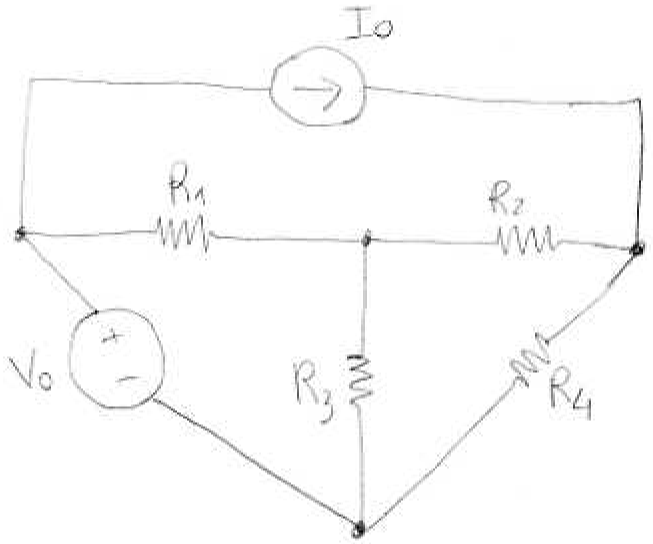
EE40, SUMMER 2004, MIDTERM 1

A) CIRCUIT ANALYSIS



$R_1 = R_2 = 2\ \Omega$

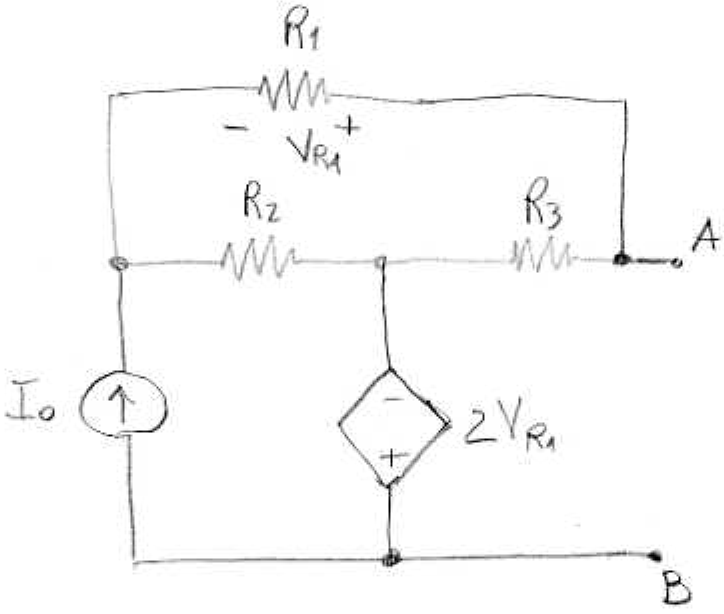
$R_3 = R_4 = 1\ \Omega$

$V_0 = 2\ V$

$I_0 = \frac{6}{7}\ A$

A.1) Using the method that you like most, solve the circuit.

