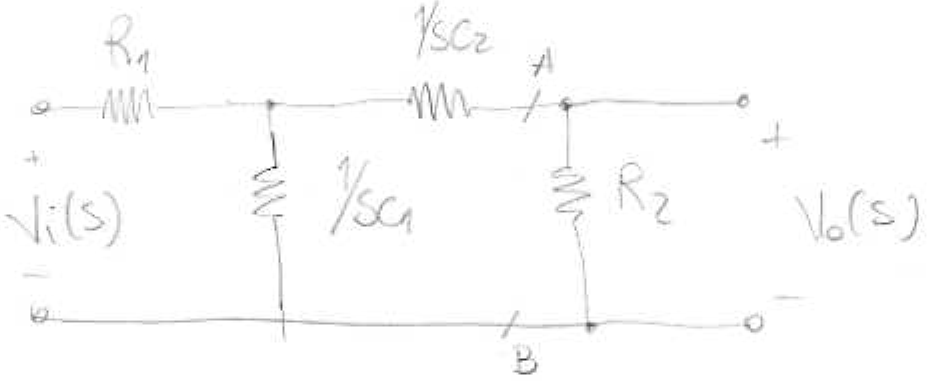
HW3 SOLUTION

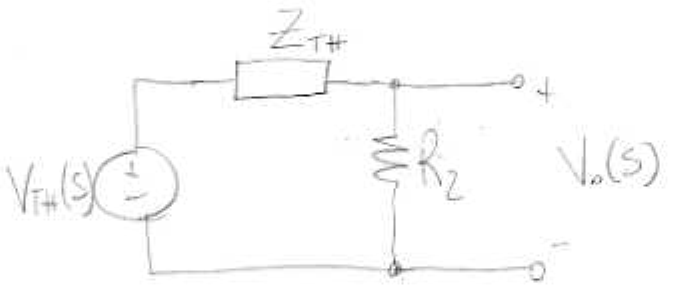
A.1)



Th. eq.

$$V_{TH}(s) = V_i(s)\,\frac{1/sC_1}{R_1 + 1/sC_1} = \frac{1}{sC_1R_1+1}\,V_i(s)$$

$$Z_{TH} = R_1 \,//\, 1/sC_1 + 1/sC_2$$



$$V_o(s) = V_{TH}(s)\,\frac{R_2}{Z_{TH}+R_2} = \frac{R_2}{Z_{TH}+R_2}\,\frac{1}{sC_1R_1+1}\,V_i(s)$$

$$Z_{TH} = \frac{R_1\,1/sC_1}{R_1+\frac{1}{sC_1}} + \frac{1}{sC_2} = \frac{R_1}{R_1sC_1+1} + \frac{1}{sC_2} =$$

$$= \frac{R_1sC_2 + R_1sC_1 + 1}{(R_1sC_1+1)sC_2} = \frac{sR_1(C_1+C_2)+1}{sC_2(sC_1R_1+1)}$$

$$V_0(s) = \frac{R_2}{\frac{sR_1(C_1+C_2)+1}{sC_2(sC_1R_1+1)} + R_2} \frac{1}{(sC_1R_1+1)} V_i(s) =$$

$$= \frac{R_2 sC_2 \cancel{(sC_1R_1+1)}}{sR_1(C_1+C_2)+1+sC_2(sC_1R_1+1)R_2} \frac{1}{\cancel{(sC_1R_1+1)}} V_i(s) =$$

$$= \frac{sR_2C_2}{\underbrace{s^2\tau_1\tau_2 + s(\tau_1+\tau_2+R_1C_2)+1}_{\text{(do you recognize this? look at HW 2)}}} V_i(s)$$

