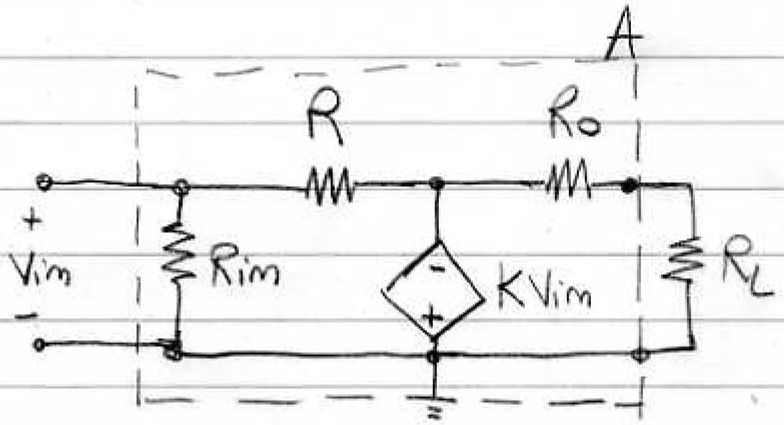
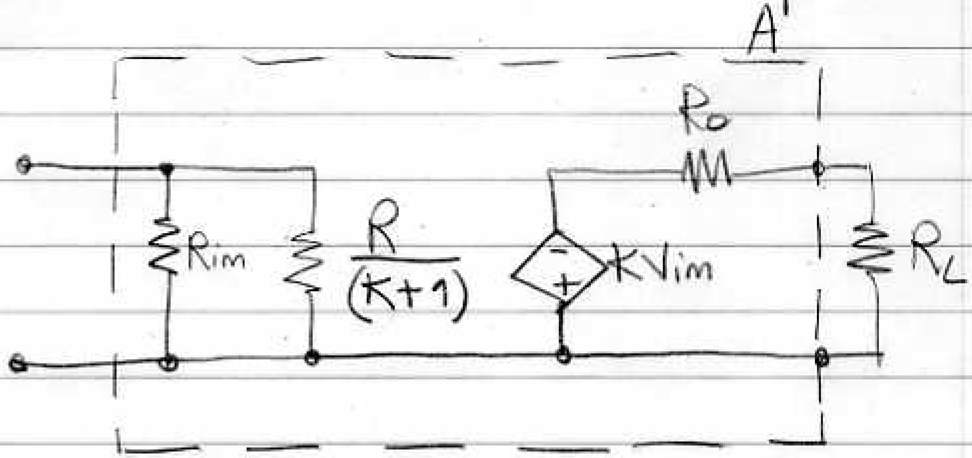
1) Consider the following circuit

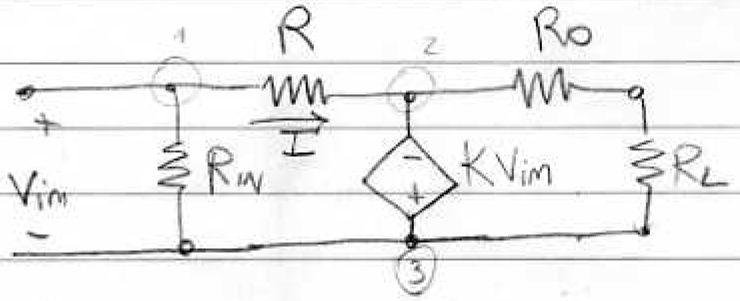


Prove that A is equivalent, from the point of view of Vin, to A'



(This is known as Miller effect. John M. Miller, 1918)