B) THEVENIN / NORTON EQUIVALENCE



$R_1 = 1\,\Omega$

$R_2 = 2\,\Omega$

$R_3 = 1\,\Omega$

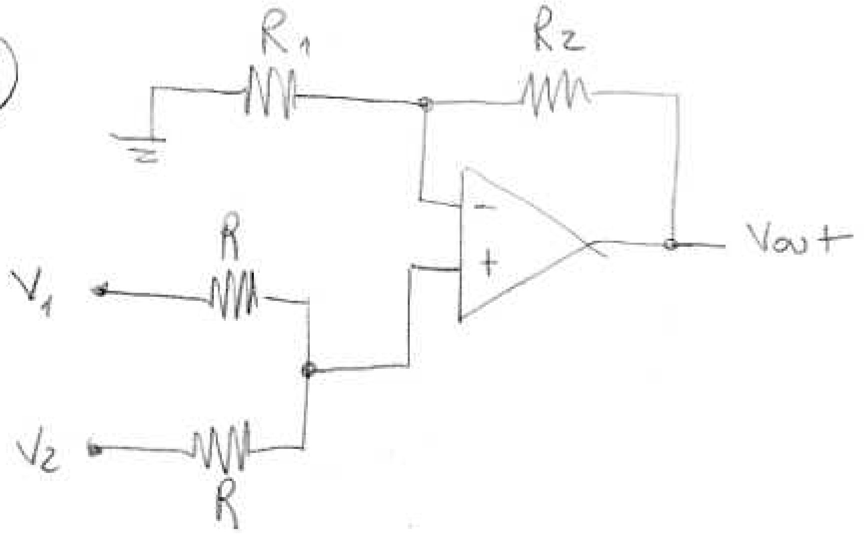
$I_0 = 2\,A$

B.1) Find the thevenin equivalent circuit at nodes A, B

B.2) Find the Norton equivalent circuit at node A, B

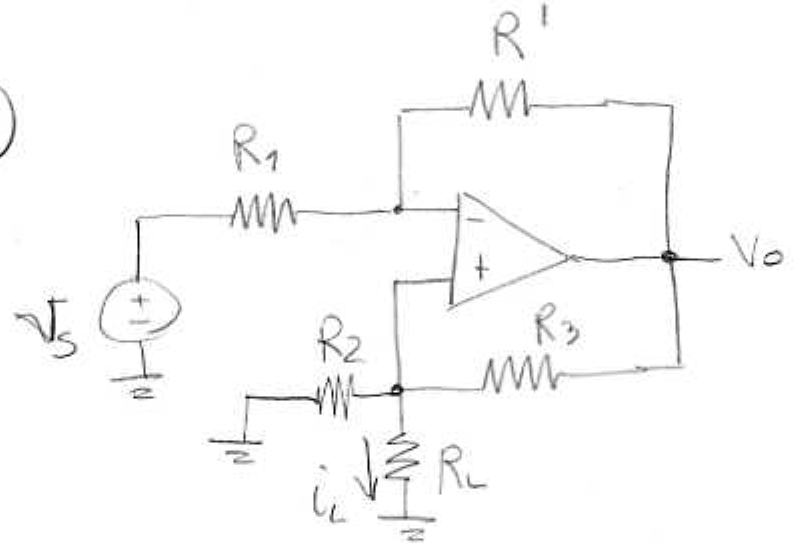
C) OP-AMPS

C.1)



compute Vout

C.2)



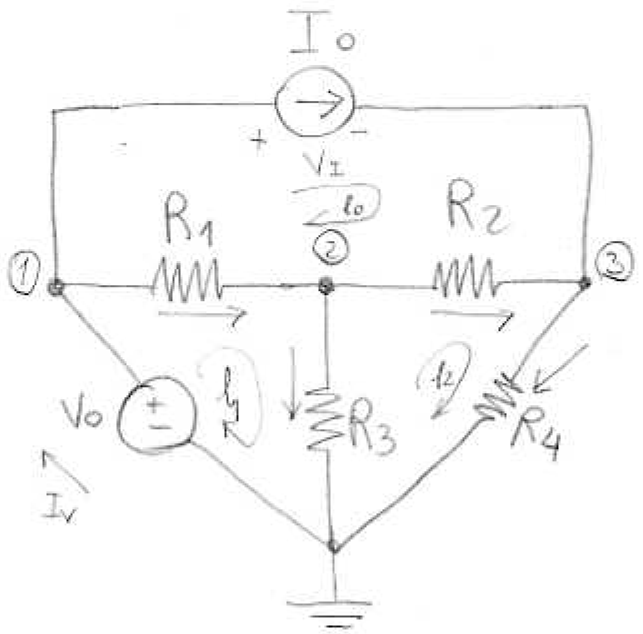
Prove that if $\frac{R_3}{R_2} = \frac{R'}{R_1}$ then

$$i_L = -\frac{v_s}{R_2}$$

(Of course, you shold first comput $i_L = f(v_s)$)