$$\boxed{F(s) = \frac{s\tau_2}{s^2\tau_1\tau_2 + s\tau^* + 1}}$$

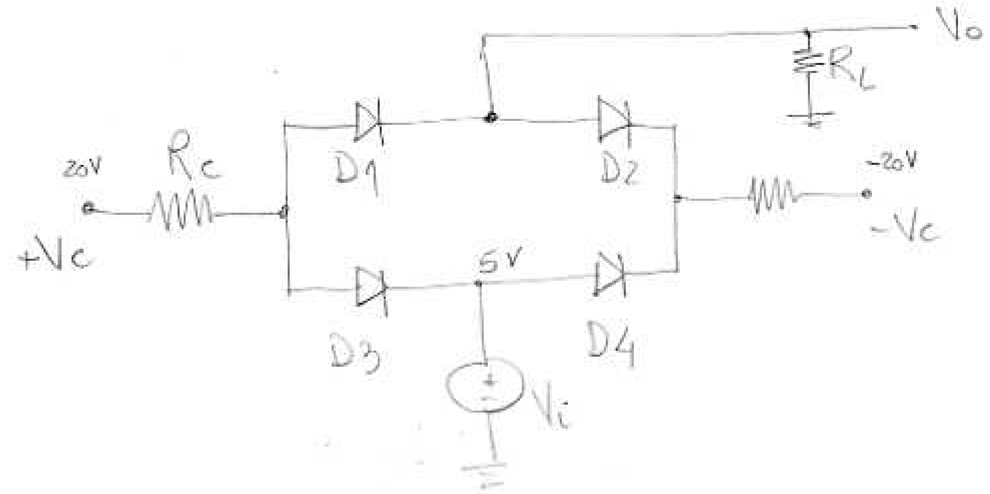
$$F(j\omega) = \frac{j\omega\tau_2}{j\omega\tau^* - \omega^2\tau_1\tau_2 + 1}$$

$$|F(j\omega)| = \frac{\omega\tau_2}{\sqrt{(1-\omega^2\tau_1\tau_2)^2 + \omega^2\tau^{*2}}}$$

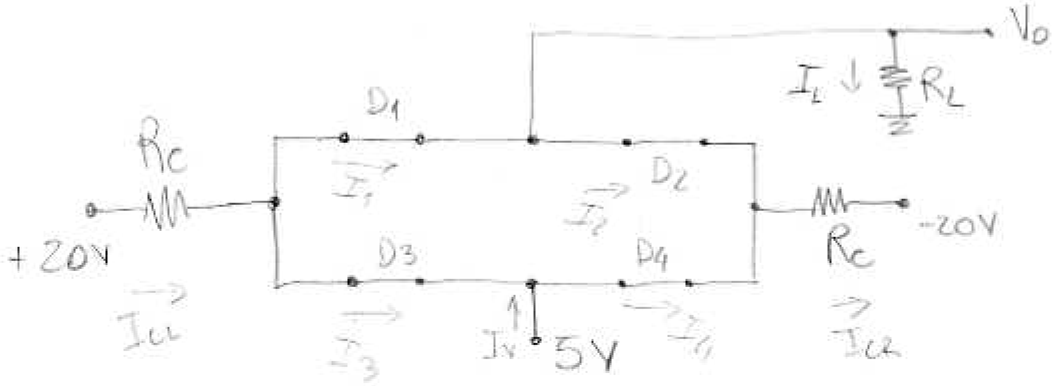
for $\omega = 0$ $|F(j\omega)| = 0$, for $\omega \to \infty$ $|F(j\omega)| \to 0$, also $|F(j\omega)|$ is always positive so intuitively it must have a maximum (that can be computed easily).

There is a standard method to plot $|F(j\omega)|$ and $\measuredangle F(j\omega)$ which is called Bode diagrams but we have not covered it.

