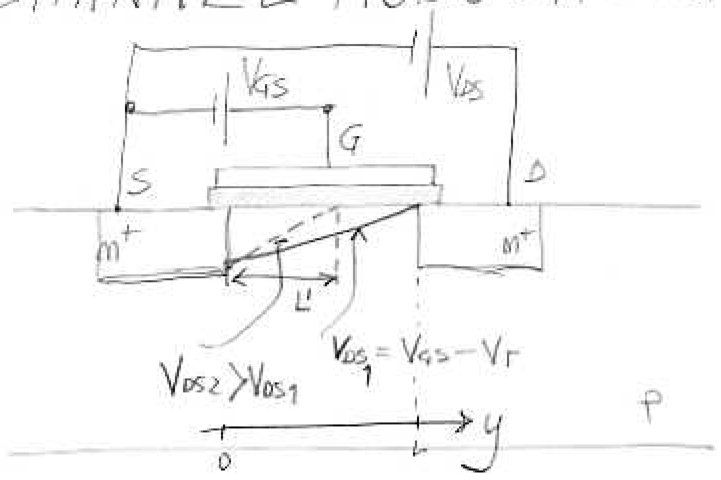
CHANNEL MODULATION EFFECT



Consider $V_{GS} > V_T$. If we increase V_{DS} we know that at some point a phenomenon called pinch-off takes place. It appears as soon as $V_{DS} = V_{GS} - V_T$.

If we keep on increasing V_{DS}, the pinch-off point (which is the point at which $V(y) = V_T$) will move left.

The expression of the mosfet current is:

$$I_D = \mu_n C_{ox} \frac{W}{L} \left((V_{GS} - V_T) V_{DS} - \frac{1}{2} V_{DS}^2 \right)$$

if V_{DS} increases, the effective (or actual) channel lenght decreases (from L to L' in our figure).

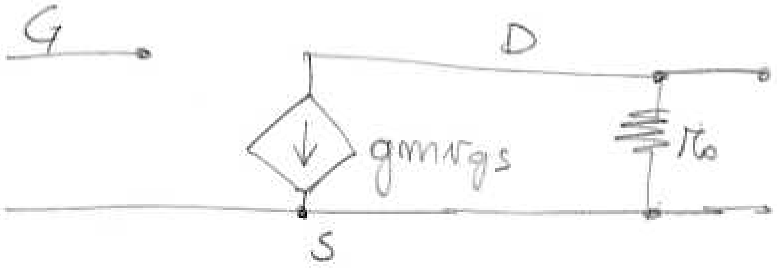
Since I_D is inversely proportional to L, the current I_D increases with V_{DS}.

Even if the dependency can be computed analytically, the expressions that come

but are too complicated to be used in practice.
The channel modulation effect is modeled using one parameter:

$$I_D = I_D'\left(1+\lambda V_{DS}\right)$$

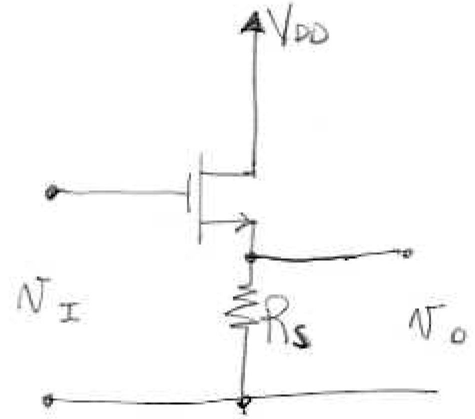
In the small signal model this means to add a resistor with value $1/\lambda$ at the output (between drain and source):



This model is more accurate than the first one we have seen and which was not considering the channel modulation effect.