MIDTERM 1 SOLUTIONS

A) Using Node-Voltage



$R_1 = R_2 = 2\,\Omega$

$R_3 = R_4 = 1\,\Omega$

$V_0 = 2\,V$

$I_0 = \frac{6}{7}\,A$

$$\begin{cases} I_V - I_1 - I_0 = 0 \\ I_1 - I_3 - I_2 = 0 \\ I_2 - I_4 + I_0 = 0 \end{cases} \qquad \begin{cases} I_V - \frac{V_0 - V_2}{R_1} - I_0 = 0 \\ \frac{V_0 - V_2}{R_1} - \frac{V_2}{R_3} - \frac{V_2 - V_3}{R_2} = 0 \\ \frac{V_2 - V_3}{R_2} - \frac{V_3}{R_4} + I_0 = 0 \end{cases}$$

$$\begin{cases} I_V - \frac{V_0 - V_2}{R_1} - I_0 = 0 \\ 1 - V_2/2 - V_2 - V_2/2 + V_3/2 = 0 \\ V_2/2 - \frac{V_3}{2} - V_3 + 6/7 = 0 \end{cases} \qquad \begin{cases} I_V - V_0/2 + V_2/2 - 6/7 = 0 \\ 1 - 2V_2 - V_3/2 = 0 \\ V_2/2 = -6/7 + 3/2\,V_3 \end{cases}$$

$$\begin{cases} I_v - 1 + V_2/2 - 6/7 = 0 \\ V_3 = 4V_2 - 2 \\ V_2/2 = 6/7 + 3/2\,(4V_2 - 2) \Rightarrow -\frac{11}{2}V_2 + \frac{27}{7} = 0 \end{cases}$$

$$\Rightarrow V_2 = \frac{27}{7}\,\frac{2}{11} = \frac{54}{77}$$

$$V_3 = 4\,\frac{54}{77} - 2 = \frac{4 \cdot 54 - 154}{77} = \frac{216 - 154}{77} = \frac{62}{77}$$

$$I_v = 1 - \frac{54}{77}\,\frac{1}{2} + \frac{6}{7} = \frac{77 - 27 + 66}{77} = \frac{116}{77}$$

$$I_1 = \left(2 - \frac{54}{77}\right)\frac{1}{R_1} = \frac{154 - 54}{77}\,\frac{1}{2} = \frac{50}{77}$$

$$I_3 = \frac{V_2}{R_3} = \frac{54}{77}$$

$$I_2 = \frac{V_2 - V_3}{R_2} = \frac{1}{2}\left(\frac{54}{77} - \frac{62}{77}\right) = -\frac{4}{77}$$

$$V_I = V_1 - V_3 = 2 - \frac{62}{77} = \frac{154 - 62}{77} = \frac{92}{77}$$

Using mesh-current

$$\begin{cases} V_I - R_2 I_2 - R_1 I_1 = 0 \\ R_1 I_1 + R_3 I_3 - V_o = 0 \\ R_2 I_2 + R_4 I_4 - R_3 I_3 = 0 \end{cases} \quad \begin{cases} V_I - R_2(I_{l_1} - I_{l_0}) - R_1(I_{l_1} - I_{l_0}) = 0 \\ R_1(I_{l_1} - I_{l_0}) + R_3(I_{l_1} - I_{l_2}) - V_o = 0 \\ R_2(I_{l_2} - I_{l_0}) + R_4(I_{l_2}) - R_3(I_{l_1} - I_{l_2}) = 0 \end{cases}$$