B.1) $V_i = 5V$, $V_c = 20V$, $R_c = R_L$

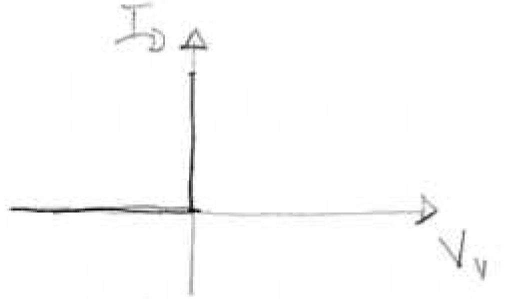


By intuition let's assume that the state is {ON, ON, ON, ON}



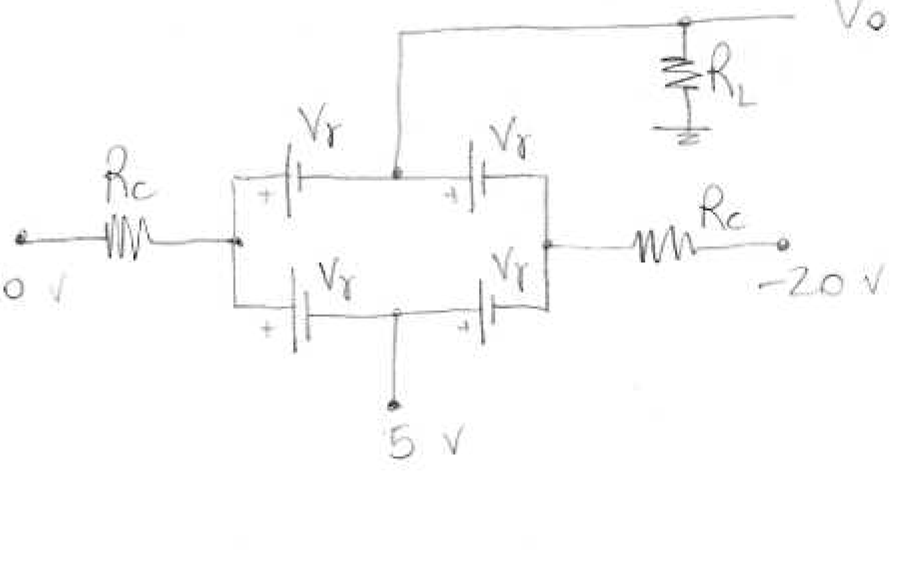
we have to verify that I_1, I_2, I_3, I_4 are all ≥ 0.

The problem that we have in determining the currents is the following. Instead of considering the model like in fig B.1.a consider the model in fig B.1.b



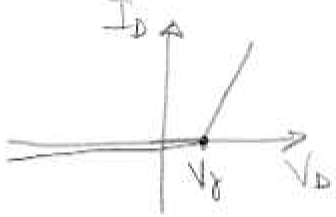
B.1.a

Then if all diodes are on:



There is a loop of voltage sources.
Their values are all equal to V_γ so KVL is satisfied but the current In the loop cannot be determined.
Model B.1.a is just equa to B.1.b if $V_\gamma \to 0$, so using the model B.1.a will not solve the problem.
We have then two choices:

1) Changing model and use

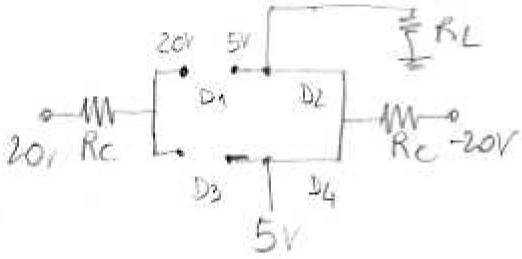


which means using

for all diodes in forward polarization

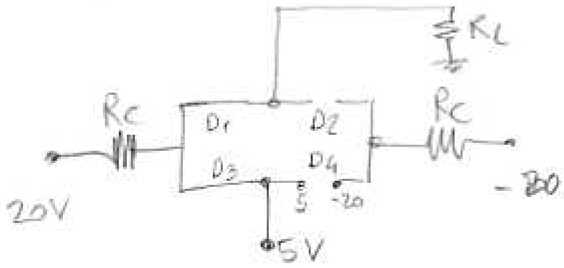
2) Verify that $\forall$ states $S \neq \{ON, ON, ON, ON\}$ we obtain a contradiction.

2) is tedious but feasible. I will draw the configurations for the states that are different from $\{ON, ON, ON, ON\}$ an write voltages but the answer could be incomplete:



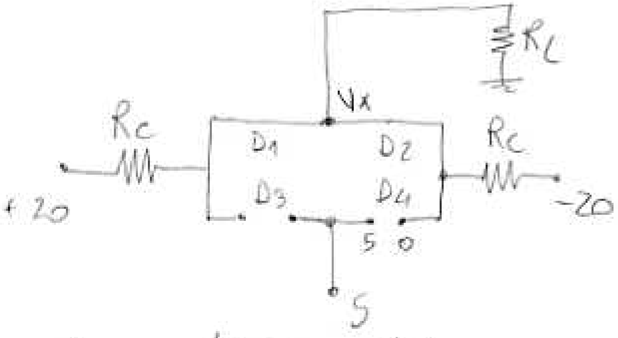
$S_1 = \{$ OFF, ON, OFF, ON $\}$

$V_{D1} > 0$, contr.



