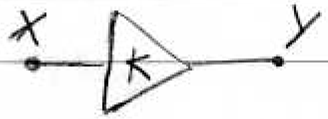
- FEEDBACK

Consider an amplifier



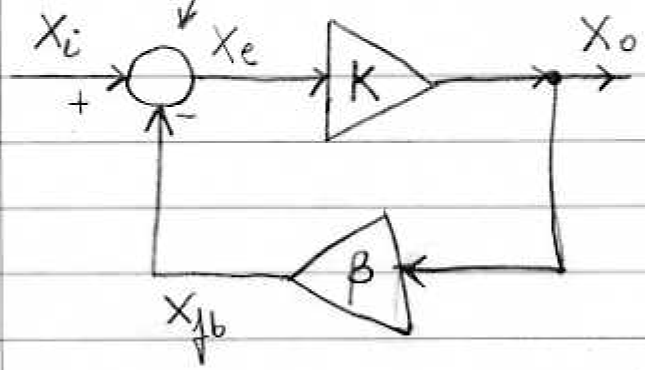
$$y = Kx$$

$$K = \frac{y}{x}$$

K is the gain by definition. Note that we are not specifying the nature of x and y, they can be generic quantities. So the gain is the ratio of the output to the input.

If we can observe the output then we can check if the amplifier is doing somenthing that it is not supposed to do.
We ask ourselves if there is a way of obtaining the desired outupt but adjusting the amplifiere input depending on the output.
This means feeding back the output to the input.

Perticular choice of combiner. It is a linear combiner.



We use part of the output (usually $\beta < 1$) to control the amplifier input.

What is the gain $\frac{X_o}{X_i}$?

$$X_{fb} = X_o \beta$$

$$X_e = X_i - X_{fb}$$

$$X_o = K X_e = K(X_i - X_{fb}) = K(X_i - X_o\beta)$$

$$\Rightarrow \quad X_o(1 + K\beta) = K X_i \Rightarrow \boxed{\frac{X_o}{X_i} = \frac{K}{1+K\beta}} = K'$$

!! Negative feedback because $K' < K$ →

K' is called closed-loop gain

$K\beta$ is called loop gain

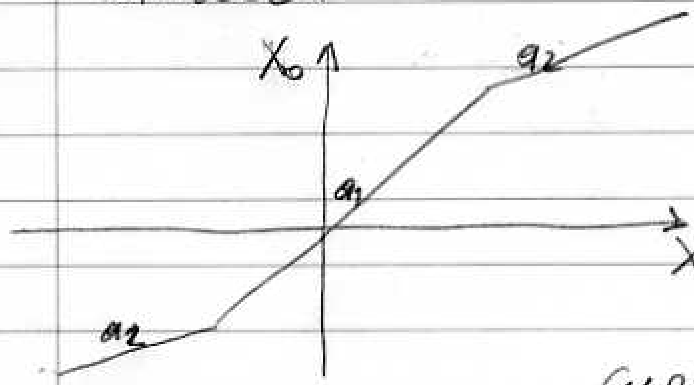
K is the open-loop gain

What happens if $K\beta \gg 1$?

$$K' = \frac{K}{1+K\beta} \simeq \frac{K}{K\beta} = \frac{1}{\beta}$$

the gain becomes independent on the open-loop gain but depends only on the "feedback network"!!

One immediate consequence is that distorsions are reduced.
By distorsion I mean the fact tha the amplifier gain is not perfectly linear:



The gain depends on the input signal. Consider the case where this curve can be approximated by three lines using two coefficients a_1 an a_2:

$$K_1' = \frac{a_1}{1+a_1\beta} \simeq \frac{1}{\beta} \qquad K_2' = \frac{a_2}{1+a_2\beta} \simeq \frac{1}{\beta}$$

The closed-loop gain is a constant!!
(well almost, but definitely better than the open-loop)

In general, gain sensitivity is reduced. It means that if K changes due to temperature, aging etc. the overall amplifier performance will stay the same:

$$\frac{dK'}{dK} = \frac{(1+K\beta) - K\beta}{(1+\beta K)^2} = \frac{1}{(1+K\beta)^2}$$

$$\Rightarrow \quad \delta K' = \frac{\delta K}{(1+K\beta)^2}$$

$$= \quad \frac{\delta K'}{K'} = \frac{\cancel{1+K\beta}}{K} \frac{\delta K}{(1+\beta K)^{\cancel{2}}} = \frac{\delta K / K}{1+K\beta}$$