$I_{l_0} = I_o$

from the last two:

$$\begin{cases} I_{l_1}(R_1 + R_3) - I_{l_2} R_3 = V_o + R_1 I_o \\ -I_{l_1} R_3 + I_{l_2}(R_2 + R_4 + R_3) = R_2 I_o \end{cases}$$

$$\begin{cases} 3I_{l_1} - I_{l_2} = 2 + \frac{12}{7} \\ -I_{l_1} + 4I_{l_2} = \frac{12}{7} \end{cases} \quad \begin{cases} 12 I_{l_2} - \frac{36}{7} - I_{l_2} = \frac{26}{7} \\ I_{l_1} = 4I_{l_2} - \frac{12}{7} \end{cases}$$

$$11 I_{l_2} = \frac{62}{7} \Rightarrow I_{l_2} = \frac{62}{77}$$

$$I_{l_1} = 4\frac{62}{77} - \frac{12}{7} = \frac{4 \cdot 62 - 11 \cdot 12}{77} = \frac{248 - 132}{77} = \frac{116}{77}$$

$$I_v = I_{e_1} = \frac{116}{77}$$

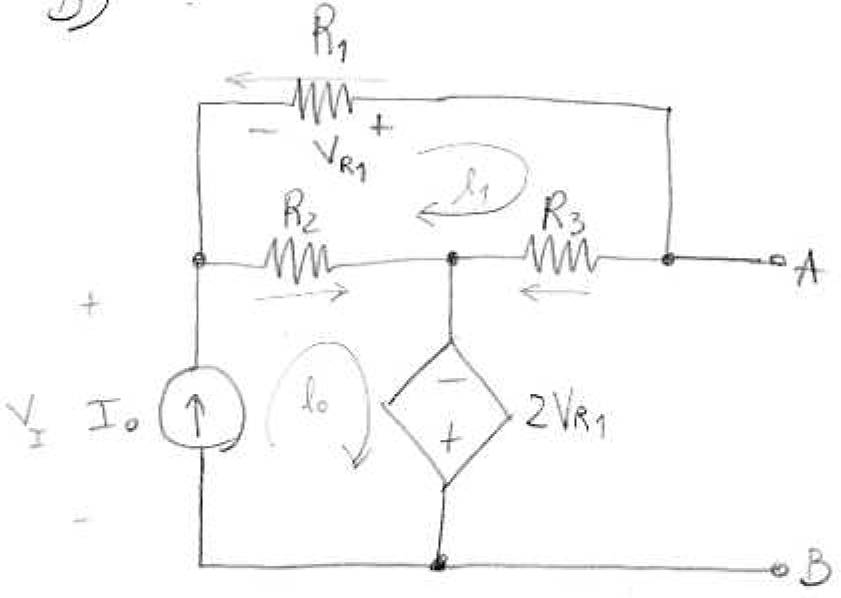
$$I_1 = I_{e_1} - I_{e_0} = \frac{116}{77} - \frac{6}{7} = \frac{116 - 66}{77} = \frac{50}{77}$$

$$I_3 = I_{e_1} - I_{e_2} = \frac{116}{77} - \frac{62}{77} = \frac{54}{77}$$

$$I_2 = I_{e_2} - I_0 = \frac{62}{77} - \frac{66}{77} = -\frac{4}{77}$$

$$I_4 = I_{e_2} = \frac{62}{77}$$

B)



B.1) Compute V_{oc} which is the V_{TH}:

$$\begin{cases} -V_I + R_2 I_2 - 2V_{R_1} = 0 \\ -V_{R_1} + R_3 I_3 - R_2 I_2 = 0 \end{cases} \quad \begin{cases} -V_I + R_2\left(I_{\ell_0} - I_{\ell_1}\right) - 2R_1\left(-I_{\ell_1}\right) = 0 \\ +R_1 I_{\ell_1} + R_3 I_{\ell_1} - R_2\left(I_{\ell_0} - I_{\ell_1}\right) = 0 \end{cases}$$