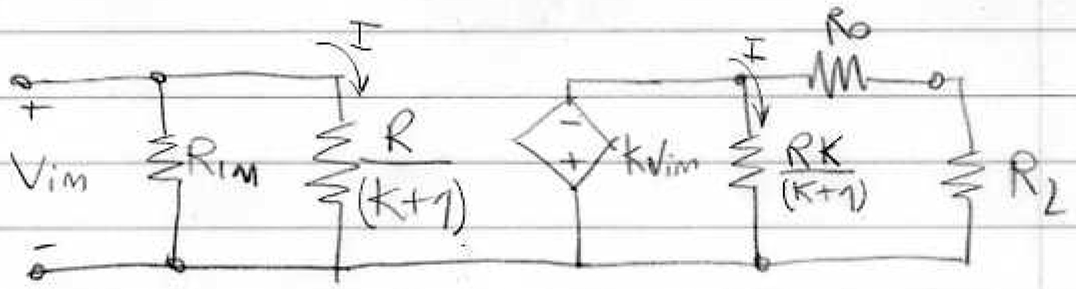
1)



$$I = \frac{(Vim + KVim)}{R} = \frac{Vim\,(1+K)}{R}$$

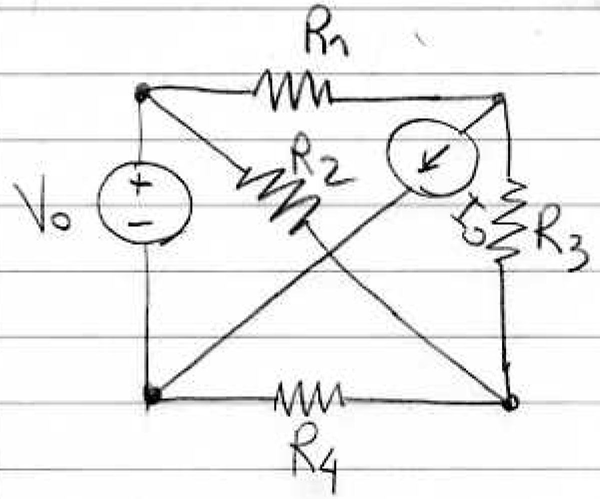
$$\frac{Vim}{I} = \frac{R}{K+1}$$

We want to add two elements, one between node 1 an 3 and the other between 2 and 3 in order to have the same currents.



the Miller theorem

2) PLANAR CIRCUITS



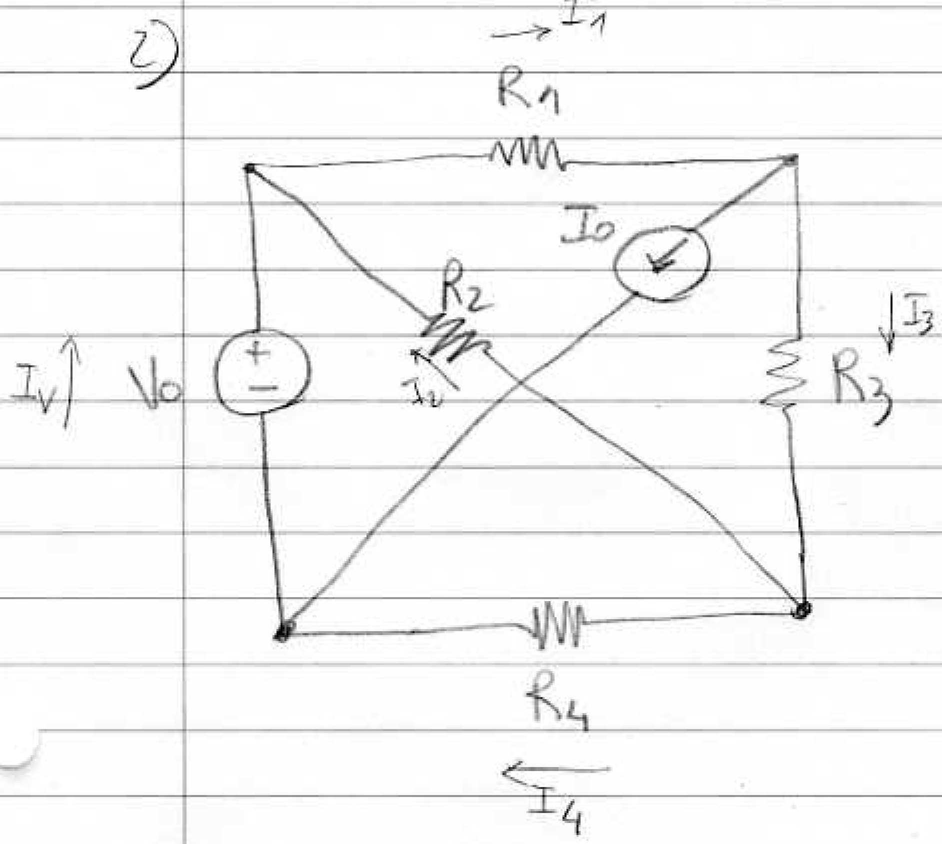
$R_1 = R_2 = 1\,\Omega$
$R_3 = R_4 = 2\,\Omega$
$I_0 = 2\,A$
$V_0 = 1\,V$

