you included), have adapted to this new way of life. Rugged and tiring though it may be, you are all happy.

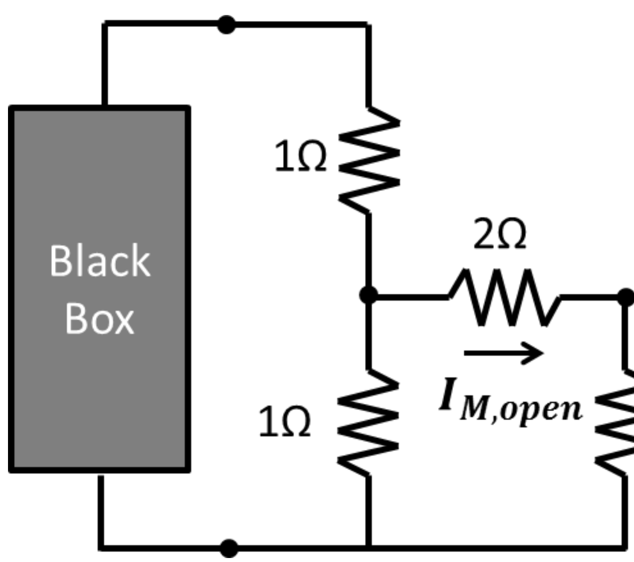
One day, you are brought a two terminal black box and it is extremely important to the future of the surviving members of the human race to know what is inside of this thing. Luckily, you know that (based on an old and faded but highly trustworthy label) that it contains only resistors and linear sources.

Thus, you know you can find the Thevenin equivalent of the circuit inside by applying tests using your lab equipment. "Piece of cake", you say", "I'll measure the open circuit voltage and the short circuit current, and then I will enjoy the gratitude and praise of my fellow man!"

Unfortunately, a dastardly dude has stolen all of your measurement equipment, and you are left with only the following asinine measuring device, abandoned in a corner, dusty and almost forgotten, designed for purposes unfathomable by persons unknown.

The device works as follows. It has two settings. In the first setting, it applies a voltage and gives you back the current through the 2Ω resistor, giving you $I_{M,5}$. In the other setting, it completely disconnects the voltage source, and still gives you the current through the 2Ω resistor, giving you $I_{M,open}$.





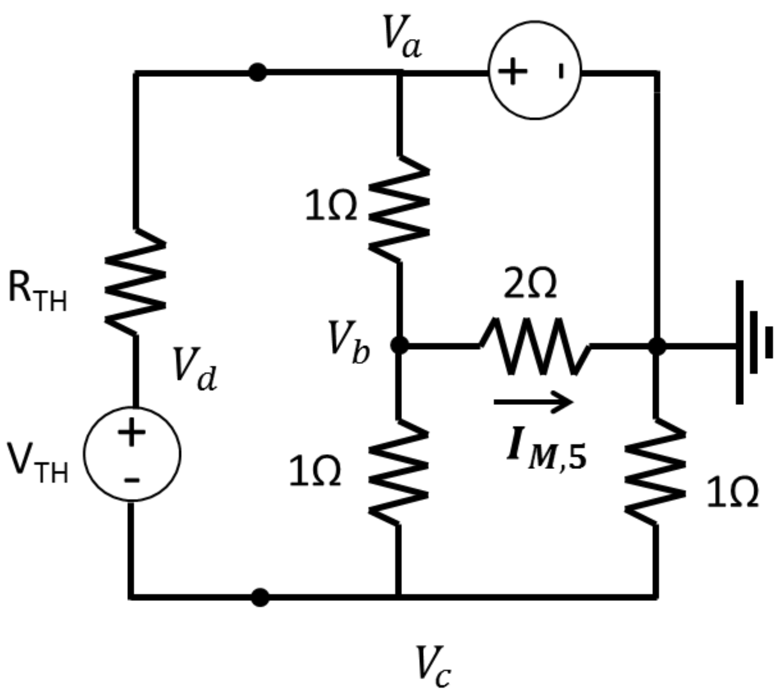
Name: ____________________

You try the first setting and find that $I_{M,5}$=1 ampere, and then when you try the other setting, you get that $I_{M,open}$ =1 ampere as well.

Using this information, determine what is in the box.