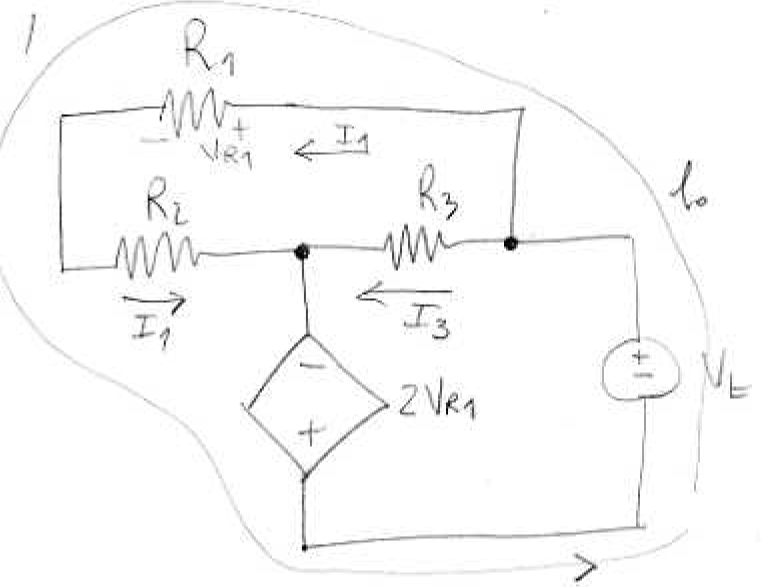
$I_{\ell_0} = I_0$

$$\begin{cases} -V_I + R_2 I_0 + I_{\ell_1}\left(+2R_1 - R_2\right) = 0 \\ I_{\ell_1}\left(R_1 + R_3 + R_2\right) - R_2 I_0 = 0 \end{cases} \Rightarrow I_{\ell_1} = \frac{R_2 I_0}{R_1 + R_3 + R_2}$$

$$= \frac{2 \cdot 2}{4} = 1$$

$$V_{oc} = R_3 I_{\ell_1} - 2V_{R_1} =$$
$$= R_3 I_{\ell_1} + 2R_1 I_{\ell_1} = 3$$

1



Write KVL at l_0

$$-V_t + R_1 I_1 + R_2 I_1 - 2R_1 I_1 = 0$$

$$I_1 (R_1 + R_2 - 2R_1) = V_t \Rightarrow I_1 = \frac{V_t}{R_2 - R_1}$$

then

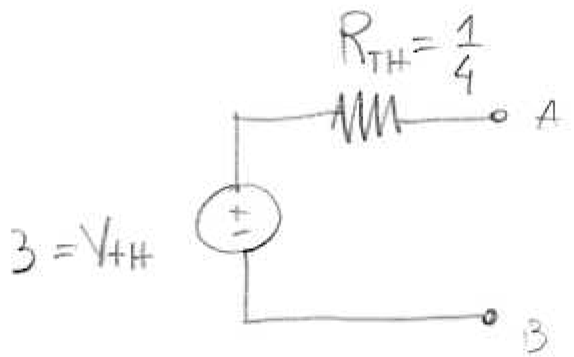
$$I_3 = \frac{V_t + 2V_{R1}}{R_3} = \frac{V_t + 2R_1 I_1}{R_3} = \frac{V_t + \frac{2R_1 V_t}{R_2 - R_1}}{R_3} =$$

$$= V_t \frac{R_2 + R_1}{R_3(R_2 - R_1)} =$$

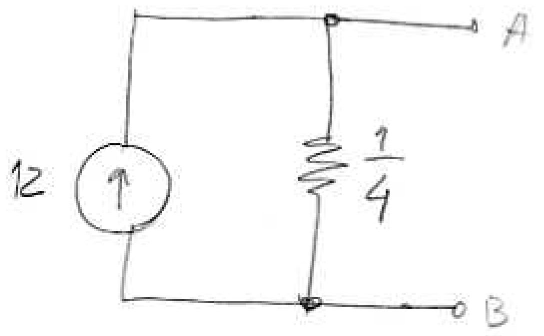
$$\Rightarrow I_t = I_1 + I_3 = V_t \left(\frac{1}{R_2 - R_1} + \frac{R_1 + R_2}{R_3(R_2 - R_1)} \right) =$$