SOURCE FOLLOWER (COMMON DRAIN)

A circuit that is usually used as analog buffer is the source follower:



Here I'm considering

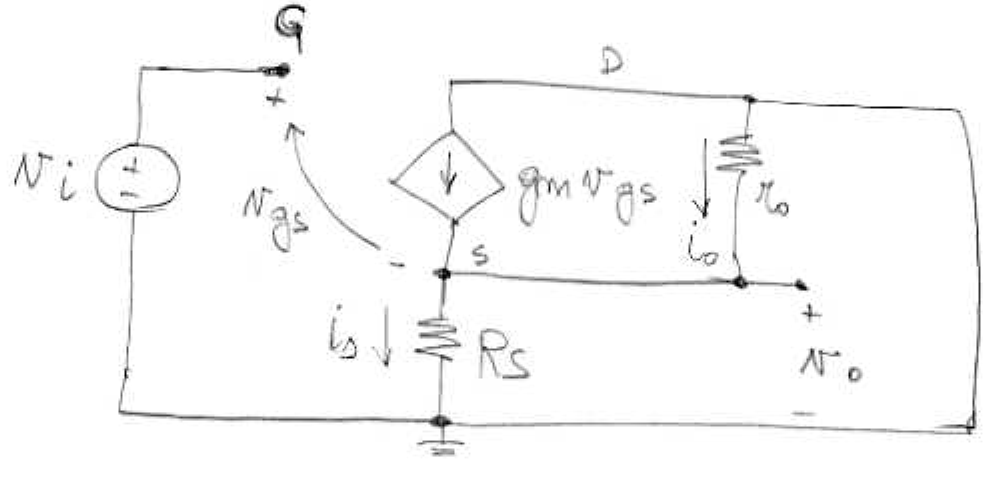
$v_I = V_I + v_i$

the sum of the DC and small AC signal.

The reason is that this circuit is usually used without an input coupling capacitance.

Consider $V_I > V_T$ and $V_{DS} \geq V_{GS} - V_T$ so that the NMOS is in saturation.

We can then perform the small signal analysis:



$$i_s = g_m v_{gs} + i_o = g_m(v_i - R_s i_s) - \frac{v_o}{r_o}$$

$$i_s(1 + g_m R_s) = g_m v_i - \frac{v_o}{r_o}$$

but $v_o = R_s i_s$

$$\underbrace{R_s i_s}_{v_o}(1 + g_m R_s) = g_m R_s v_i - v_o \frac{R_s}{r_o}$$

$$v_o\left(1 + g_m R_s + \frac{R_s}{r_o}\right) = g_m R_s v_i$$

$$\Rightarrow \quad \frac{v_o}{v_i} = \frac{g_m R_s}{1 + g_m R_s + \frac{R_s}{r_o}}$$

r_o is very big compared to $R_s \Rightarrow \frac{R_s}{r_o}$ is negligible

also $g_m R_s \gg 1 \Rightarrow$

$$\frac{v_o}{v_i} \simeq \frac{g_m R_s}{g_m R_s} = 1$$

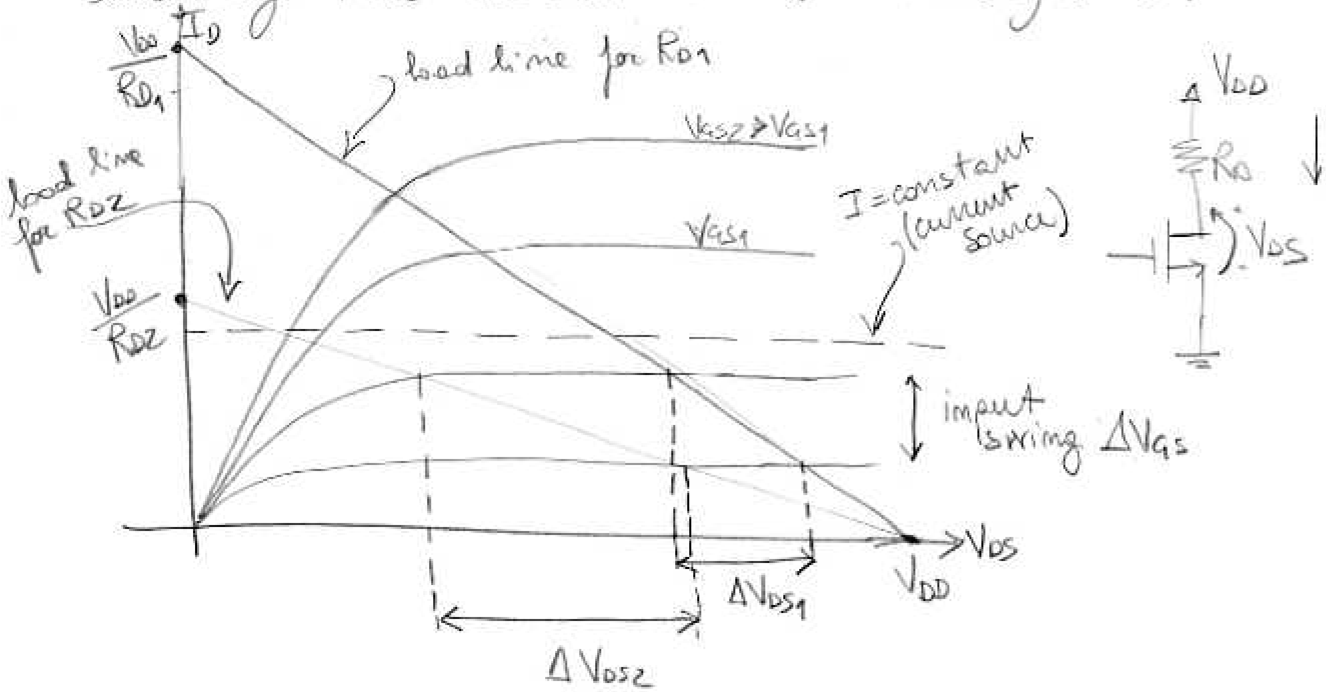
So $v_o = v_i$, input resistance $R_i \to \infty$