$S_2 = \{$ ON, OFF, ON, OFF $\}$

$V_{D4} > 0$, contr.

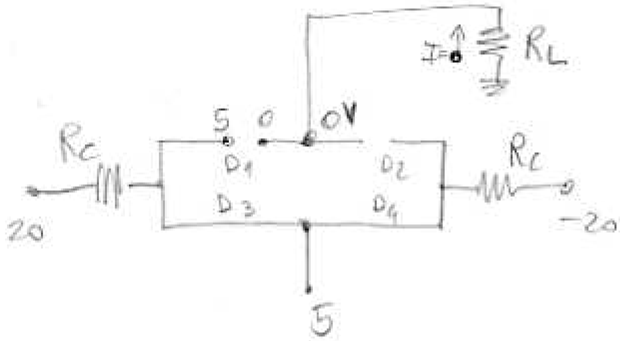


by antisymmetry

$V_x = 0$

$S_3 = \{$ ON, ON, OF, OF $\}$

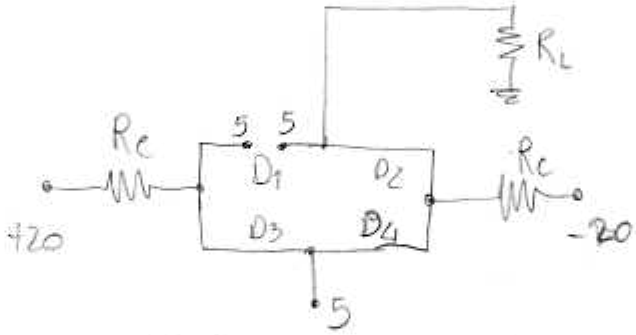
$V_{DS} > 0$, contr.



$S_4 = \{$ OFF, OFF, ON, ON $\}$

$V_{D4} > 0$, contr.

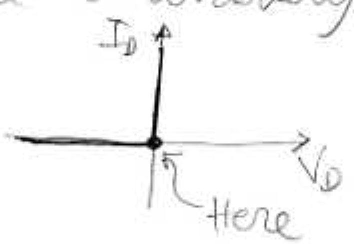
The states where only one diode is ON and all the others off (4 states) are easy to check and they all lead to a contr.



$S_9 = \{$ OFF, ON, ON, ON $\}$

$V_{D1} = 0$

this case is at the boundary



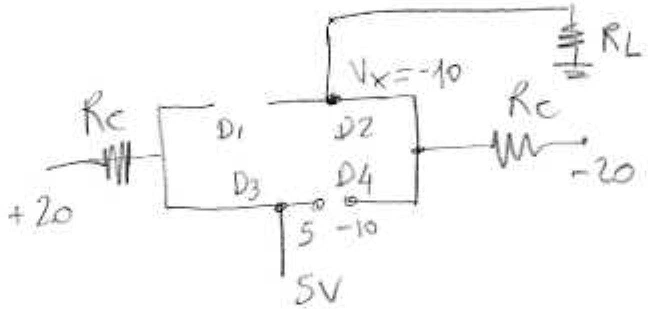
The diode can be considered both on or OFF. So we can either say that S_9 is a solution or that it is not a solution. (this will not change the final answer).

The same happens for

$S_{10} = \{ON, OFF, ON, ON\}$

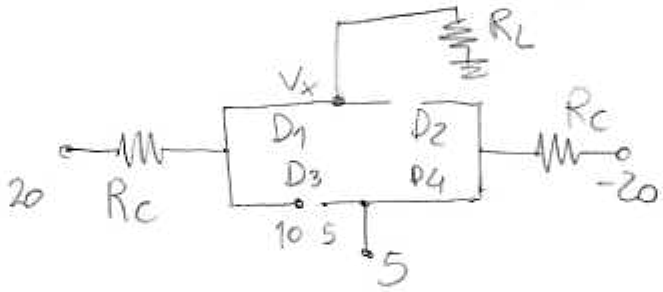
$S_{11} = \{ON, ON, OFF, ON\}$

$S_{12} = \{ON, ON, ON, OFF\}$



$$V_x = (-20-0)\frac{R_L}{R_c+R_L} = -20\frac{R_L}{2R_L} = -10$$

$S_{13} = \{OFF, ON, ON, OFF\}$ $\quad V_{D4} > 0$, contr



$$V_x = 20\frac{R_L}{R_c+R_L} = 10$$

$V_{D3} > 0$, contr.