$$= V_t (1 + 3) = V_t 4 \Rightarrow \frac{V_t}{I_t} = \frac{1}{4} = R_{TH}$$

The thevenin equivalent circuit is then

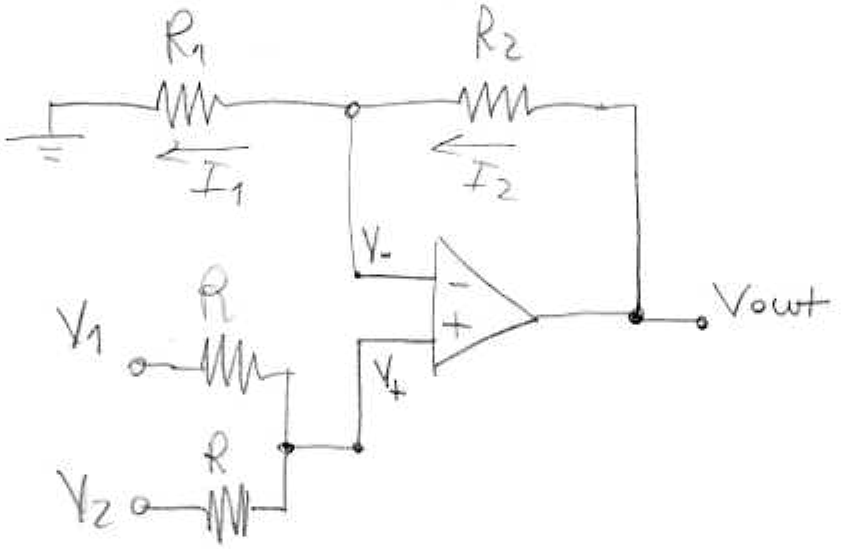


Hence the norton eq. circuit is



C)

C.1)



$$V_+ = V_1 \frac{R}{R+R} + V_2 \frac{R}{R+R} = \frac{V_1}{2} + \frac{V_2}{2}$$

↑ by superposition of V_1 and V_2

Virtual short circuit $V_+ = V_-$

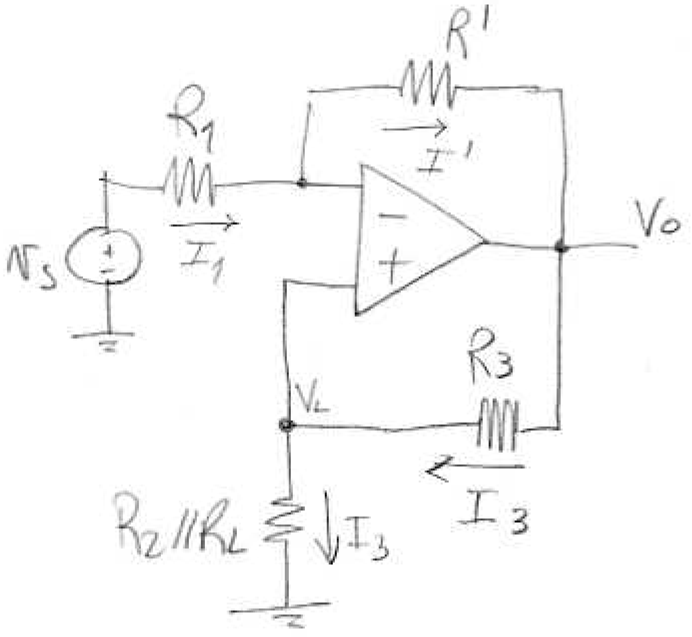
$$I_1 = \frac{V_-}{R_1} = \frac{V_+}{R_1} = \frac{1}{R_1}\left(\frac{V_1}{2} + \frac{V_2}{2}\right)$$

$$I_2 = I_1 \Rightarrow V_{out} = (R_1 + R_2) I_1 =$$

$$= \frac{R_1 + R_2}{2R_1}(V_1 + V_2) = \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right)(V_1 + V_2)$$

