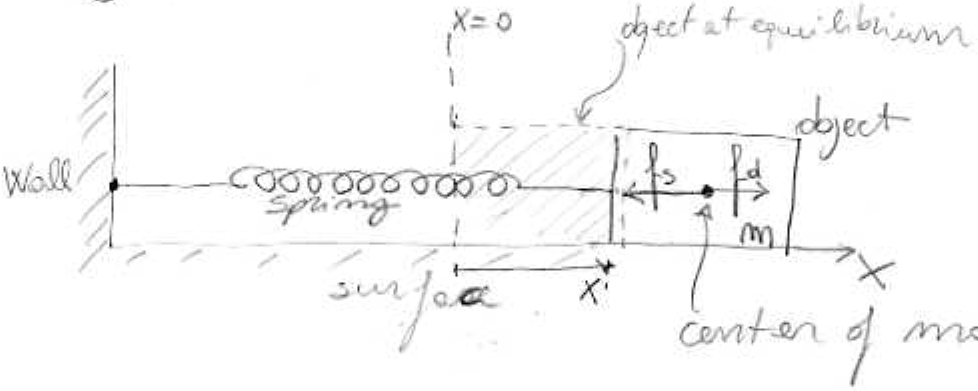
A) On Laplace transform

Before starting, review lecture 9 and also the textbook at pag. 181-204, 244-259.

A.1) We now apply the Laplace tranform to a system that is not electrical. This is yust to understant how it works.

We consider the following mechanical system:



You have probably seen this system before. A spring is attached to a wall. An object of mass m is attached to the spring. At equilibrium, meaning when the object is at $x=0$, the spring force f_s is equal to 0. If we move the object to the left the spring force increases. If we release the object, then the spring force will be directed towards the equilibrium point and the object will move to te right. But it is moving on a surface wich applies a damping force wich opposes the object motion. The spring force is:

$$f_s = -K_s X$$

meaning that it is proportional to the displacement of the object to the right direction. The minus sign means the the force direction is to the left.
The damping force is opposite to the motion and proportional to the object speed:

$$f_d = K_d V = K_d \frac{dX}{dt}$$

So the system equation is:

$$m a = K_s X - K_d \frac{dX}{dt} \Rightarrow$$

$$\Rightarrow m \frac{d^2X}{dt^2} - K_s X + K_d \frac{dX}{dt} = 0$$

We need the initial conditions.
If we stretch the spring until $x=1$ and the release the object, the initial conditions are:

$$X(0) = 1 \quad , \quad \left.\frac{dx}{dt}\right|_{t=0} = \dot{x}(0) = 0$$

The system that we have is then:

$$\begin{cases} m\dfrac{d^2x}{dt^2} - K_s X + K_d \dfrac{dX}{dt} = 0 & (1) \\ X(0) = 1 \\ \dot{X}(0) = 0 \end{cases}$$

A.1.1) Using the fact that:

$$\mathcal{L}\left[\frac{dx(t)}{dt}\right] = \left[sX(s) - x(0)\right]$$

and hence

$$\mathcal{L}\left[\frac{d^2x(t)}{dt^2}\right] = s^2X(s) - sx(0) - \dot{x}(0)$$

equation (1) in the Laplace domain and call this equation (1)'

A.1.2) From eq. (1)', find an expression for $X(s)$. It is a fractional function of s.
What is the order of the numerator and denominator?

A.1.3) A second order polynomial
ax^2+bx+c can be written using the two solutions of $ax^2+bx+c=0$.
If the two solutions are x_1 and x_2, then

$$ax^2+bx+c=a(x-x_1)(x-x_2)$$

Write $X(s)$ by using that expression for the denominator. You should compute also the two solutions.