The fractional change of K' is reduced by a factor $1+K\beta$ compared to the fractional change in K

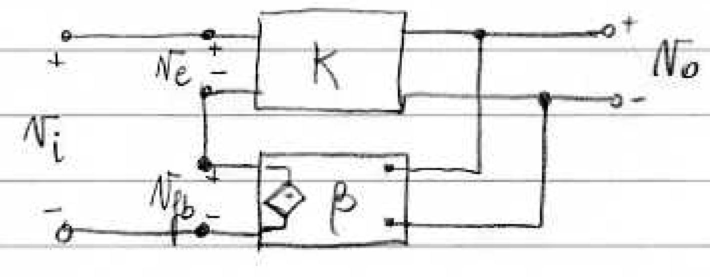
If $1+K\beta = 10^3$ then if our original gain K reduces by 50%, the closed-loop gain reduces only by 0,05% !

We will see other advantages but all this good things are not for free. The price that we pay is that the original amplification is reduced by a factor $1+K\beta$.

- FEEDBACK CONFIGURATIONS (look at pag 5-bis first)

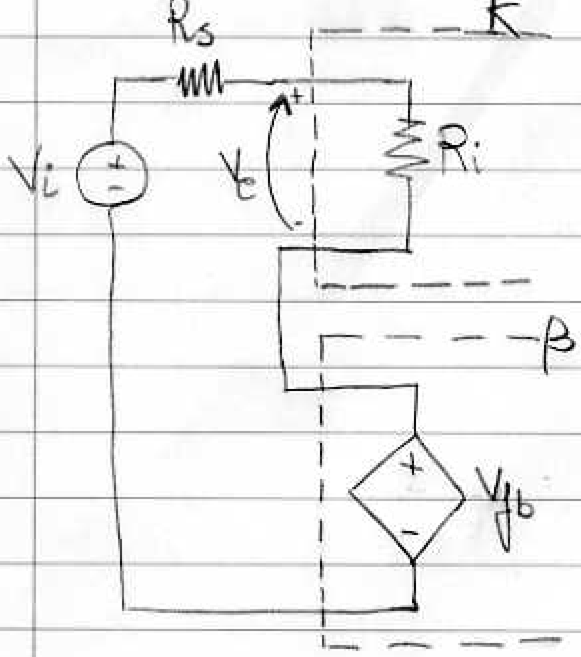
After the general introduction, we now consider the application to electronic circuit. We consider voltages an currents.
Depending on the output quantity and the input quantity we can have 4 configurations.

- <u>Series-Shunt</u> (or Voltage-Voltage)

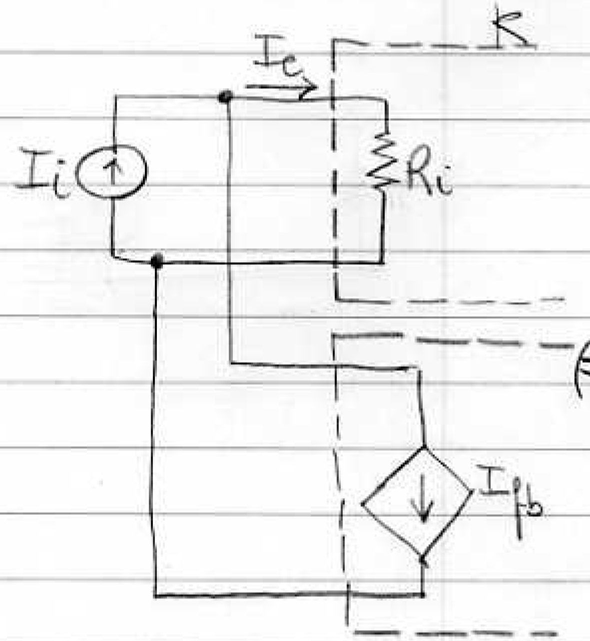


Depending on the quantity we take from the output and the quantity we feedback to the input we can have different connections:

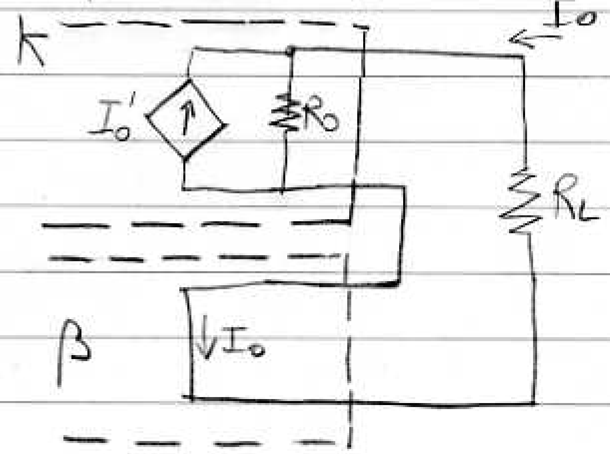
INPUT SERIES



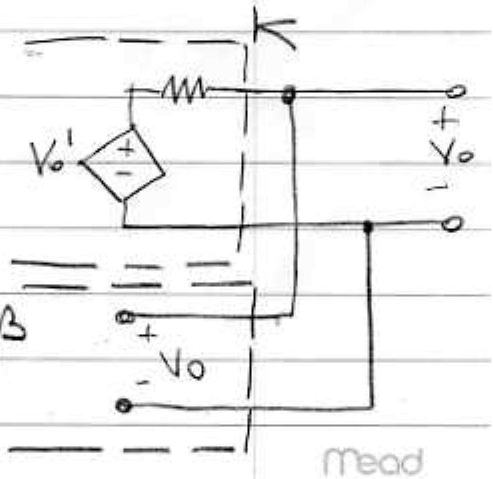
INPUT SHUNT



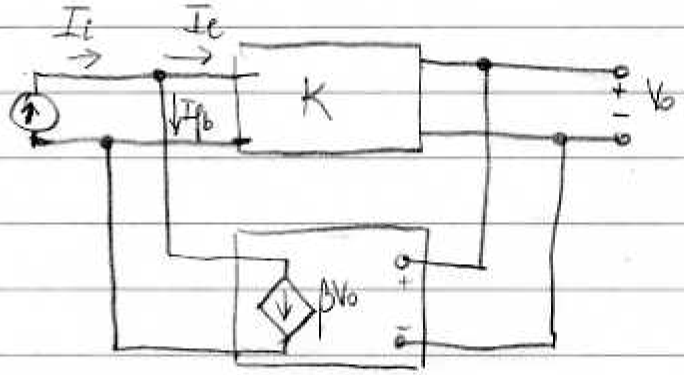
OUTPUT SERIES



OUTPUT SHUNT

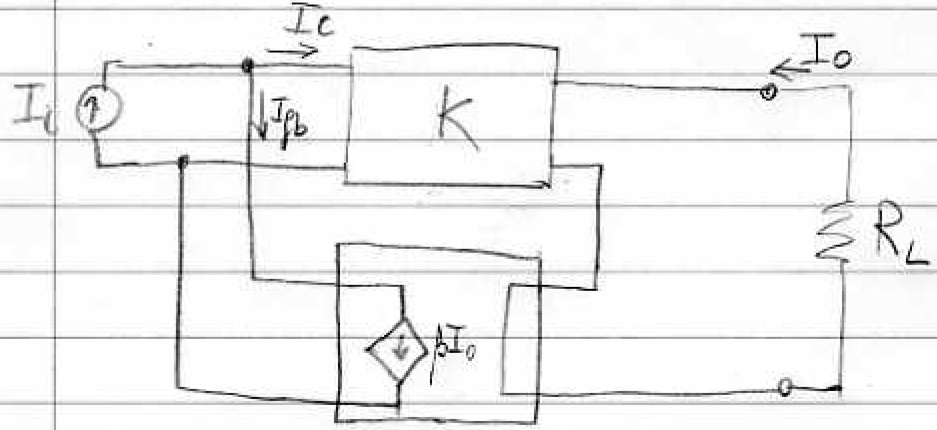


- Shunt-Shunt (or Current-Voltage)



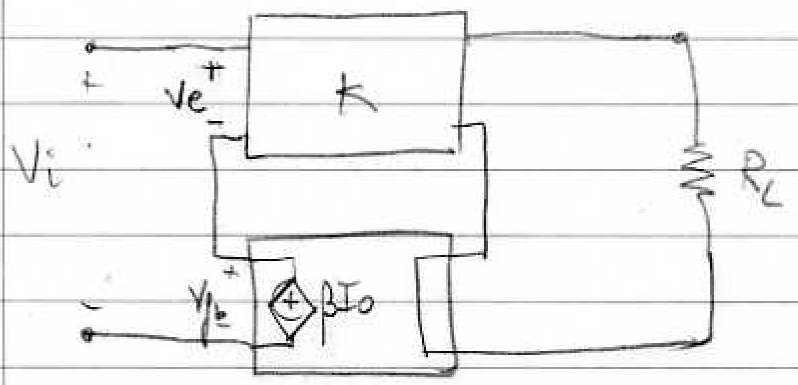
In this case K is a current controlled voltage source, its output is $V_o = K I_e$.