and output resistance $R_o = R_s \,//\, r_o \,//\, 1/g_m$ that can be made very small ($\simeq 1/g_m$)

ACTIVE LOADS

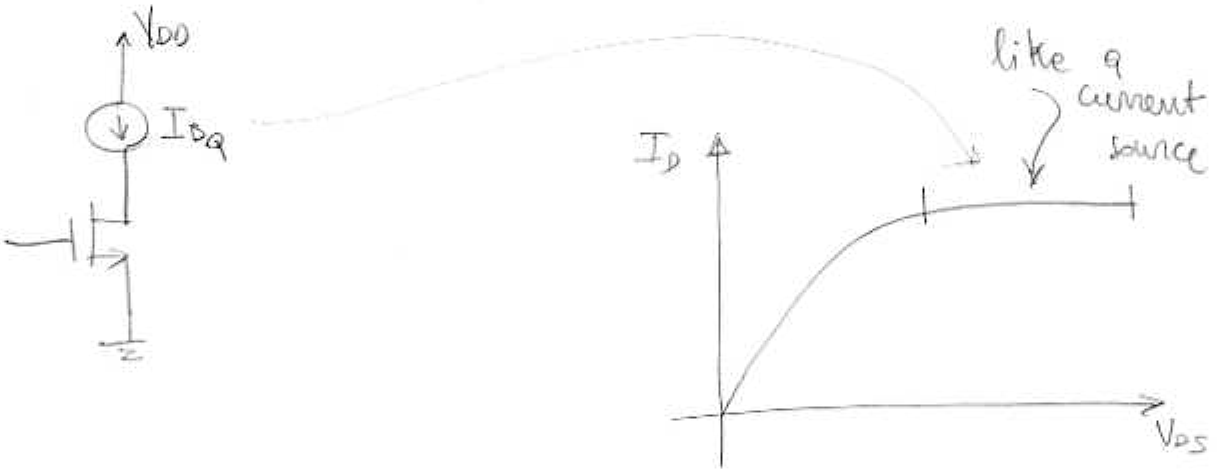
Until now we have decorated transistors with resistors to build amplifiers. There are two potential problems in doing this: first of all an integrated resistor is built by using silicon and it is usually very big in terms of area. The other problem, that has an impact on the first one, is that, for instance, the gain of a common source amplifier is $-g_m R_D$ so in order to achieve a big gain the resistor R_D has to be big.