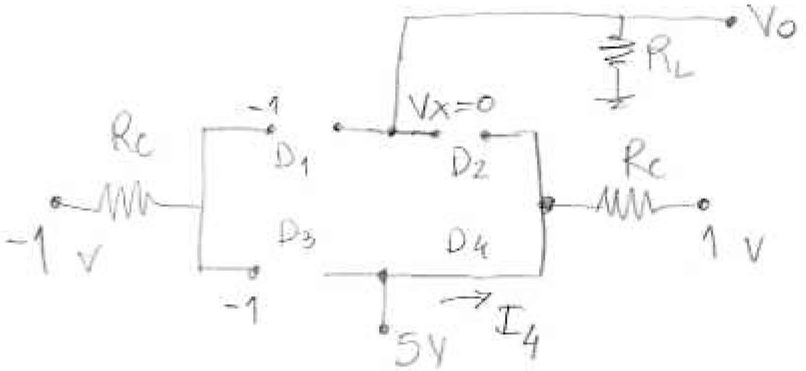
$S_{14} = \{ON, OFF, OFF, ON\}$

the last two states are $S_{15} = \{OFF, OFF, OFF, OFF\}$ that is easy to check and

$S_{16} = \{ON, ON, ON, ON\}$ which was our original guess. So, if we consider the states like S_9 to be a solution then $V_o = 5V$. If we consider the states like S_9 to be a contradiction the the only possible state is S_{16} and $V_o = 5V$

II) $V_c = -1$

By intuition let's try $S = \{OFF, OFF, OFF, ON\}$



$V_x = 0$

$V_{D1} = -1 - 0 = -1V < 0$ (ok)

$V_{D3} = -1 - 5 = -6V < 0$ (ok)

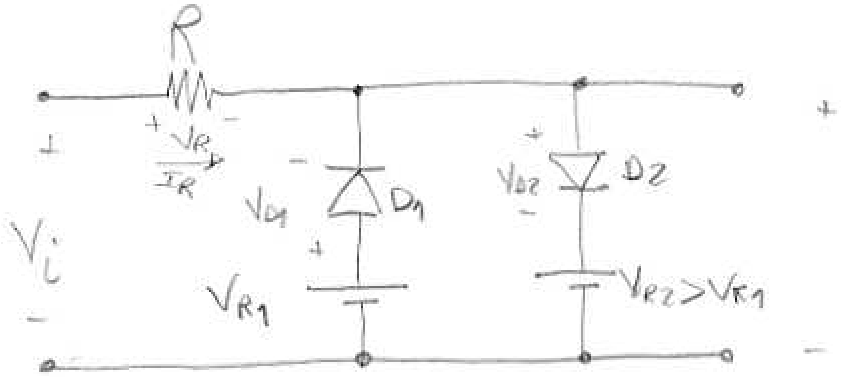
$V_{D2} = 0 - 5 = -5V < 0$ (ok)

$I_4 = \frac{5-1}{R_c} = \frac{4}{R_c} > 0$ (ok)

This state is a solution and

$V_o = V_x = 0$ V

C.1)



considere both D_1 and D_2 <u>off</u> :

$$V_{D1} - V_R + V_i - V_{R1} = 0 \Rightarrow V_{D1} = -V_i + V_{R1} + V_R$$

$$-V_{D2} - V_R + V_i - V_{R2} = 0 \Rightarrow V_{D2} = V_i - V_{R2} - V_R$$

also since $I_R = 0$ (because they are both OFF) $V_R = 0$, then

$$V_{D1} = -V_i + V_{R1}$$
$$V_{D2} = V_i - V_{R2}$$

In order for the state {OFF, OFF} to be valid :

$$V_{D1} < 0 \Rightarrow -V_i + V_{R1} < 0 \Rightarrow V_i > V_{R1}$$
$$V_{D2} < 0 \Rightarrow V_i - V_{R2} < 0 \Rightarrow V_i < V_{R2}$$