A.1.4) Now we want to know what happens when the dumping force is zero, meaning $K_d=0$.
Write $X(s)$ in the case of $K_d=0$. Put $X(s)$ in the form

$$X(s)=\frac{N(s)}{c(s-s_1)(s-s_2)}$$

← a function of s

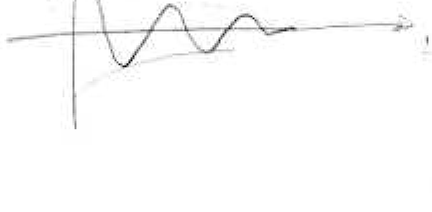
Characterize the two solutions s_1 and s_2 (p2.5
if they are complex numbers then
what is the real and imaginary parts)

A.1.5) Now multiply the two factors $(s-s_1)$
and $(s-s_2)$ obtained in A.1.4.
Looking at the table I gave you in
class, what is $x(t)$? (look at lec.9, pag 10)
What is the conclusion?

A.1.6) Consider now $K_d^2 - 4mK_s > 0$.
Are the two solutions s_1 and s_2 complex?

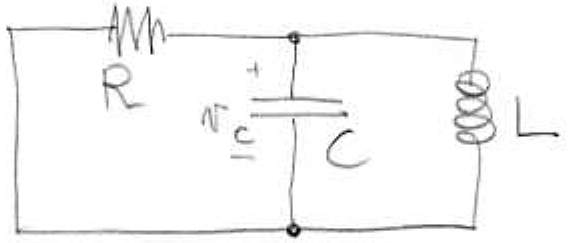
A.1.7) Using the partial fraction method
compute $x(t)$ in the case of A.1.6
(look at lecture 9 pag. 14).

A.1.8) If you have time try the
case when $K_d^2 - 4mK_s < 0$
you should get:



$e^{-t/\alpha} \cos(\omega_0 t)$

A.2) The circuit:



is equivalent to the system in question A.1 when:

R is the damping force

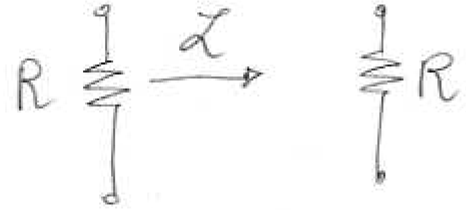
v_c is the position x

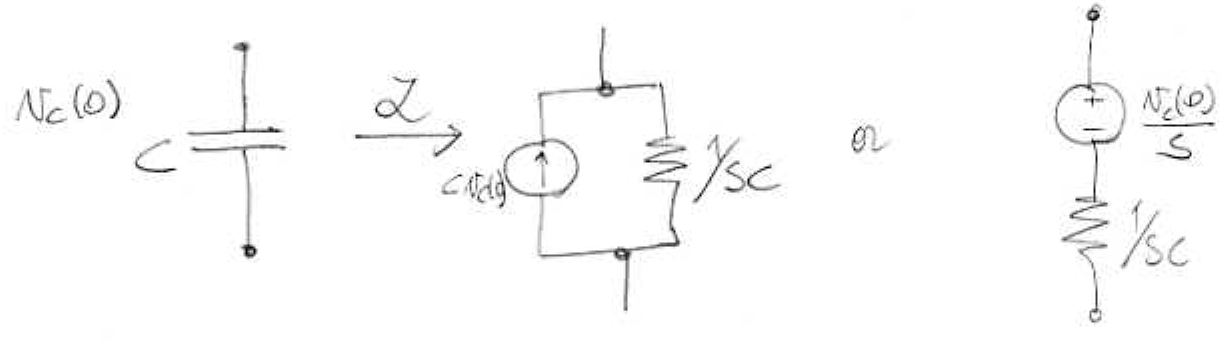
... you can continue the analogy after solving the problem