- Shunt-Series (or Current-Current)



K is a current amplifier $I_o = K I_e$

• Series - Series (or Voltage-current)

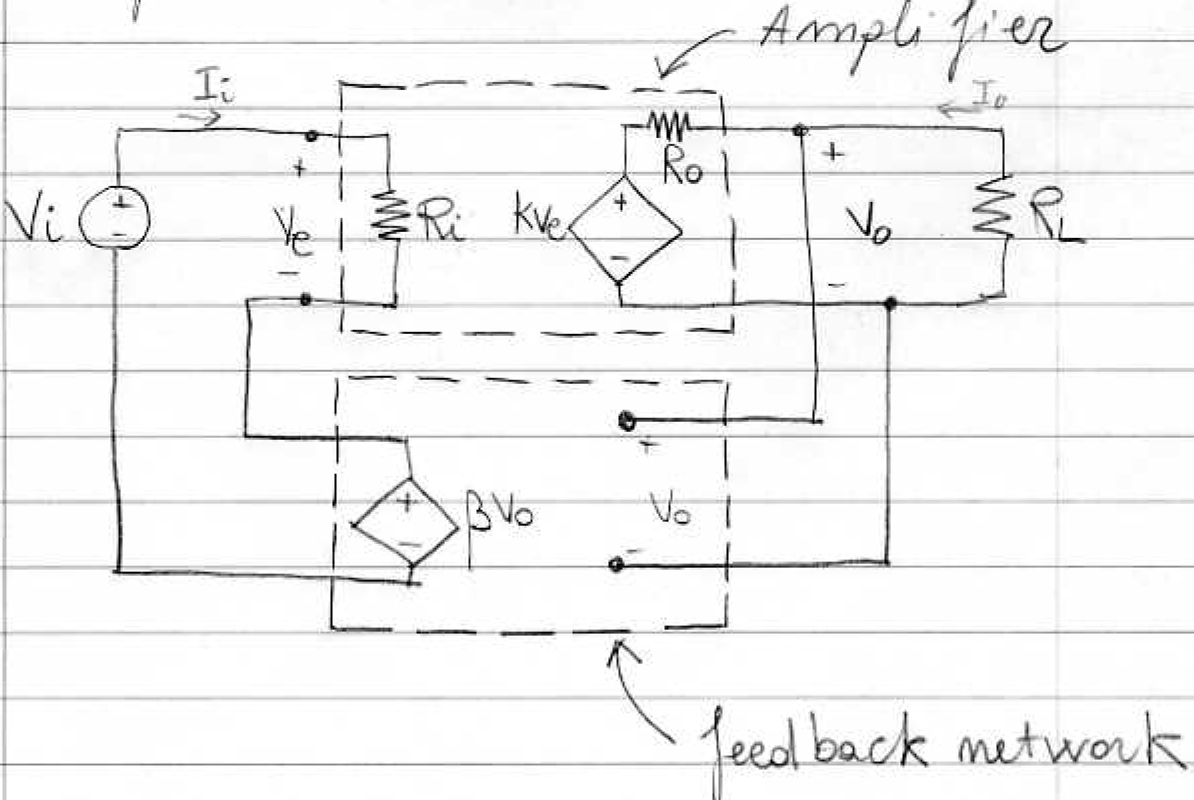


In the connections I have shown here I have not explicitly defined K but you can understand how it looks like case by case.

For instance in the Series-Series connection, we mesure a current I_o and we feed back a voltage V_{fb} so K takes as input a Voltage and outputs a current (because those are the quantity we are considering).

- Effect on the input resistance

We have a clear idea of what an ideal voltage amplifier should look like.
We know that the input resistance shoud be very high.
Consider now a voltage amplifier (modelled as a controlled source) an let's compute the effect of a Series-Shunt feedback on the input resistance:



Input resistance computation

$$R_{in} = \frac{V_i}{I_i} = \frac{V_i}{V_e/R_i} = R_i \frac{V_i}{V_e}$$

$$V_e = V_i - \beta V_o = V_i - \beta K V_e \frac{R_L}{R_o + R_L}$$

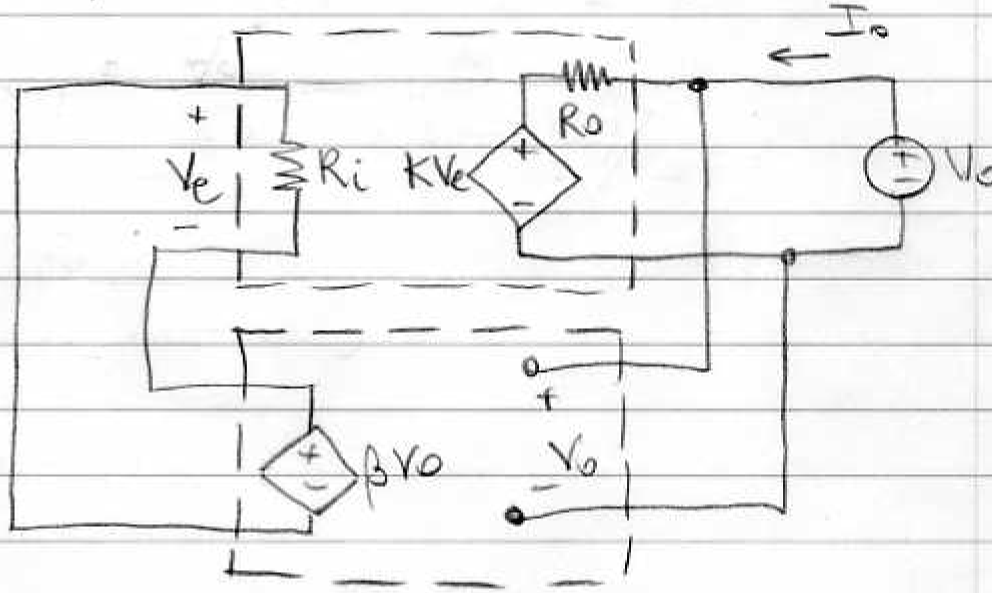
Consider $R_L \gg R_o$ (remember that R_o must be small for a voltage amplifier)

$$V_e \simeq V_i - \beta K V_e \Rightarrow V_e(1+\beta K) = V_i$$

$$R_{in} = \frac{V_i}{I_i} = R_i \frac{V_e(1+\beta K)}{V_e} = R_i(1+\beta K)$$

The input resistance increases by $1+\beta K$! this is good for a voltage amplifier!

Now we compute the output Resistance. It is the thevenin resistance so let's set $V_i = 0$ and use a test voltage source at the output instead of R_L



$$R_{o} = \frac{V_o}{I_o} = \frac{V_o}{(V_o - KV_e)/R_o} = R_o \frac{V_o}{V_o - KV_e}$$

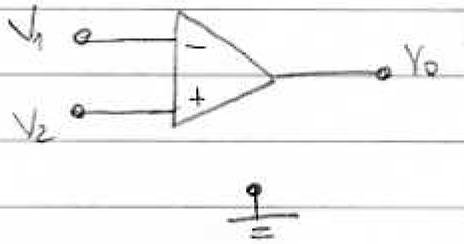
$$V_e = -\beta V_o$$

$$\Rightarrow \quad R_{o} = R_o \frac{V_o}{V_o + K\beta V_o} = \frac{R_o}{1 + K\beta}$$

The out resistance decreases by a factor $1+\beta K$ which is again good for a voltage amplifier!!

Going back to op-Amp.

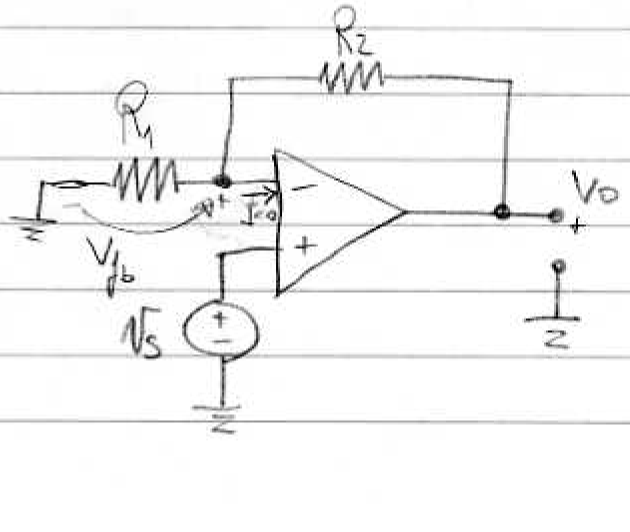
We are given this device:



$$V_0 = -K(V_1 - V_2)$$

This is great, there is already a voltog combiner.