C.2)

1st method (direct method):



$$V_L = V_o \frac{R_2//R_L}{R_3 + R_2//R_L} \quad *$$

$$V_o - V_L = -R'I' = -R'I_1 =$$

$$= -R' \frac{(V_S - V_L)}{R_1} \Rightarrow$$

$$\Rightarrow V_o = V_L\left(1 + \frac{R'}{R_1}\right) - \frac{R'}{R_1} V_S$$

Substituting in $*$:

$$V_L = V_L\left(1 + \frac{R'}{R_1}\right)\frac{R_2//R_L}{R_3 + R_2//R_L} - \frac{R'}{R_1}\frac{R_2//R_L}{R_3 + R_2//R_L} V_S$$

$$V_L\left(1-\left(1+\frac{R'}{R_1}\right)\frac{R_2/\!/R_L}{R_3+R_2/\!/R_L}\right)=-\frac{R'}{R_1}\,\frac{R_2/\!/R_L}{R_3+R_2/\!/R_L}\,V_S$$

$$V_L=-\frac{\dfrac{R'}{R_1}\,\dfrac{R_2/\!/R_L}{R_3+R_2/\!/R_L}}{1-\left(1+\dfrac{R'}{R_1}\right)\dfrac{R_2/\!/R_L}{R_3+R_2/\!/R_L}}\,V_S$$

Now $$\frac{R_2/\!/R_L}{R_3+R_2/\!/R_L}=\frac{R_2R_L}{R_3(R_2+R_L)+R_2R_L}$$

$$V_L=-\frac{\dfrac{R'}{R_1}\,\dfrac{R_2R_L}{R_3(R_2+R_L)+R_2R_L}}{1-\left(1+\dfrac{R'}{R_1}\right)\dfrac{R_2R_L}{R_3(R_2+R_L)+R_2R_L}}\,V_S$$

$$=-\frac{\dfrac{R'}{R_1}}{\dfrac{R_3(R_2+R_L)+R_2R_L}{R_2R_L}-\left(1+\dfrac{R'}{R_1}\right)}\,V_S=$$

$$=-\frac{\dfrac{R'}{R_1}\,R_2R_LR_1}{R_1R_3(R_2+R_L)+\cancel{R_2R_LR_1}-\cancel{R_2R_LR_1}-R_2R_LR'}\,V_S=$$

$$= -\frac{\frac{R'}{R_1} R_2 R_L R_1}{R_1 R_3 R_2 + R_1 R_3 R_L - R_2 R_L R'} V_S =$$