As announced in class, the bottom right resistor should be 1Ω.

Since the black box can be written as a Thevenin equivalent, we should replace the black box with a Thevenin circuit to make things more clear. Selecting a ground and making up node labels, this brings us to:



Where $I_{M,5}$ is just 1A.

We can try to find V_{TH} and R_{TH} just using this data. We could do node voltage, but we can just do a big series of KCL and Ohm's law steps to avoid having to do any algebra. First, we note that we know V_a is just 5V, and that $V_b$is just 1A*2Ω=2V (because we know $I_{M,5}$).

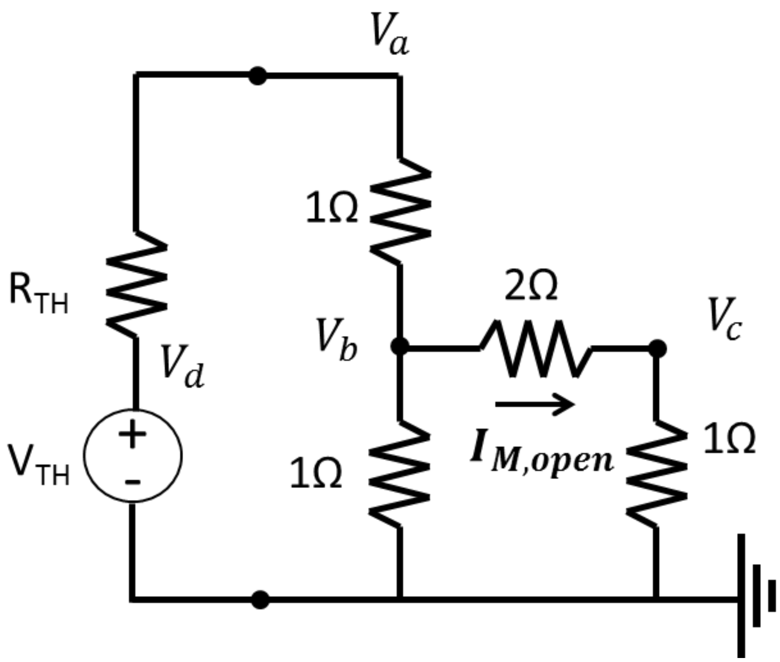
Next, we note that $i_{A\to B}$ is $\frac{(V_a-V_b)}{1\Omega} = 3A$. Since 1A goes to $I_{M,5}$, this means $i_{b\to c}$ must be 3A-1A=2A. This in turn gives us that $V_c = 2V - 2A \times 1\Omega = 0V$, and finally that $V_d = V_{th} + V_c = V_{th}$

Thus the total current coming from the Thevenin source is just $i_{b\to c} + i_{0\to c} = 2A$. Then, focusing on V_{th} and R_{th}, this gives us that $\frac{V_d-V_a}{R_{th}} = \frac{V_{th}-5}{R_{th}} = 2A$, or alternately that $V_{th} = 2R_{th} + 5$.

The basic picture there was that we were just trying to get a relationship between V_{th} and R_{th}. We know that since we only have one data point so far, we can't get both at once, so we settle for a relationship between them. If you got confused with that long sequence of KCL and Ohm's law steps, you can do node voltage instead, it's just a little slower.

Name: ______________________

So we next consider the other data point:



Here, we have immediately that V_c is 1V, and V_b is 3V. This means that total source current is just $\frac{V_b}{1\Omega} + \frac{V_b}{3\Omega}$=4A. This in turn means $V_a = V_b + 4A * 1\Omega = 3V + 4V = 7V$.

Then finally looking at the current from the source, we have $\frac{V_{th}-7}{R_{th}} = 4A$. So this gives us the V_{th} R_{th} relationship given by:

$V_{th} = 4R_{th} + 7$

We then just take our earlier constraint $V_{th} = 2R_{th\_} + 5$, and find where they intersect:

$V_{th} = 4R_{th} + 7 = 2R_{th} + 5$

This gives us that $R_{th} = -1$, and then we just plug into either equation and get V_{th}=3V

Done!

+1 pt for finding at least one useful quantity (total current provided by the source, for example)
+5 to 7 pts for making some progress
+22 points for getting the write equations, but not completing the algebra correctly