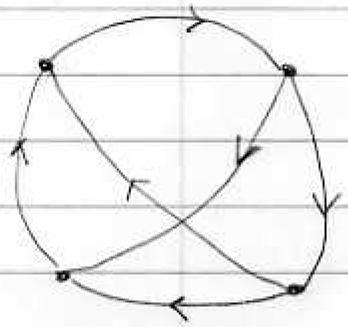
Solve the circuit using mesh current analysis

2.1) How many nodes do you have?
2.2) How many branches?
2.3) How many branches are going to be in the Tree?
2.4) How many branches are going to be in the cotree?
2.5) How many equations will you have?

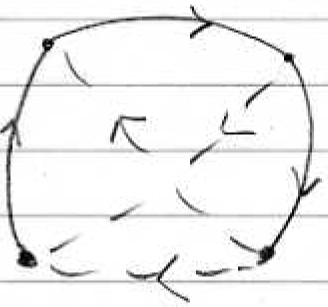
After solving the circuit, try to understand what is happening, how currents are flowing!

2)



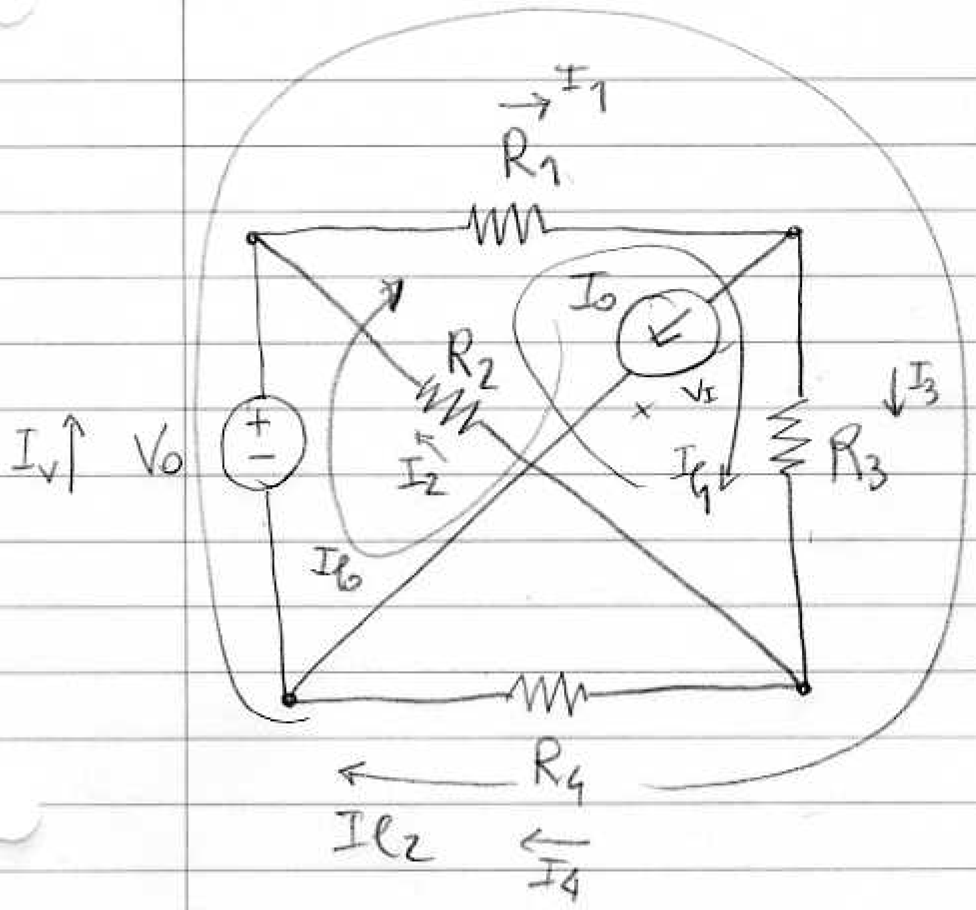


A smart choice is to have the current source in the cotree because it is going to fix a value for the loop-current.



— tree

- - - co-tree



2.1) 4 (N)
2.2) 6 (R)
2.3) 4−1=3 (N−1)
2.4) 6−3=3 (R−(N−1))
2.5) 3

$$\begin{cases} -V_0 + R_1 I_{\ell_0} + R_1 I_{\ell_2} + R_1 I_{\ell_1} - V_I = 0 \\ I_{\ell_1}(R_1+R_2+R_3) + I_{\ell_2}(R_1+R_3) + I_{\ell_0} R_1 = 0 \\ -V_0 + I_{\ell_2}(R_1+R_3+R_4) + I_{\ell_0} R_1 + I_{\ell_1} R_3 = 0 \\ I_{\ell_0} = I_0 \end{cases}$$

$$\begin{cases} -V_0 + R_1 I_0 + R_1 I_{e_1} + R_1 I_{e_2} - V_I = 0 \\ I_0 R_1 + I_{e_1}(R_1 + R_2 + R_3) + I_{e_2}(R_1 + R_3) = 0 \\ -V_0 + I_0 R_1 + I_{e_1}(R_1 + R_3) + I_{e_2}(R_1 + R_3 + R_4) = 0 \end{cases}$$

$$\begin{cases} I_{e_1} + I_{e_2} - V_I = -1 \\ 4 I_{e_1} + 3 I_{e_2} = -2 \\ 3 I_{e_1} + 5 I_{e_2} = -1 \end{cases}$$

$$\begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} I_{e_1} \\ I_{e_2} \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$I_{e_1} = \frac{-1 - 5 I_{e_2}}{3} = -\frac{1}{3} - \frac{5}{3} I_{e_2}$$

$$-\frac{4}{3} - \frac{20}{3} I_{e_2} + 3 I_{e_2} = -2 \Rightarrow -\frac{4}{3} - \frac{11}{3} I_{e_2} = -2$$

$$\Rightarrow I_{e_2} = \frac{2}{\cancel{3}} \frac{3}{11} = \frac{2}{11}$$

$$I_{e_1} = -\frac{1}{3} - \frac{5}{3} \frac{2}{11} = -\frac{1}{3} - \frac{10}{33} = \frac{-11 - 10}{33} = -\frac{21}{33} = -\frac{7}{11}$$

$$-\frac{7}{11} + \frac{2}{11} - V_I = -1 \Rightarrow -V_I = -1 + \frac{5}{11} \Rightarrow V_I = \frac{6}{11}$$

$$I_v = 2 + I_{e_2} = 2 + \frac{2}{11} = \frac{24}{11}$$

$$I_1 = I_0 + I_{e_1} + I_{e_2} = 2 + \frac{2}{11} - \frac{7}{11} = \frac{22+2-7}{11} = \frac{17}{11}$$

$$I_2 = I_{e_1} = -\frac{7}{11}$$

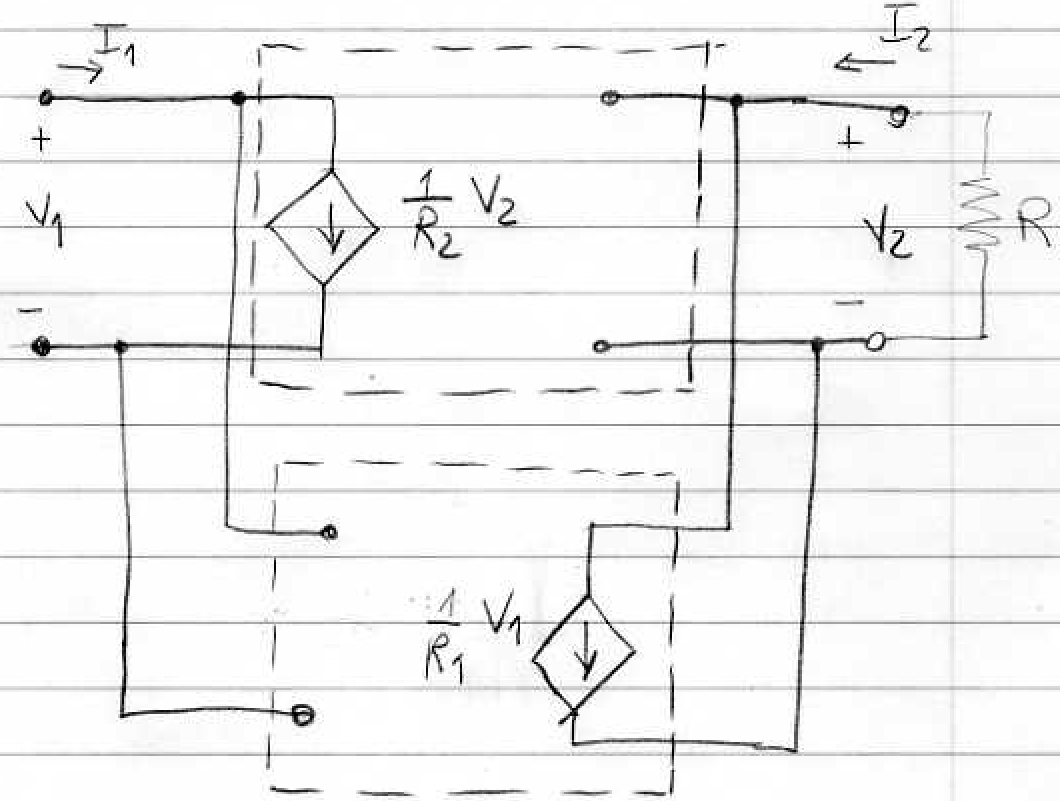
$$I_3 = I_{e_2} + I_{e_1} = -\frac{5}{11}$$

$$I_4 = I_{e_2} = \frac{2}{11}$$

$$V_I = \frac{6}{11}$$

3) Negative impedance converter

Using controlled sources we can change a passive resistor R in an active resistor $-R'$



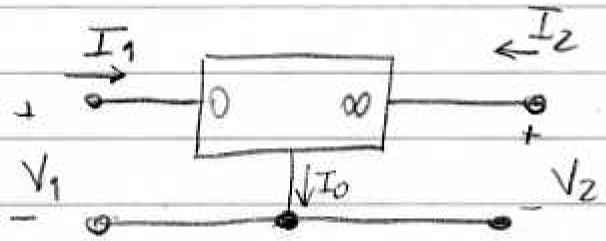
Compute $\frac{V_1}{I_1}$

3) $I_1 = \frac{V_2}{R_2} = -\frac{R I_2}{R_2} = -R\frac{V_1}{R_1 R_2}$

$\frac{V_1}{I_1} = -\frac{R_1 R_2}{R} = -R'$

4) NULLOR

Define a new component:



the nullor

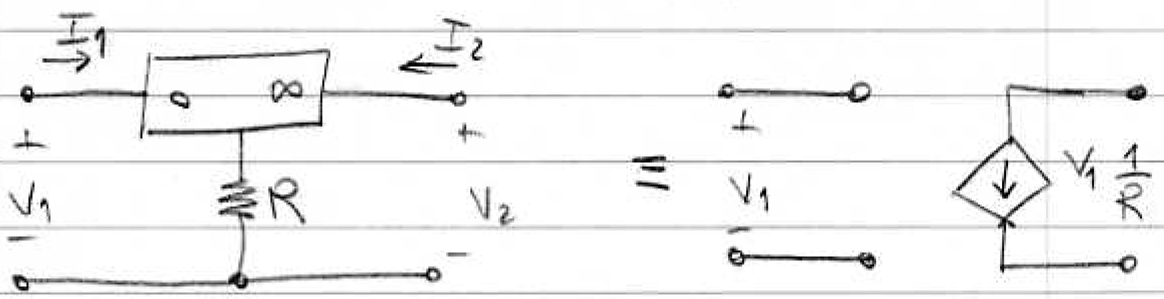
The costitutive equations are

$$V_1 = 0 \ , \ I_1 = 0$$

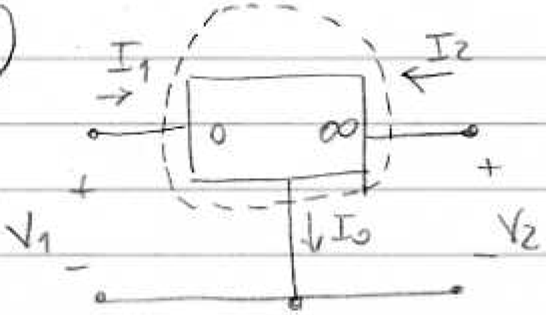
V_2 an I_2 are determined by the rest of the circuit.

4.1) Using the generalization of KCL compute I_0.

4.2) Show that the two circuits are equivalent

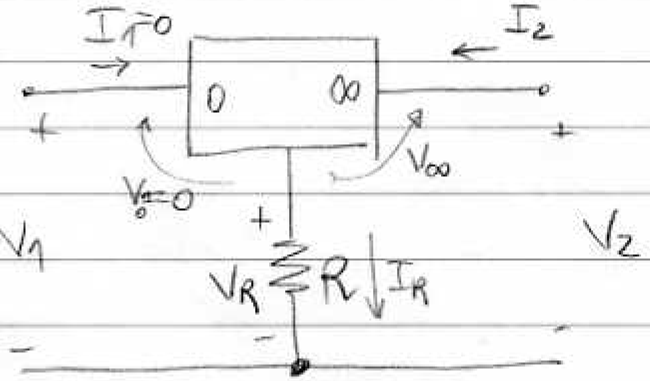


4.1)



$I_0 = I_1 + I_2 = I_2$ because $I_1 = 0$

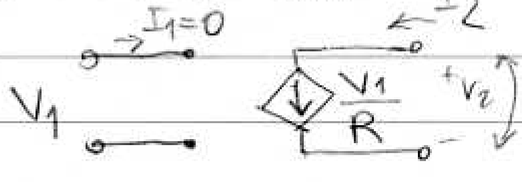
4.2)



$V_R + V_0 = V_1 \Rightarrow V_R = V_1 \Rightarrow I_R = \dfrac{V_1}{R}$

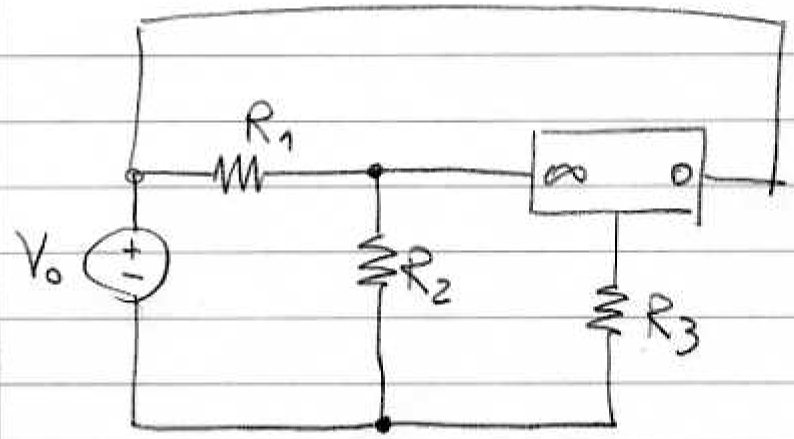
$I_R = I_2 \Rightarrow I_2 = \dfrac{V_1}{R}$

$V_2 = V_R + V_{\infty}$ but V_{∞} is determined by the rest of the circuit and so is V_2.



determined by the rest of the circuit

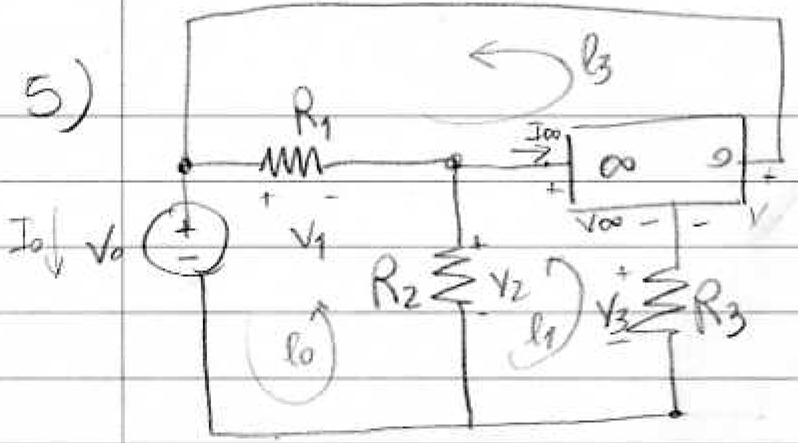
5)



$V_0 = 2 \ V$
$R_1 = 1 \ \Omega$
$R_2 = 1/3 \ \Omega$
$R_3 = 1/10 \ \Omega$

5.1) Solve the circuit

5)



Using mesh-current

$$\begin{cases} l_0 : & V_0 - V_2 - V_1 = 0 \\ l_1 : & -V_3 - V_\infty + V_2 = 0 \\ l_3 : & (V_\infty - V) + V_1 = 0 \end{cases}$$

$$\begin{cases} V_0 - R_2(I_{l_1} - I_{l_0}) - R_1(I_{l_3} - I_{l_0}) = 0 \\ +I_{l_1}R_3 - V_\infty + R_2(I_{l_1} - I_{l_0}) = 0 \\ (V_\infty - V) + R_1(I_{l_3} - I_{l_0}) = 0 \end{cases}$$

but $I_{l_3} = 0$ and $V = 0$

$$\begin{array}{l} l_0 \\ l_1 \\ l_3 \end{array} \begin{cases} V_0 - R_2(I_{l_1} - I_{l_0}) - R_1 I_{l_0} = 0 \\ I_{l_1}R_3 - V_\infty + R_2(I_{l_1} - I_{l_0}) = 0 \\ V_\infty - R_1 I_{l_0} = 0 \end{cases}$$

We have 3 eqns in 3 unknowns

$V_{00} = R_1 I_{\ell_0}$ (from ℓ_3)

$$\begin{cases} V_0 - R_2 I_{\ell_1} + I_{\ell_0}(R_2 - R_1) = 0 \\ I_{\ell_1}(R_3 + R_2) - R_2 I_{\ell_0} - R_1 I_{\ell_0} = 0 \end{cases}$$

$$I_{\ell_1} = I_{\ell_0} \frac{(R_2 + R_1)}{R_2 + R_3} = I_{\ell_0} \frac{4/3}{13/30} = I_{\ell_0} \frac{4}{\cancel{3}} \frac{3\cancel{0}}{13} =$$

$$= I_{\ell_0} \frac{40}{13}$$

$$V_0 - R_2 \frac{40}{13} I_{\ell_0} - \frac{2 I_{\ell_0}}{3} = 0 \Rightarrow$$

$$\Rightarrow 2 - I_{\ell_0}\left(\frac{40}{39} + \frac{2}{3}\right) = 0 \Rightarrow I_{\ell_0} \frac{66}{39} = 2$$

$$\Rightarrow I_{\ell_0} = \frac{39}{33} = \frac{13}{11}$$

$$I_{\ell_1} = \frac{13}{11} \frac{40}{13} = \frac{40}{11}$$

$I_0 = \frac{13}{11}$; $I_1 = -\frac{13}{11}$; $I_2 = \frac{40}{11} - \frac{13}{11} = \frac{27}{11}$

$I_3 = -\frac{40}{11}$ $I_\infty = -\frac{40}{11}$

Also you can compute all the voltages.