Let's build a feedback amplifier. We have to take parte of the output voltage and subtract it from the input. Part of the output voltage means a voltage divider!



NON-INVERTING CONFIGURATION

The current entering the operational amplifier is zero so:

$$V_{fb} = \frac{R_1}{R_1 + R_2} V_0$$

Since $K \gg 1$, then $K' \simeq \frac{1}{\beta}$ and

We have computed $\beta = \frac{R_1}{R_1+R_2}$

hence $K' = \frac{R_1+R_2}{R_1}$

By direct computation

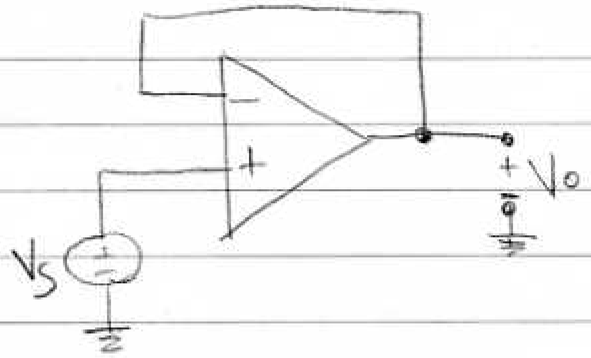
$$V_0 = (V_s - V_{fb})K = \left(V_s - \frac{R_1}{R_1+R_2} V_0\right)K$$

$$\Rightarrow \frac{V_0}{K} + \frac{R_1}{R_1+R_2} V_0 = V_s$$

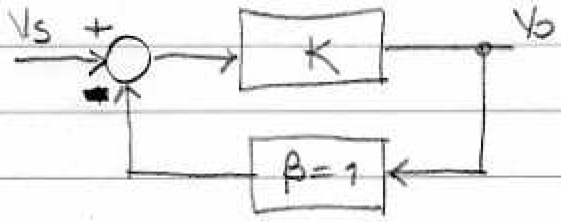
$$\left(\frac{1}{K} + \frac{R_1}{R_2+R_2}\right) V_0 = V_s \Rightarrow$$

$$\frac{V_0}{V_S} = \frac{1}{\frac{1}{K} + \frac{R_1}{R_1+R_2}} \simeq \frac{R_1+R_2}{R_1}$$

VOLTAGE FOLLOWER



This is like



$$V_o = (V_s - V_o)K \Rightarrow V_o(1+K) = V_s$$

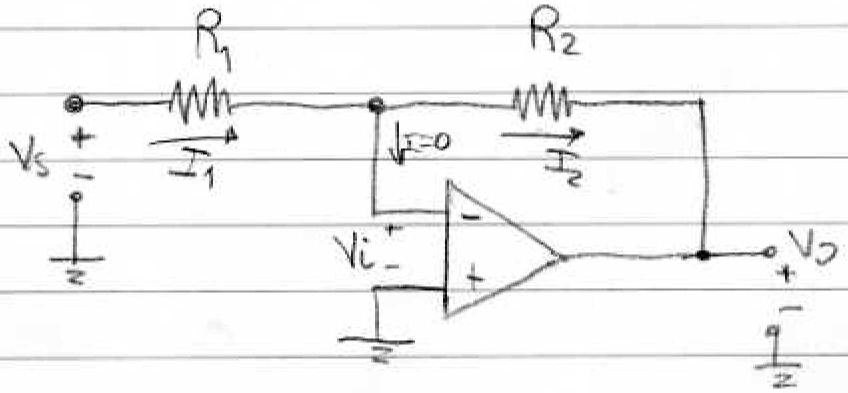
$$\frac{V_o}{V_s} = \frac{1}{1+K} \qquad \lim_{K\to\infty} \frac{V_o}{V_o} = 1$$

$$\Rightarrow V_o = V_s$$

$R_{in} \to \infty$ because is the input resistance of the ideal amplifier

$R_o \to 0$ because is the output resistance of the ideal amplifier

INVERTING CONFIGURATION



by KCL $\quad I_1 = I_2$

$$\frac{V_s - V_i}{R_1} = \frac{V_i - V_o}{R_2} \qquad V_o = -K V_i$$

$$V_i = -\frac{V_o}{K}$$

$$\frac{V_s - \frac{V_o}{k}}{R_1} = \frac{\frac{V_o}{K} - V_o}{R_2} \Rightarrow V_s + \frac{V_o}{K} = -\frac{R}{R_2} V_o\left(1 + \frac{1}{k}\right)$$