Announcements

- Midterm 2 this Thursday 11:30 to 1pm
  - 145 Dwinelle
  - Lectures 8-13, HW #4,5 (no small signal model)
  - Review Session this Wednesday 5-8pm in Cory 277

Lecture #14

OUTLINE

- Load Line and Small signal analyses of:
  - Common source amplifier
  - Source follower
  - Common gate amplifier

Reference Reading

- Hambley: Chapter 12.1-12.5
MOSFET Circuit

- First look at DC case to find Q point
  - Use load line technique
  - All capacitors are open circuit
  - From Q-point, get $g_m$ and $r_d$ for small signal AC model

- AC Small signal analysis
  - DC source is AC ground (because there is no AC signal variation).
  - All capacitors are short circuit (unless otherwise specified).

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Common Source Amplifier

![Common Source Amplifier Diagram](image-url)
Step 1: find Q point

\[ V_G = V_{DD} \frac{R_2}{R_1 + R_2} \]

\[ V_{GS} = V_G - I_D R_S \]

\[ V_{DD} = I_D (R_D + R_S) + V_{DS} \]

Load line:

From load lines, we get \( I_D \rightarrow \) and hence \( g_m \) and \( r_d \)
Small Signal Model:

\[ v_g = v_{in}, v_s = 0 \rightarrow v_{gs} = v_{in} \]

\[ v_o = \frac{R_1 R_D}{R_L + R_D} (-g_{m} v_{gs}) \]

\[ A_v = \frac{v_o}{v_{in}} = -g_{m} \frac{R_L R_D}{R_L + R_D} \]

\[ R_{in} = \frac{v_{in}}{i_{in}} = \frac{R_1 R_2}{R_1 + R_2} \]

For output impedance \( R_{out} \):

1. Turn off all independent sources.
2. Take away load impedance \( R_L \)

\[ v_{in} = 0, v_{gs} = 0, g_{m} v_{gs} = 0 \]

\[ R_{out} = \frac{r_g R_D}{r_d + R_D} \]

Source Follower

[Diagram of a Source Follower]
**Step 1: find Q point**

\[ V_G = V_{DD} \frac{R_2}{R_1 + R_2} \]

\[ V_{GS} = V_G - I_D R_S \]

\[ V_{DD} = I_D R_S + V_{DS} \]

From load lines, we get \( I_D \) and hence \( g_m \) and \( r_d \)
Small Signal Model:

\[ R'_L = \frac{1}{r_{d}^{-1} + r_{s}^{-1} + R_L^{-1}} \]
\[ v_{gs} = v_m - v_o \]
\[ v_s = g_m v_{gs} R'_L \]
\[ v_m = v_g (1 + g_m R'_L) \]
\[ A_i = \frac{v_m}{v_o} = \frac{g_m R'_L}{1 + g_m R'_L} \]
\[ R_m = \frac{v_m}{i_m} = \frac{R_i R_S}{R_i + R_s} \]

For output impedance \( R_{out} \):
1. Turn off all independent sources.
2. Take away \( R_L \)
3. Add \( V_x \) and find \( i_x \)
\[ v_x = v_o, v_y = 0, v_g = -v_x \]

\[ R'_L = \frac{r_{d} R_S}{r_{d} + R_S} \]
\[ i_x = \frac{v_x}{R'_L} - g_m (-v_x) = v_x \left( R'_L + g_m \right) \]
\[ R_{out} = \frac{1}{g_m + r_{d}^{-1} + R'_L} \]

Common Gate Amplifier

[Diagram of a common gate amplifier]
Step 1: find Q point

\[ V_{GS} = 0 - I_D R_S + V_{SS} \]
\[ V_{DD} + V_{SS} = I_D (R_D + R_S) + V_{DS} \]

Load line

The only difference in all three circuits are the intercepts at the axes.
Again from load lines, we get \( I_D \) and hence \( g_m \) and \( r_d \).
Small Signal Model:

\[
R'_e = \frac{1}{R_e^{-1} + R_n^{-1}}
\]
\[
v_{ip} = -v_{in}
\]
\[
v_c = -g_{ma}v_p R'_e
\]
\[
A_c = \frac{v_c}{v_{in}} = g_{ma} R'_e
\]
\[
i_n = -(g_{ma} v_p + \frac{v_{ip}}{R'}_e)
\]
\[
R_n = \frac{v_{in}}{i_n} = \frac{1}{g_{ma} + R'_e^{-1}}
\]

For output impedance \(R_{out}\):

1. Turn off all independent sources.
2. Take away \(R_L\).
3. Add \(V_x\) and find \(i_x\)

\[
R' = \frac{RR}{R + R_e}
\]
\[
i = \frac{v}{R} + g_{ma}v_p = v \left( R_e^{-1} + g_{ma} \right)
\]
\[
v_p = -g_{ma}v_p R', \text{ but } g_{ma}R' \neq 1 \therefore v_p = 0
\]
\[
R_{out} = R_p
\]