Since $\frac{R_3}{R_2} = \frac{R'}{R_1} \Rightarrow R_3 R_1 = R_2 R'$

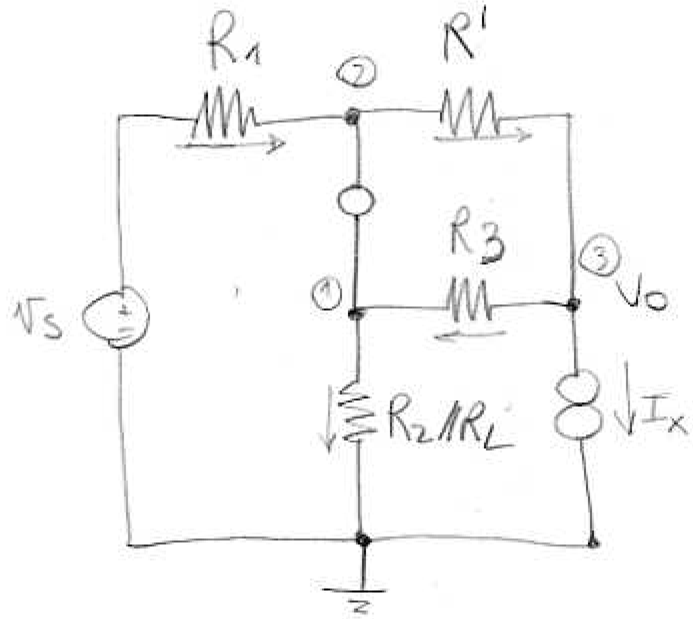
$$V_L = -\frac{\frac{R'}{R_1} R_2 R_L R_1}{\cancel{R_2 R' R_L} - \cancel{R_2 R_L R'} + R_1 R_3 R_2} V_S = -\frac{R' R_L}{R_1 R_3} \cancel{V_S} =$$

$$= -R_L V_S \frac{R'}{R_1} \frac{1}{R_3} = -V_S R_L \frac{R_3}{R_2} \frac{1}{R_3} =$$

$$= -\frac{R_L}{R_2} V_S$$

Now $I_L = \frac{V_L}{R_L} = -\frac{V_S}{R_2}$

2^{nd} method (nullator, norator):



node-Voltage Analysis:

$$\begin{cases} -\dfrac{V_1}{R_2 /\!/ R_L} + \dfrac{V_3 - V_1}{R_3} = 0 \\[2ex] \dfrac{V_s - V_2}{R_1} - \dfrac{V_2 - V_3}{R'} = 0 \\[2ex] -\dfrac{V_3 - V_1}{R_3} + \dfrac{V_2 - V_3}{R'} - I_x = 0 \end{cases}$$

$V_2 = V_1$ (this is imposed by the nullator

$$\begin{cases} -V_1 / R_2 /\!/ R_L + \dfrac{V_3 - V_1}{R_3} = 0 \\[2ex] \dfrac{V_s - V_1}{R_1} - \dfrac{V_1 - V_3}{R'} = 0 \\[2ex] -\dfrac{V_3 - V_1}{R_3} + \dfrac{V_1 - V_3}{R'} - I_x = 0 \end{cases}$$

Using the first two equations:

$$V_3 = R_3\left(\frac{V_1}{R_2/\!/R_L} + \frac{1}{R_3}\right) = V_1\left(1+\frac{R_3}{R_2/\!/R_L}\right)$$

and substituting into the second equation:

$$\frac{V_S - V_1}{R_1} - \frac{V_1}{R'} + \frac{V_3}{R'} = 0$$

$$\Downarrow$$

$$\frac{V_S - V_1}{R_1} - \frac{V_1}{R'} + \frac{V_1}{R'}\left(1+\frac{R_3}{R_2/\!/R_L}\right) = 0$$

$$V_1\left(\cancel{\frac{1}{R'}} + \frac{R_3}{R'(R_2/\!/R_L)} - \cancel{\frac{1}{R'}} - \frac{1}{R_1}\right) = -\frac{V_S}{R_1}$$

$$V_1\left(\frac{R_3(R_2+R_L)}{R'\,R_2R_L} - \frac{1}{R_1}\right) = -\frac{V_S}{R_1}$$

$$V_1\left(\frac{R_3R_1R_2 + \cancel{R_LR_1R_3 - R'R_2R_L}}{R'\,R_2\,R_L\cancel{R_1}}\right) = -\frac{V_S}{\cancel{R_1}}$$

$$V_1 = -V_S\,\frac{R'\cancel{R_2}R_L}{R_1\cancel{R_2}R_3} = -V_S\,\frac{R_L}{R_2} \Rightarrow I_L = \frac{V_1}{R_L} = -\frac{V_S}{R_2}$$