It can be graphically explained using the load line analysis:

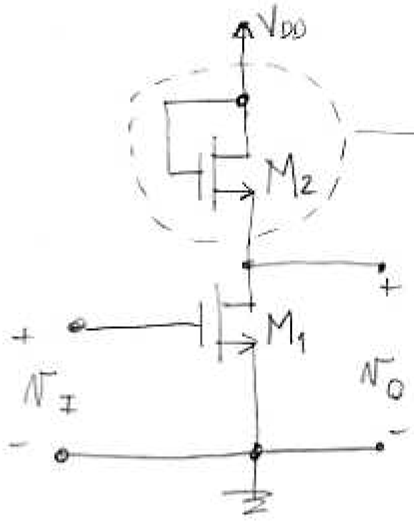


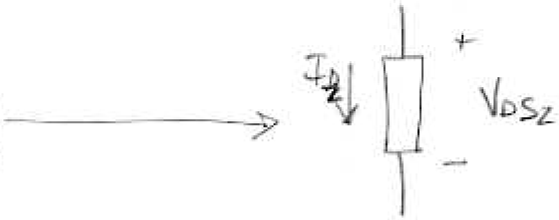
Two load lines are shown in figure. $R_2 > R_1$ an we can see that for the same input swing ΔV_{GS} the output swing ΔV_{DS2} is greater than ΔV_{GS1} which means that the amplification is greater.

In order to have an amplification which is very big we would need something like an horizontal load line. For such situation, even if ΔV_{GS} is very small the output swing would be infinite. An horizontal line corresponds to a load (instead of R_D) whose current is constant which is a current source.



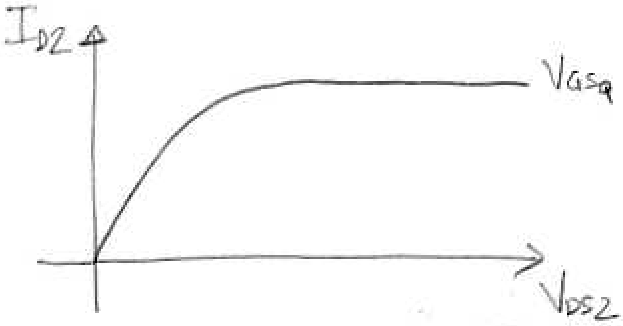
We could use a transistor in saturation region. In this conditions the current is equal to I_{DSAT} almost independent on V_{DS}, Which means almost a current source.





The transistor M_2 is like a load and it is characterized by $I_D = f(V_{DS})$

We know $I_D = f(V_{DS})$ and we can actually plot it:



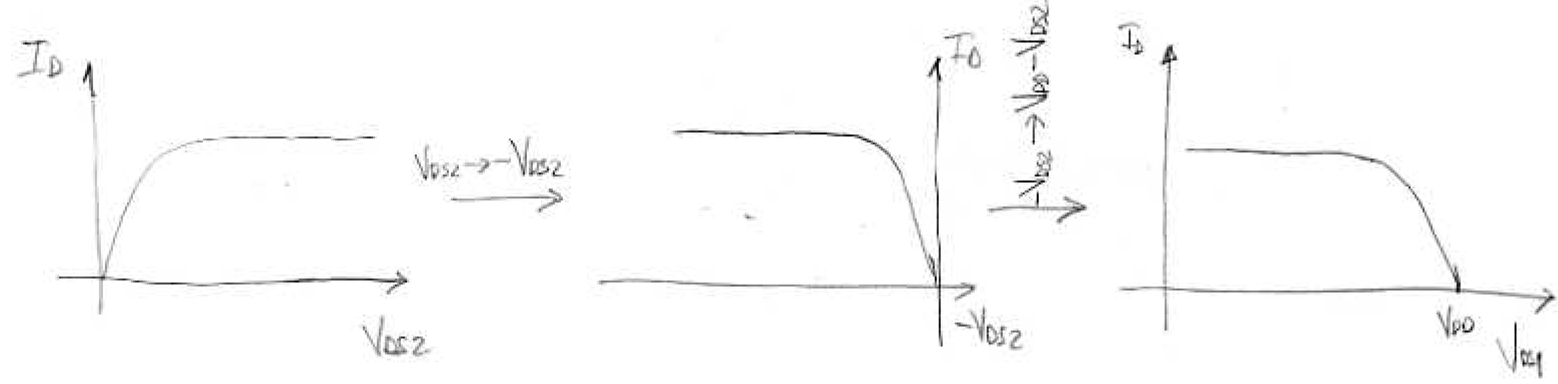
We notice that $V_{GD} = 0$ so there are only to possibilities: M_2 is in cut off or M_2 is in saturation.