Review on Bode Plots

- Transfer function \(H(f) = \frac{V_{out}}{V_{in}}\)
- It is a complex number and typically a function of frequency.

\[
H(f) = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_{in}} \angle \left( \theta_{out} - \theta_{in} \right)
\]

\[
|H(f)| \angle \theta(f)
\]

\[|H(f)|\] is the magnitude

\[
\theta(f)\] is the phase

Both are functions of frequency \(f\).
**Review on Bode Plots**

\[ H(f) = H_1(f) \cdot H_2(f) \]

\[ |H(f)| = |H_1(f)| \cdot |H_2(f)| \]

\[ \theta(f) = \theta_1(f) + \theta_2(f) \]

\[ Y = 20 \log |H(f)| = 20 \log |H_1(f)| + 20 \log |H_2(f)| \]

- Bode plots include magnitude vs. plot and phase vs. frequency plots
  - Magnitude plots are log-log (dB vs. log Hz) \( \Rightarrow Y \) vs. log f
  - Phase are linear-log (angle in degrees or radians vs. log frequency)
- Factor of 10 increase in frequency = 1 decade
- Factor of 2 increase in frequency = 1 octave
- If \( H(f) \) can be factored into \( H_1(f) \) and \( H_2(f) \), it will be easier to plot bode plots for each and then sum them up.

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**Example: Second Order Circuit**

\[
\begin{align*}
\frac{V_{out}}{V_{in}} &= \frac{1}{j\omega C + R + j\omega L} = \frac{1}{1 - j\omega^2 LC + j\omega RC} \\
&= \frac{1}{\sqrt{1 - \omega^2 LC}^2 + (\omega RC)^2} \left[ -\tan^{-1}\left( \frac{\omega RC}{1 - \omega^2 LC} \right) \right]
\end{align*}
\]

In plotting the magnitude Bode plot, we should evaluate the extreme cases: \( \omega = 0 \) and infinity.

In addition, we should examine \( 0^+ \) (small positive), and the frequency at which \( |H(f)| \) is maximum or minimum.

Examine \( |H(f)| \), we can easily see that max occurs at \( \omega = \left( \frac{1}{\sqrt{LC}} \right) \)

Recall in Lec. 9 when we talked about 2\(^{nd}\) order circuit, we had a resonance frequency, which is exactly \( \omega = \left( \frac{1}{\sqrt{LC}} \right) \)
Magnitude Response

\[ |H(\omega)| = \frac{1}{\sqrt{1 - \omega^2 LC + (\omega RC)^2}} \]

\(\omega = 0\), \(|H(0)| = 1\), \(Y = 20 \log(1) = 0\) dB

\(0 < \omega < \omega_0\), \(\omega^2 LC < 1\), as \(\omega\) increases, \(|H(\omega)|\) decreases.

At \(\omega = \omega_0, 1 - \omega^2 LC = 0\), \(|H(\omega)| = \frac{1}{2\zeta} \sqrt{\frac{1}{R/C}} = \frac{1}{2\zeta} \sqrt{\frac{L}{C}}\)

\[Y = 20 \log \frac{1}{2\zeta} = 20 \left[ \log \frac{1}{2} + \log \frac{1}{\zeta} \right]\]

Recall, \(\zeta > 1\) → critically damped, \(Y(\omega_0) = -3\) dB
\(\zeta > 1\) → over damped, \(Y(\omega) < 3\) dB
\(\zeta < 1\) → under damped, \(Y(\omega) > 3\) dB
\(\zeta < 0.5\) → under damped, \(Y(\omega) > 0\) dB

For \(\omega \gg \omega_0\), As \(\omega\) increases, \(|H(\omega)| \approx \frac{\omega_0^2}{\omega^2}\)

\(Y\) reduces by 40 dB per 10x increase in \(\omega\), i.e., slope in bode magnitude plot = \(-40\) dB/dec

Phase Response

\(\omega = 0\), \(|H(\omega)| = 1, \ \theta = 0\)

\(\omega = 0^+\), \(|H(\omega)| = 1, \ \theta = \text{negative small value}\)

As \(\omega\) increases, \(\theta\) becomes more negative.

at \(\omega = \omega_0, 1 - \omega^2 LC = 0, \theta = \frac{\pi}{2}\) or \(-\frac{\pi}{2}\)

at \(\omega = \omega_0 + \delta, 1 - \omega^2 LC > 0, \theta \rightarrow \frac{-\pi}{2}\)

at \(\omega = \omega_0 + \delta, 1 - \omega^2 LC < 0, \theta \rightarrow \frac{\pi}{2}\)

For \(\omega \gg \omega_0\), As \(\omega\) increases, \(\theta\) is positive but decreases.
as \(\omega \rightarrow \infty, \theta \rightarrow 0^\circ\)