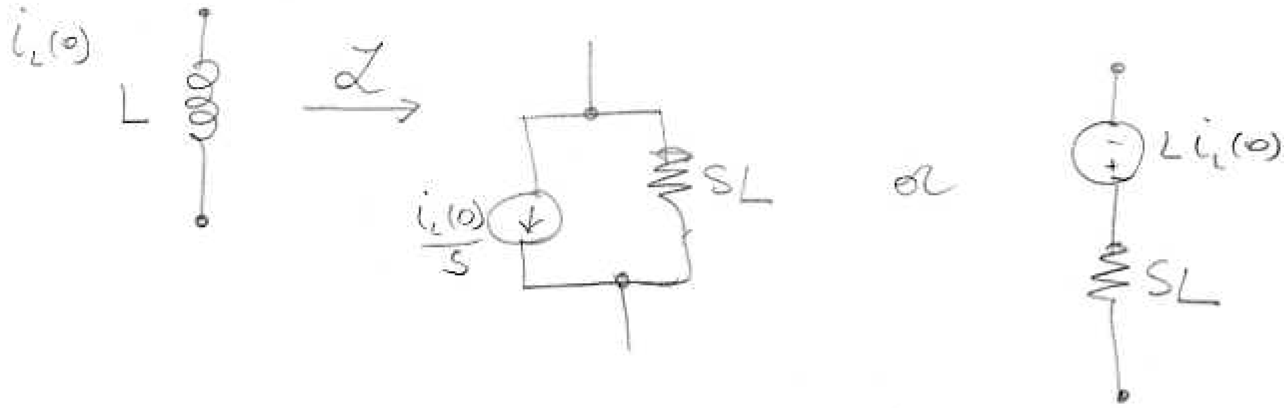
A.2.1) Solve this circuit using Laplace transform and following the same steps of problem A.1)

A.2.2) Instead of writing the differential equations first and then use Laplace transform, it is possible to do a direct analysis

Use the equivalences:



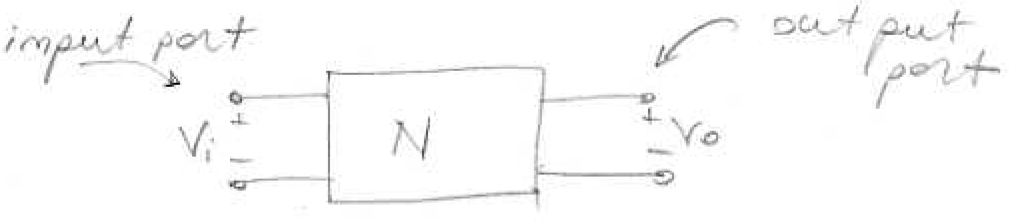




Directly write the circuit in the Laplace domain and solve it using standard techniques.

B) Transfer function

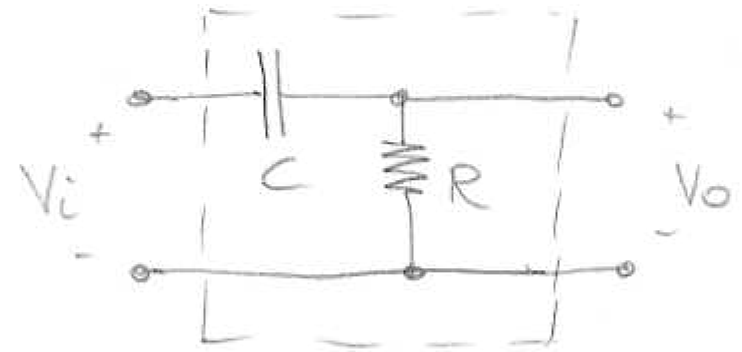
given a 2 ports network



its transfer function is by definition

$$F(s) = \frac{V_o(s)}{V_i(s)}$$

B.1) Compute the transfer function of the following circuit



B.2) We know that $F(s)$ is useful when we want to know what happens to a sinusoidal input of angular freq. ω_0.
In particular if $V_i(t) = A\cos(\omega_0 t)$
then $V_o(t) = A\,|F(j\omega_0)|\cos(\omega_0 t + \measuredangle F(j\omega_0))$
So the shape of $|F(j\omega_0)|$ is important to understand how different frequencies are attenuated. Plot $|F(j\omega_0)|$ and $\measuredangle F(j\omega_0)$ as functions of ω_0.

c) Diodes

10.9, 10.15, 10.26 of your text book