Writing the equation for the load line analysis:

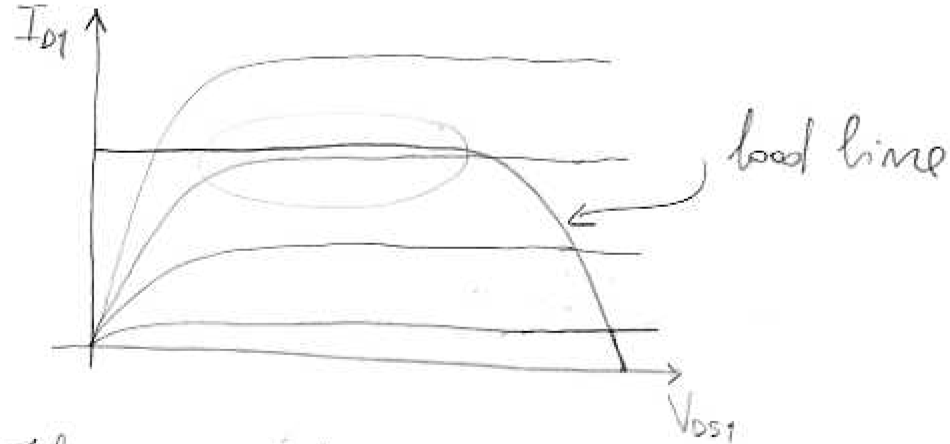
$$V_{DS2} + V_{DS1} - V_{DD} = 0$$

$$V_{DS1} = V_{DD} - V_{DS2}$$

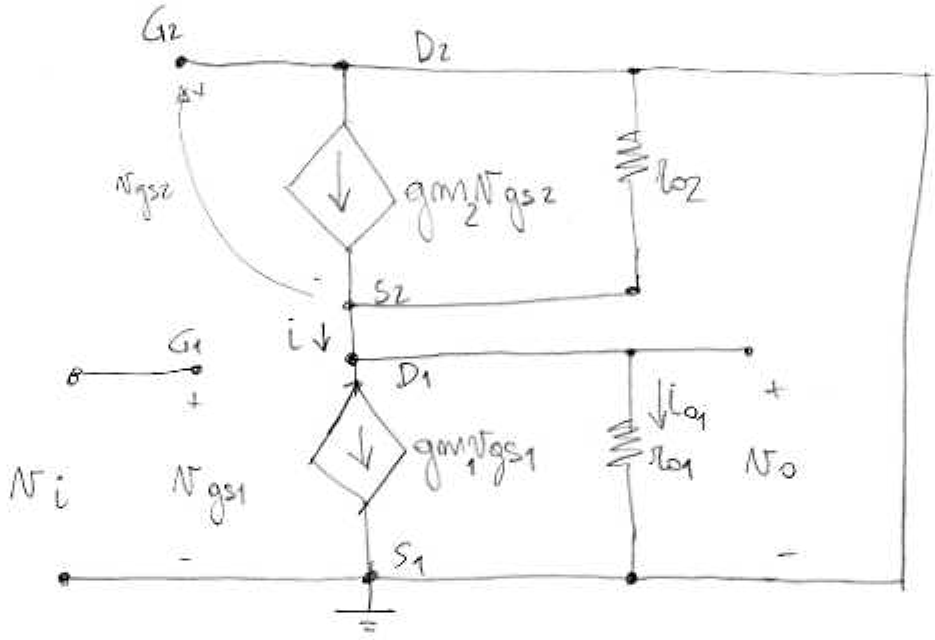
also $I_{D1} = I_{D2}$. So in order to represent both lines on the same plane we need few easy steps



Now that both lines are expressed in the $I_{D1} - V_{DS1}$ plane we can plot them together:



If we choose an operating point such that both transistors are in saturation than the following analysis holds (small signal analysis):



$$v_{gs2} = -v_o$$

$$v_o = r_{o1} i_{o1} = r_{o1}\left(i - gm_1 v_{gs1}\right) = r_{o1}\left(-gm_2 v_o - \frac{v_o}{r_{o2}} - gm_1 v_i\right)$$

$$\Rightarrow v_o\left(1 + r_{o1} gm_2 + \frac{r_{o1}}{r_{o2}}\right) = - r_{o1} gm_1 v_i$$

$$\frac{v_o}{v_i} = A_v = - \frac{r_{o1} gm_1}{1 + r_{o1} gm_2 + \frac{r_{o1}}{r_{o2}}} \simeq - \frac{gm_1}{gm_2} v_{in}$$

$$r_{o1} gm_2 \gg 1 + \frac{r_{o1}}{r_{o2}}$$

this result was somehow expected because the amplification of a common source amplifier is $-g_m R$ where R is the resistance seen by the transistor drain. Transistor M_1 sees a total resistance equal to $r_{o2} // \frac{1}{g_{m2}} \simeq \frac{1}{g_{m2}}$ and so its gain is $-\frac{g_{m1}}{g_{m2}}$.

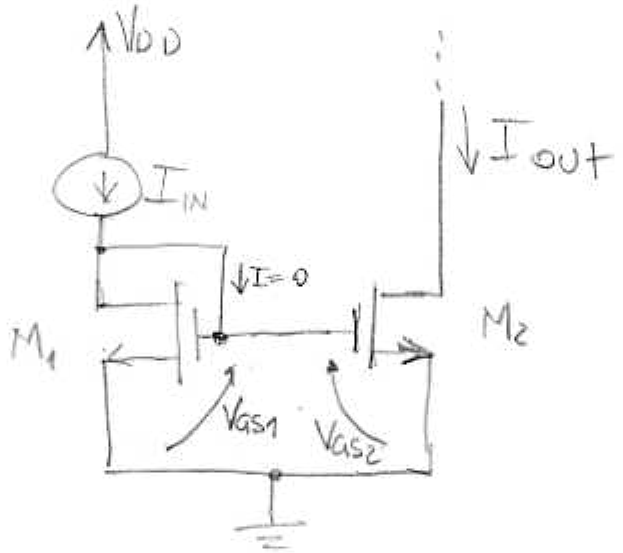
We know that $g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)$

so

$$A_v = -\frac{g_{m1}}{g_{m2}} = \frac{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS1} - V_T)}{\mu_n C_{ox} (W/L)_2 (V_{GS2} - V_T)} \propto \frac{(W/L)_1}{(W/L)_2}$$

playing with the transistors' geometry we can choose the gain of our amplifier.

CURRENT MIRRORS



We want to find the relation between I_{IN} and I_{out}. We observe that M_1 is either in saturation or in cut off.

If $I_{IN} > 0$, since $I_{D1} = I_{IN}$ transistor M_1 has to be in saturation.

For a transistor in saturation:

$$I_{D_{SAT}} = \frac{K}{2}\left(V_{GS} - V_T\right)^2 \Rightarrow V_{GS} - V_T = \sqrt{\frac{2 I_{D_{SAT}}}{K}}$$

where $K = \mu_n C_{ox} \frac{W}{L}$

So:

$$V_{GS1} - V_{T1} = \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} \quad ; \quad V_{GS2} - V_{T2} = \sqrt{\frac{2 I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}}$$

also from KVL:

$$V_{GS1} = V_{GS2} \Rightarrow$$

$$V_{T2} + \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} - V_{T1} = \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}}$$

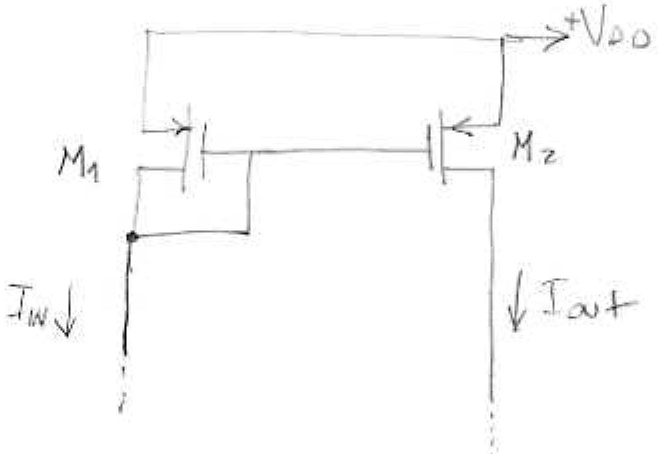
If we assume that the two transistors are built within the same technology, then $V_{T1} = V_{T2}$ and we obtain:

$$\frac{\cancel{2} I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2} = \frac{\cancel{2} I_{D1}}{\cancel{\mu_n C_{ox}}\left(\frac{W}{L}\right)_1} \Rightarrow \frac{I_{D2}}{I_{D1}} = \frac{(W/L)_2}{(W/L)_1}$$

$I_{D1} = I_{IN}$ and $I_{D2} = I_{out}$.