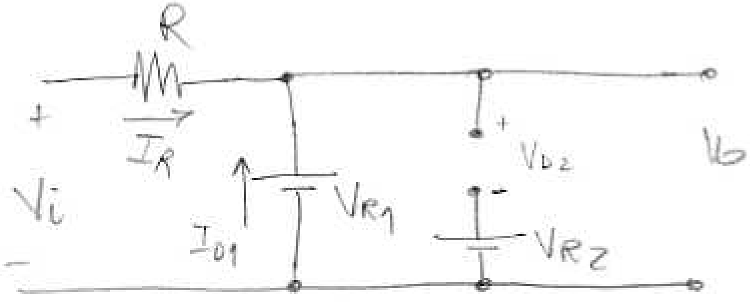
the two inequalities give a solution for V_i only if $V_{R2} > V_{R1}$

which is our case.

If $V_{R1} < V_i < V_{R2}$ then $V_o = V_i$ (because $I_R = 0$).

If $V_i < V_{R1}$ then the state {OFF, OFF} is not valid anymore because $V_{D1} > 0$.

Let's try the state {ON, OFF} then

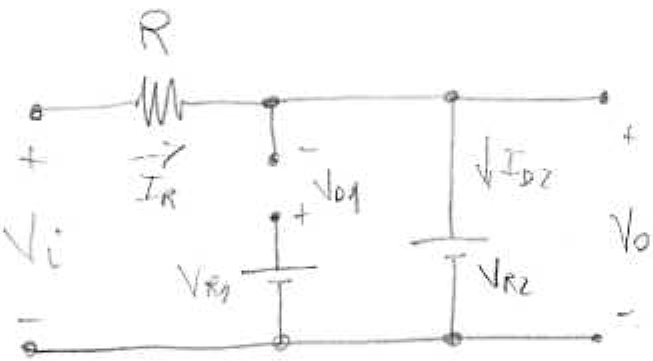


$V_{D2} = V_{R1} - V_{R2} < 0$ (ok)

$$I_R = \frac{V_i - V_{R1}}{R} = -I_{D1} \Rightarrow I_{D1} = \frac{V_{R1} - V_i}{R} > 0 \quad \text{(ok)}$$

this state is valid and $V_o = V_{R1}$

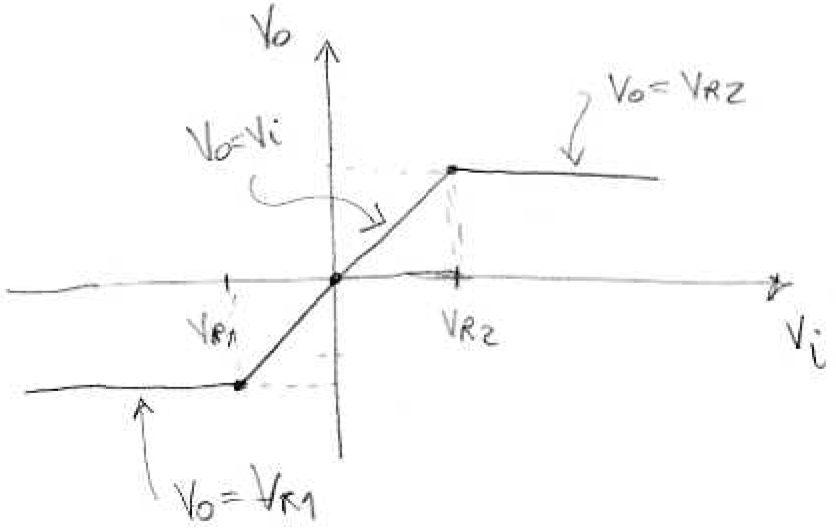
If $V_i > V_{R2}$ then let's try this other state



$V_{D1} = V_{R1} - V_{R2} < 0$ (ok)

$$I_R = I_{D2} = \frac{V_i - V_{R2}}{R} > 0 \quad \text{(ok)}$$

this state is also valid and $V_o = V_{R2}$. The transcharacteristic is then as follows



Notice that we haven't say anything about the values of V_{R1} and V_{R2} but we just know that $V_{R2} > V_{R1}$