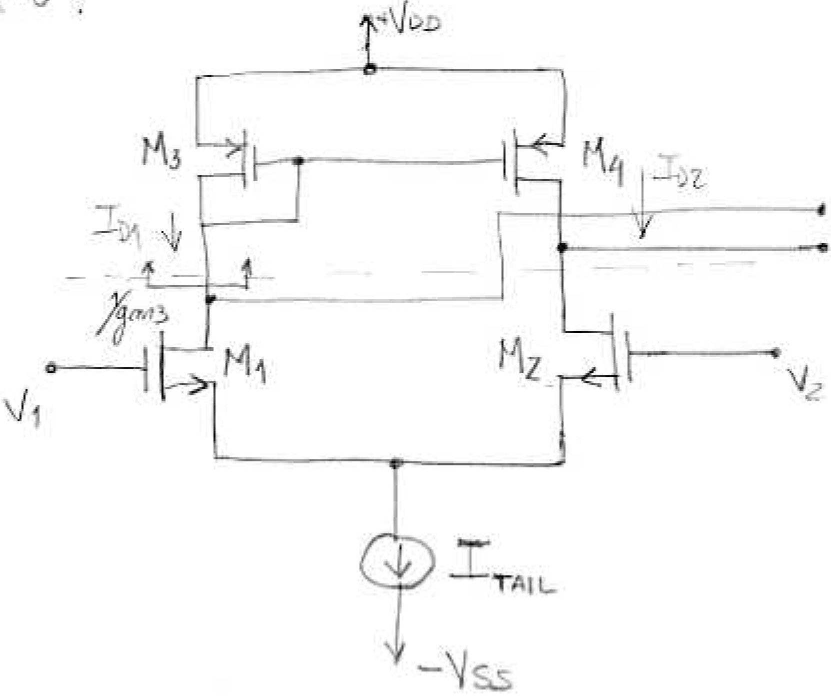
So if the two transistors are identical $I_{OUT} = I_{IN}$, otherwise we can choose their geometries in order to have a specific ratio for the two currents.

Current mirrors can be built using PMOS transistors instead of NMOS:



as we said, if $M_1 = M_2$ then $I_{out} = I_{IN}$.

this circuit is perfect for driving a differential pair because the two currents are forced to be the same and also the two transistors are in saturation so they act as active loads for the differential pair:



$$V_o = A(V_2 - V_1) =$$

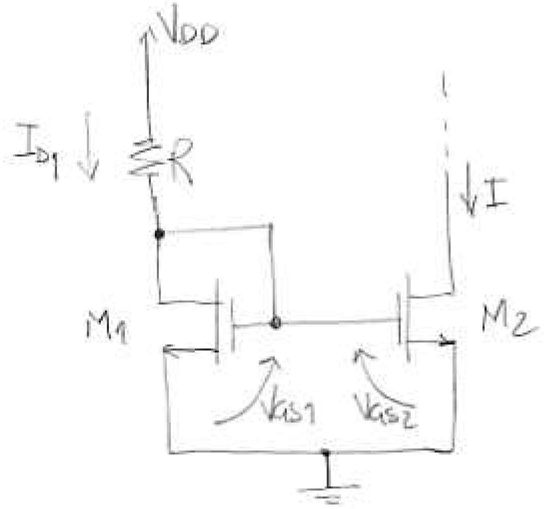
$$= \frac{g_{m_1}}{1/g_{m_3}}(V_2 - V_1)$$

(if $M_3 = M_4$ and $M_1 = M_2$)

In DC, $I_{D1} = I_{D2} = I_{TAIL}/2$ because the current mirror will force the two currents to be the same.
Also the current mirror is an active load for the differential pair making the amplification extremely big.
This is almost our op-amp because the input resistance is infinite (the gate is insulated from the rest of the circuit), the output is proportional to the difference of the inputs, and the gain is very big.
The last component that we need is the current source:

WIDLAR CURRENT SOURCE

We start by introducing a way of building a current source:



In saturation:

$$I_{D1} = \frac{K}{2}\left(V_{GS1}-V_T\right)^2 =$$

$$= \frac{K}{2}\left(V_{DS1}-V_T\right)^2 =$$

$$= \frac{K}{2}\left(V_{DD}-RI_{D1}-V_T\right)^2$$

$$\Rightarrow \quad I_{D1}\frac{2}{K} = \left(V_{DD}-V_T\right)^2 + R^2 I_{D1}^2 - 2R\left(V_{DD}-V_T\right)I_{D1}$$

$$R^2 I_{D1}^2 - \left[\frac{2}{K} + 2R\left(V_{DD}-V_T\right)\right]I_{D1} + \left(V_{DD}-V_T\right)^2 = 0$$

$$I_{D1} = \frac{\frac{2}{K} + 2R\left(V_{DD}-V_T\right) \pm \sqrt{\left(\frac{2}{K}+2R\left(V_{DD}-V_T\right)\right)^2 - 4R^2\left(V_{DD}-V_T\right)^2}}{2R^2}$$

If instead we want $R = f(I_{D1})$ then:

$$R = \frac{2\left(V_{DD}-V_T\right)I_{D1} \pm \sqrt{4\left(V_{DD}-V_T\right)^2 I_{D1}^2 - 4I_{D1}^2\left(\left(V_{DD}-V_T\right)^2 - I_{D1}\frac{2}{K}\right)}}{2I_{D1}^2}$$

$$R = \frac{(V_{DD} - V_T)}{I_{D1}} \pm \sqrt{I_{D1} \frac{2}{K}}$$

Example :

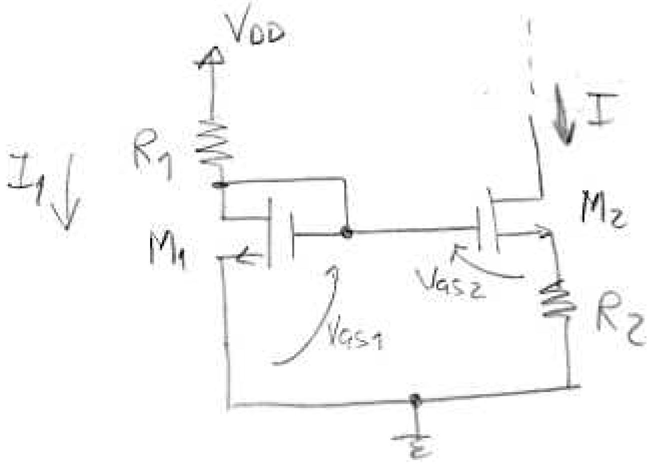
$K = 0.82 \qquad V_T = 2.9 \qquad V_{DD} = 10$

We want a current $I_{D1} = I_{D2} = 2mA = I$

$$R = \frac{7.1}{2 \cdot 10^{-3}} \pm \sqrt{\frac{2 \cdot 10^{-3} \cdot 2}{0.82}} \simeq 3550 \Omega$$

This resistor is pretty big.

The Widlar current source uses two resistors:



Here we can use the fact that the voltage drop on R_2 will decrease V_{GS1} to obtain V_{GS2}:

$$V_{G2} = V_{GS1} - R_2 I$$

It is then possible to have $I \ll I_1$.

For instance:

$V_{DD} = 10V$, $K = 0.82$, $V_T = 2.9$, $I = 2mA$

We can set $I_1 = 40mA$:

$$V_{GS1} = V_T + \sqrt{\frac{2I_1}{K}} = 3.21\,V$$

$$R_1 = \frac{V_{DD} - V_{GS1}}{I_1} = 169.75\,\Omega$$

Now:

$$V_{GS2} = V_T + \sqrt{\frac{2I}{K}} = 2.97\,V$$

$$R_2 = \frac{V_{GS1} - V_{GS2}}{I} = \frac{3.21 - 2.97}{2 \cdot 10^{-3}} = 120\,\Omega$$

As you can see R_1 an R_2 are pretty small an easy to implement.

The complete scheme of our op-amp is then: