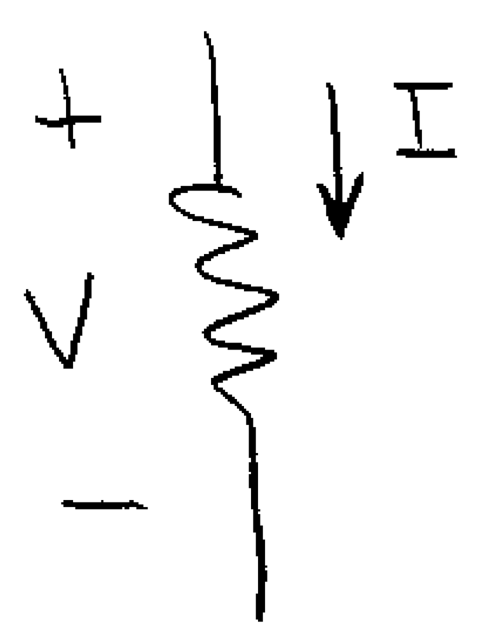
EE 40

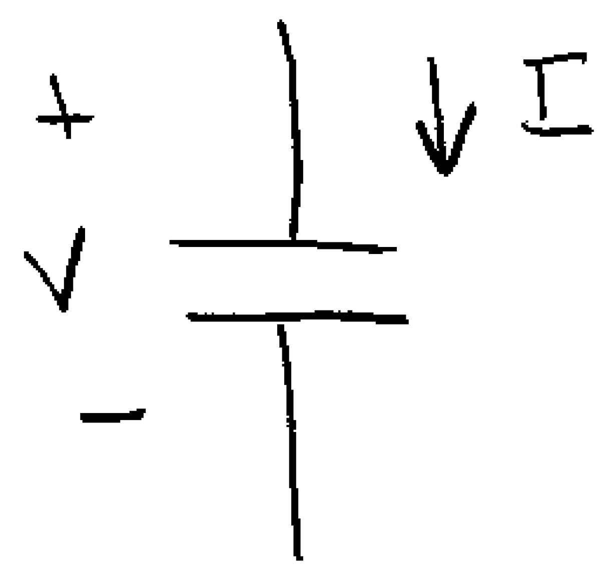
Lecture 8

Thevenin and Norton Equivalent Circuits

We have been studying the current-voltage relationships for several devices, like resistors and capacitors, for example.

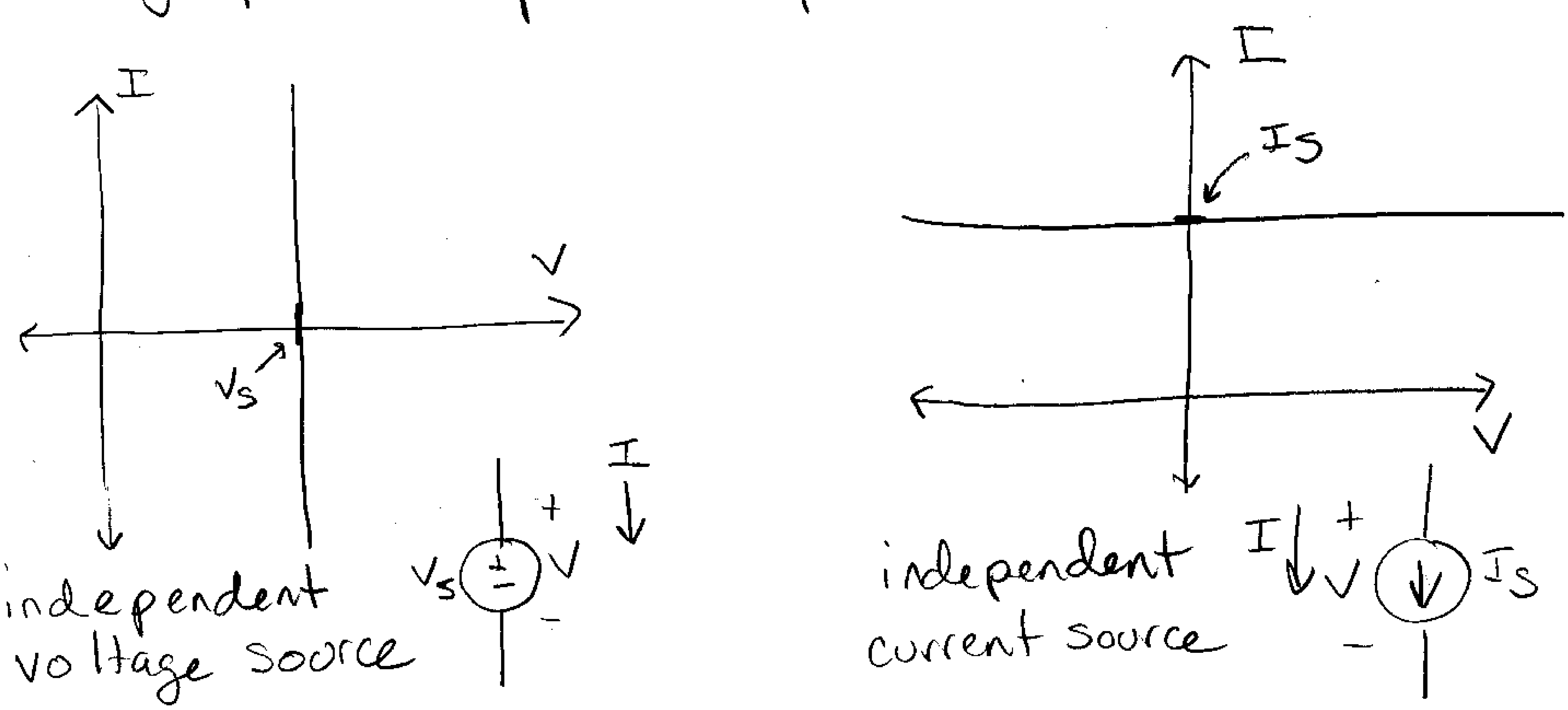


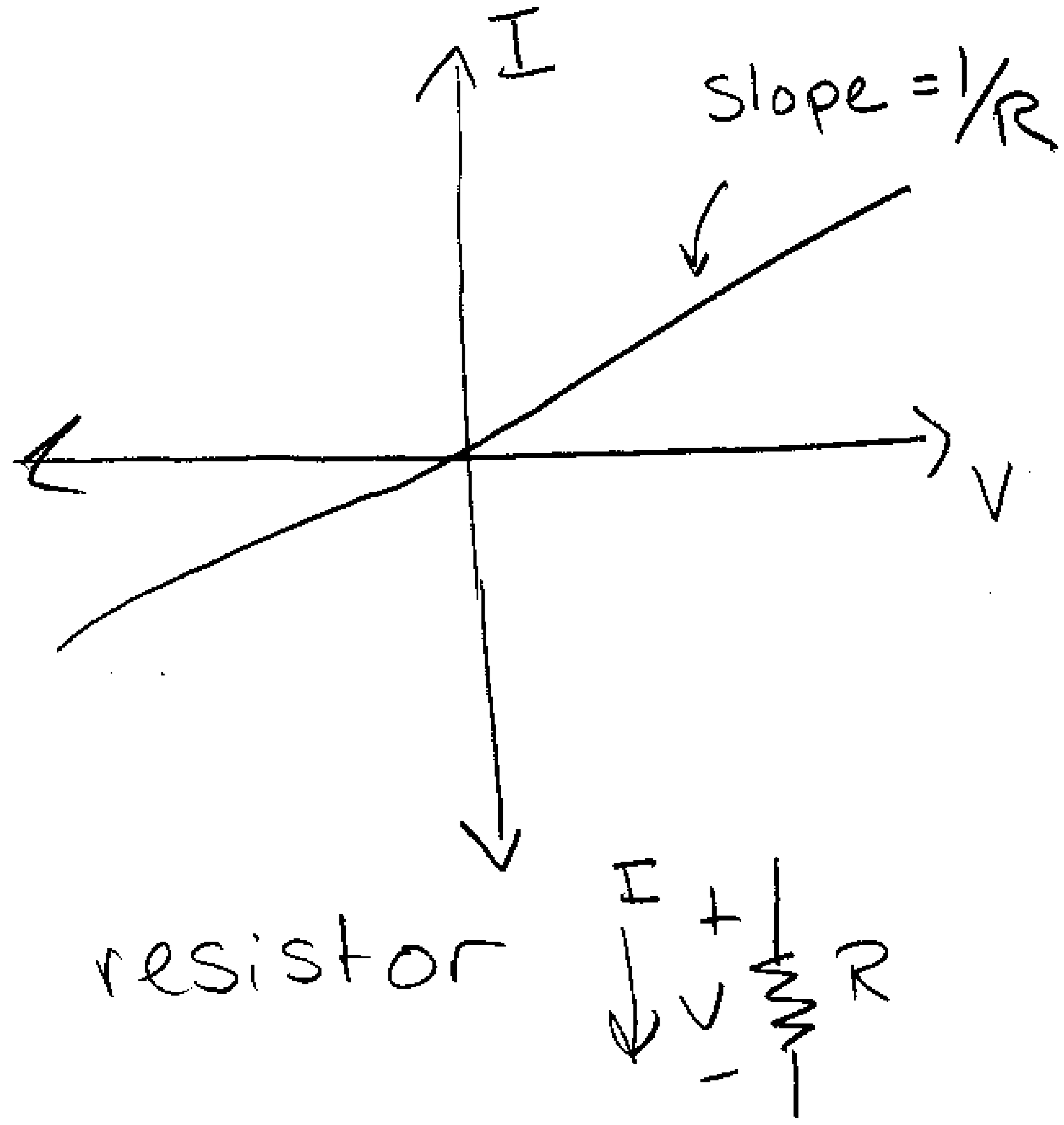
$V = RI$

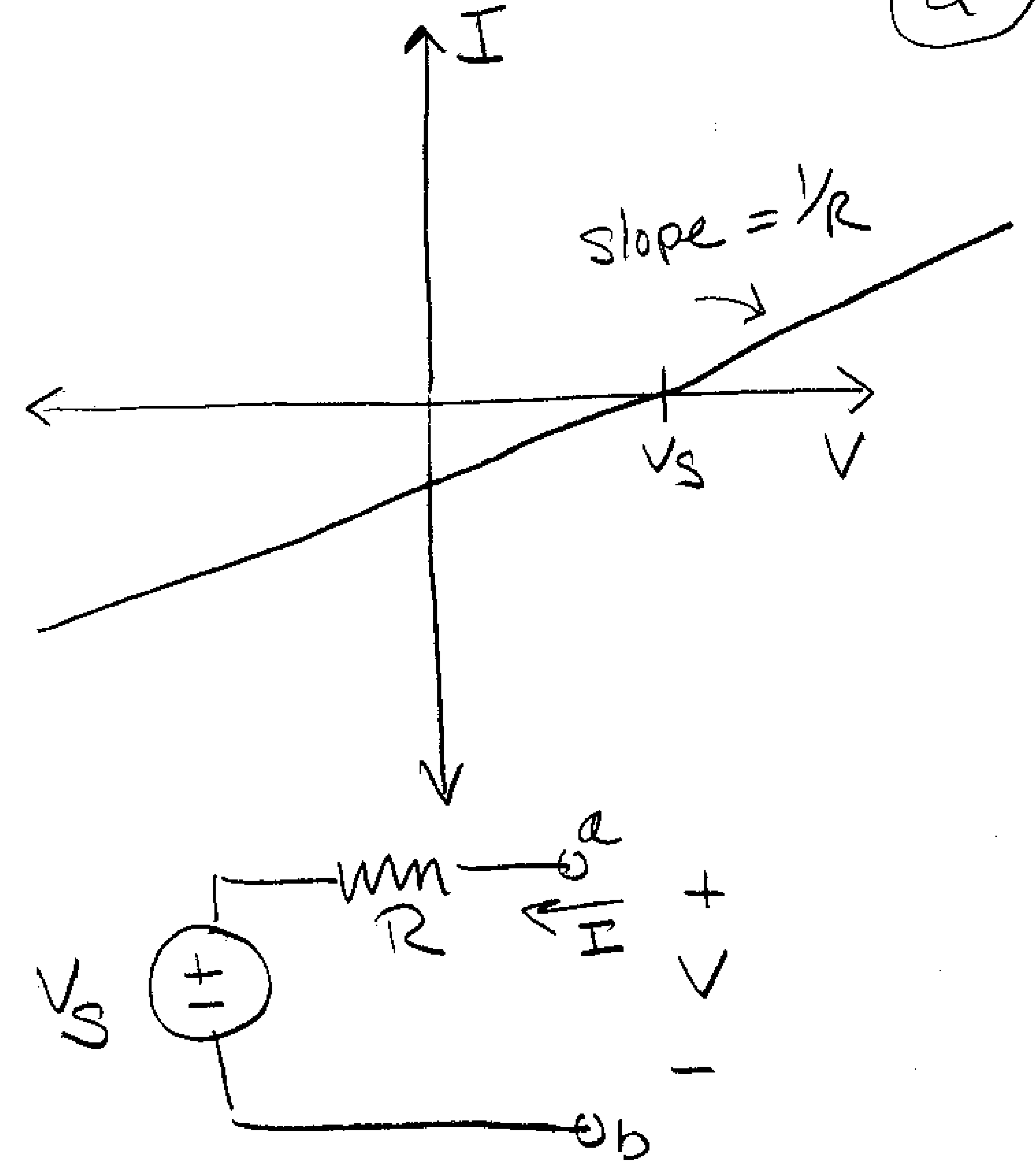


$I = C\frac{dV}{dt}$

We can characterize the I-V relationship that an entire chunk of circuit contributes to the rest of the system, by drawing a graph. Simple examples:

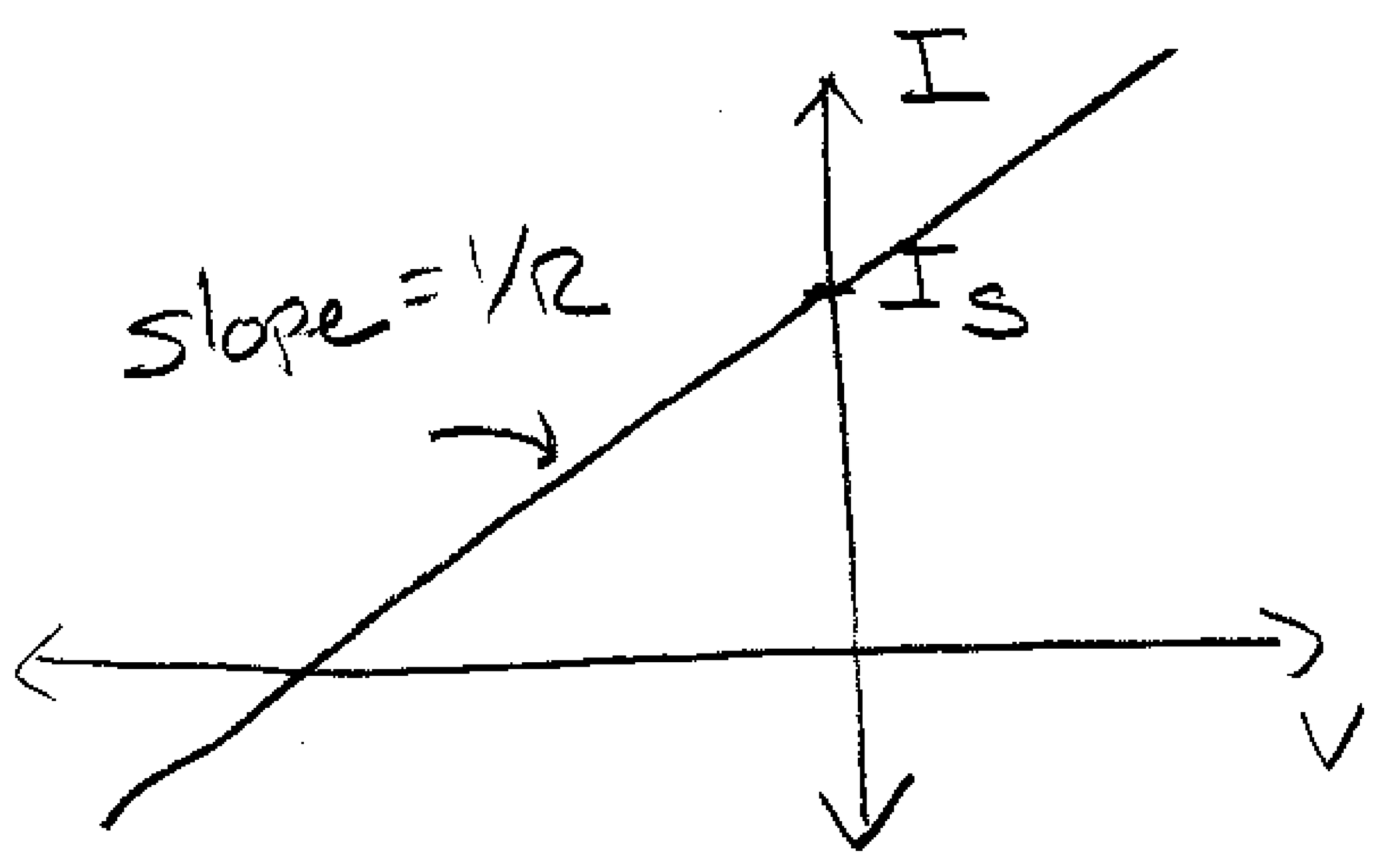




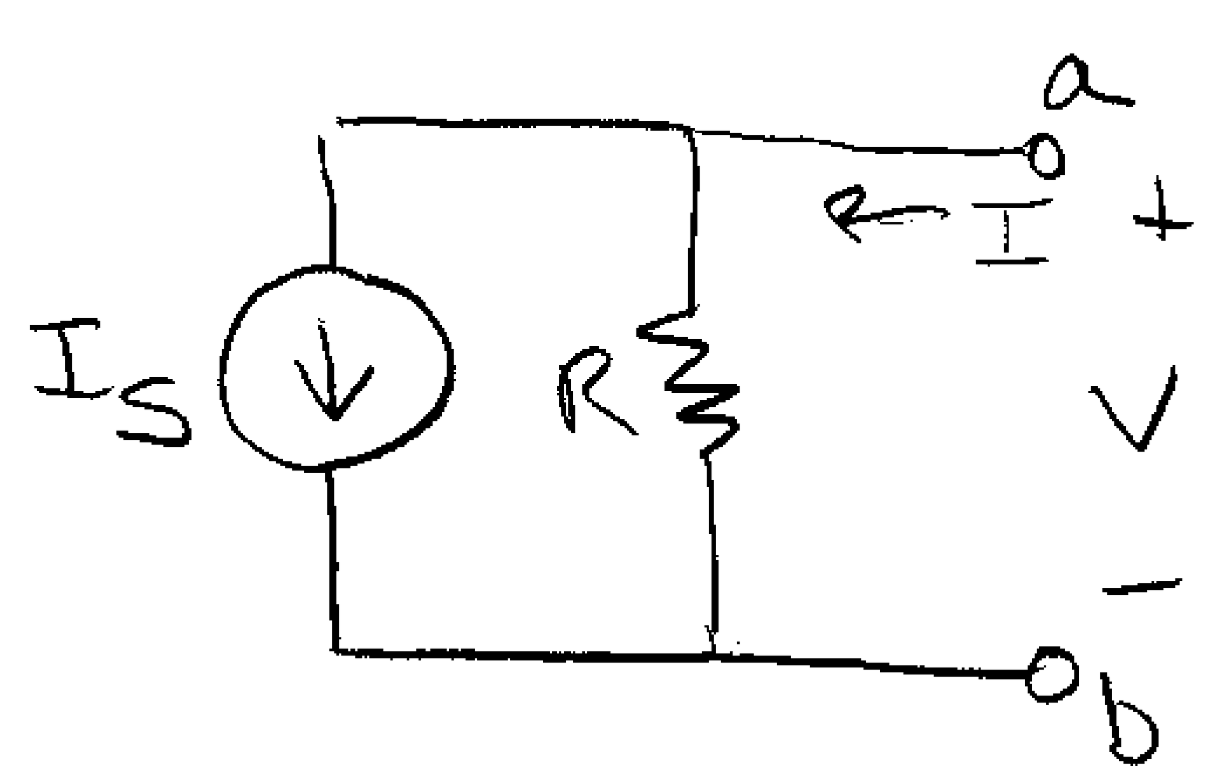


Derive the graph for the voltage source w/resistor:

$$-V + IR + V_S = 0 \qquad I = \frac{V - V_S}{R}$$



Derive the graph for the current source in parallel with resistor:

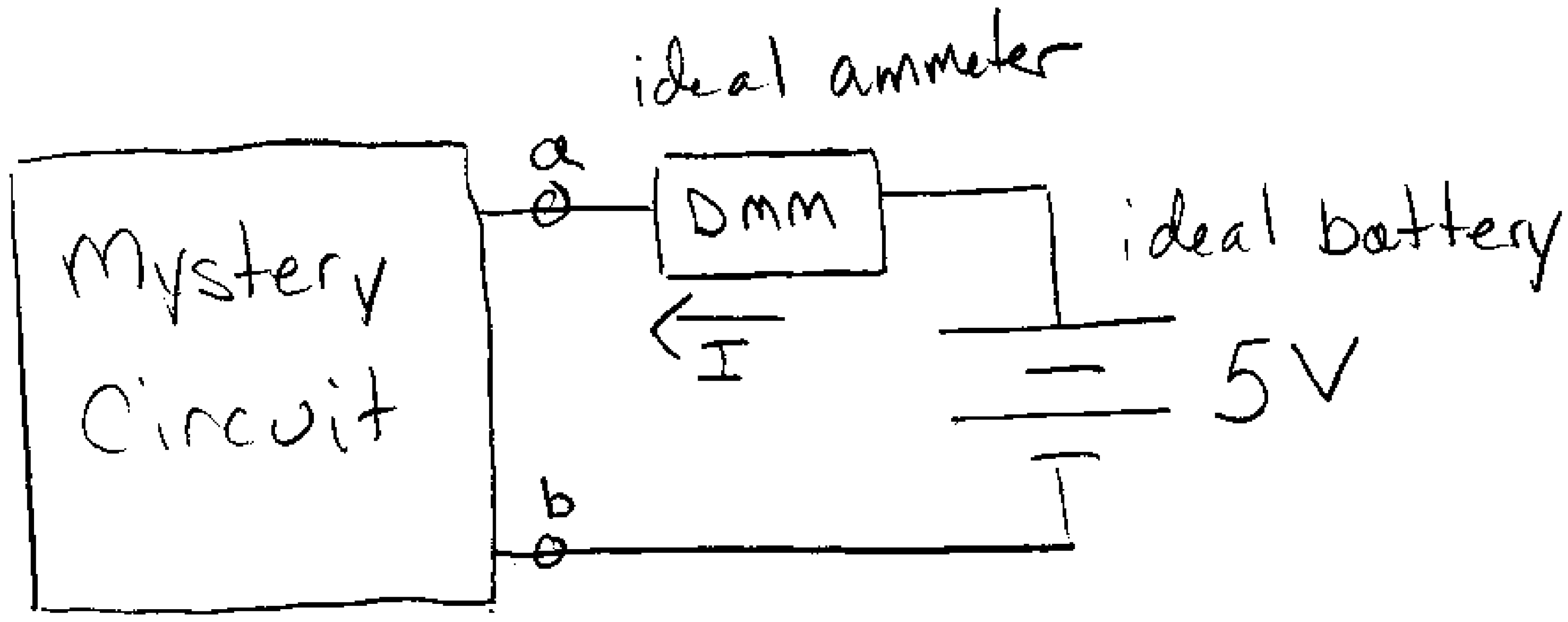


$$I = I_S + \frac{V}{R}$$

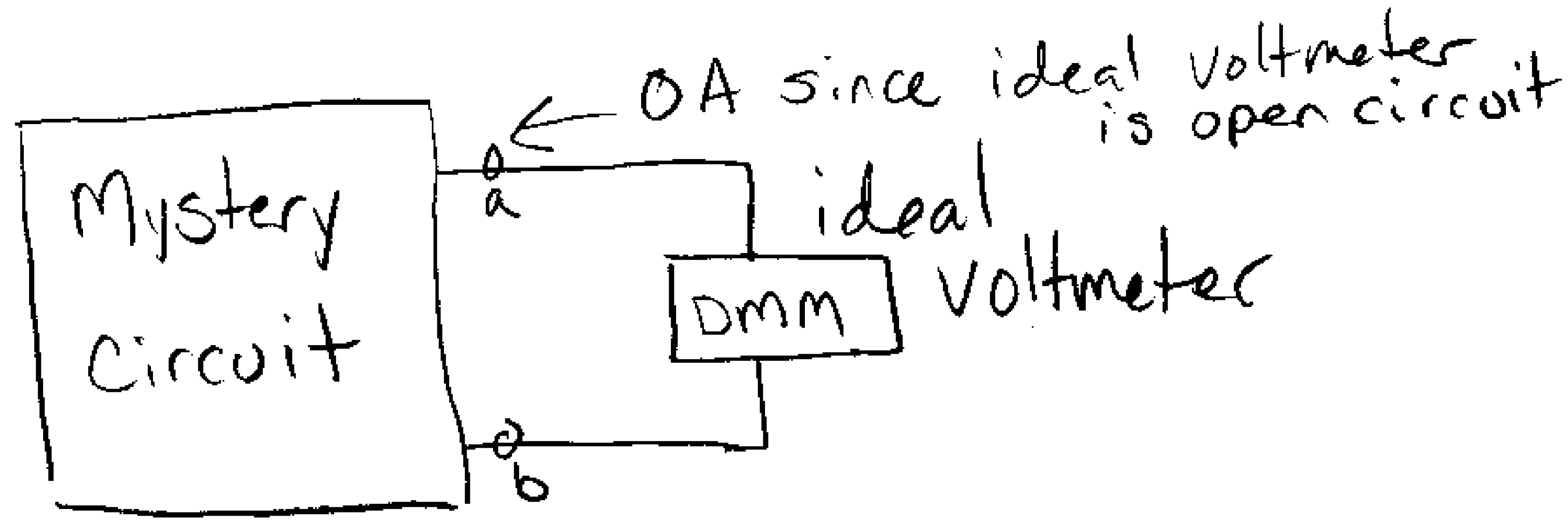
If a chunk of a circuit contains only resistors, independent sources, and linear dependent sources, then its I-V relationship will be linear.

You can graph the I-V relationship then, using only 2 current-voltage measurements.

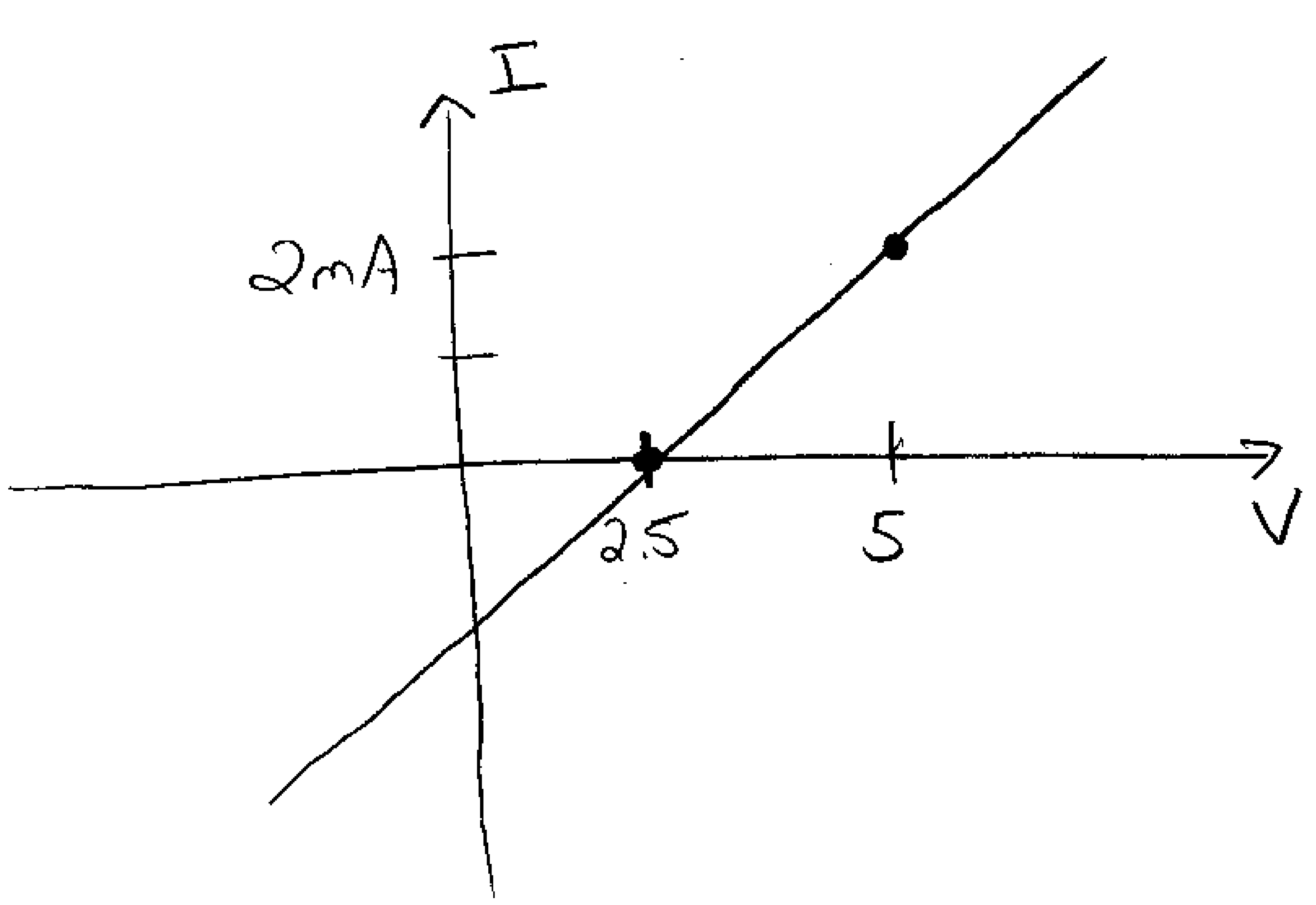
Example:



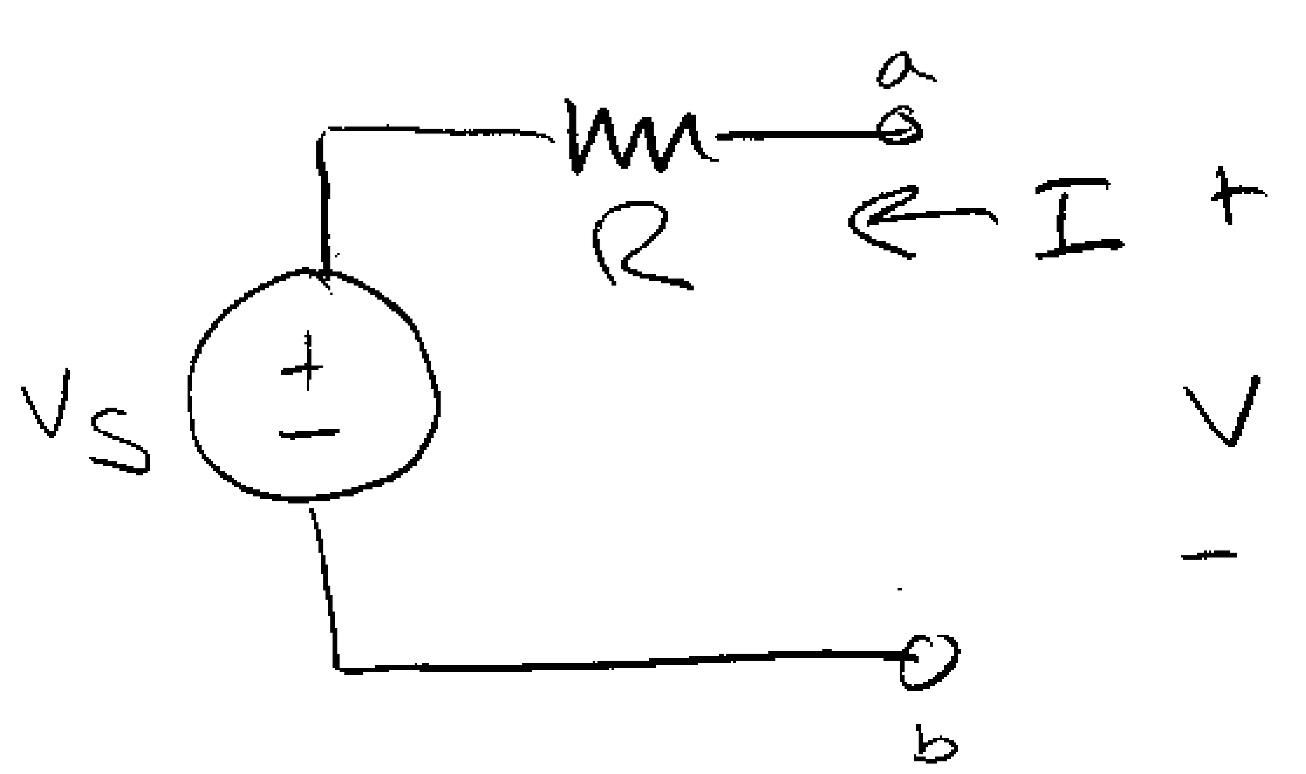
Ammeter reads 2 mA

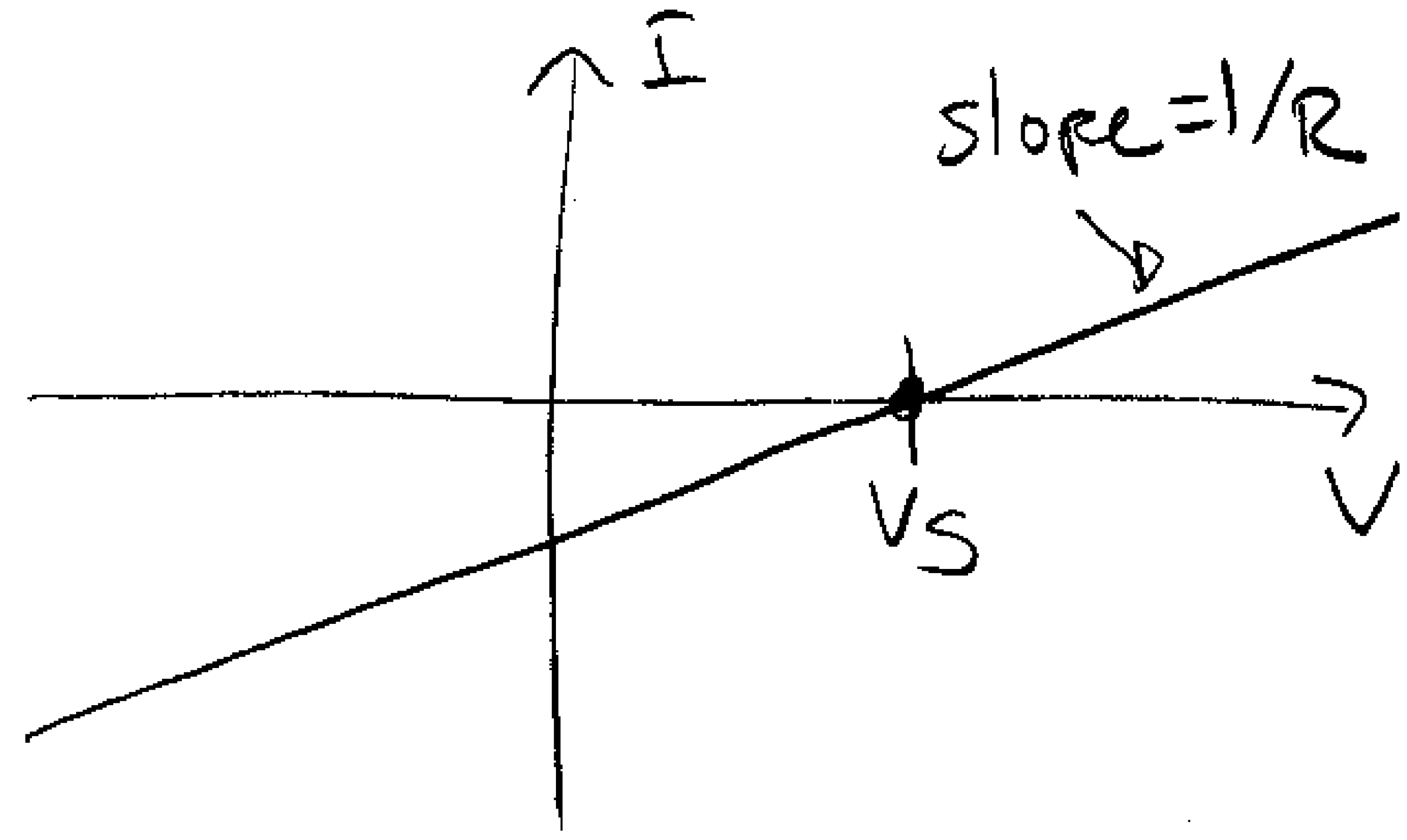


Voltmeter reads 2.5 V



Let's look back at the I-V relationship





We can replicate <u>any</u>* linear I-V characteristic graph through choice of V_S and R.

This is the <u>Thevenin</u> <u>equivalent</u> circuit.

So the way that a complicated linear circuit interacts with the outside world can be simplified down to the behavior of a voltage source in series with a resistor!

Example: Voltage source in lab. Inside, very complicated! But you only care how it interacts with your circuit. To you, it is an ideal voltage source in series with a small resistance.

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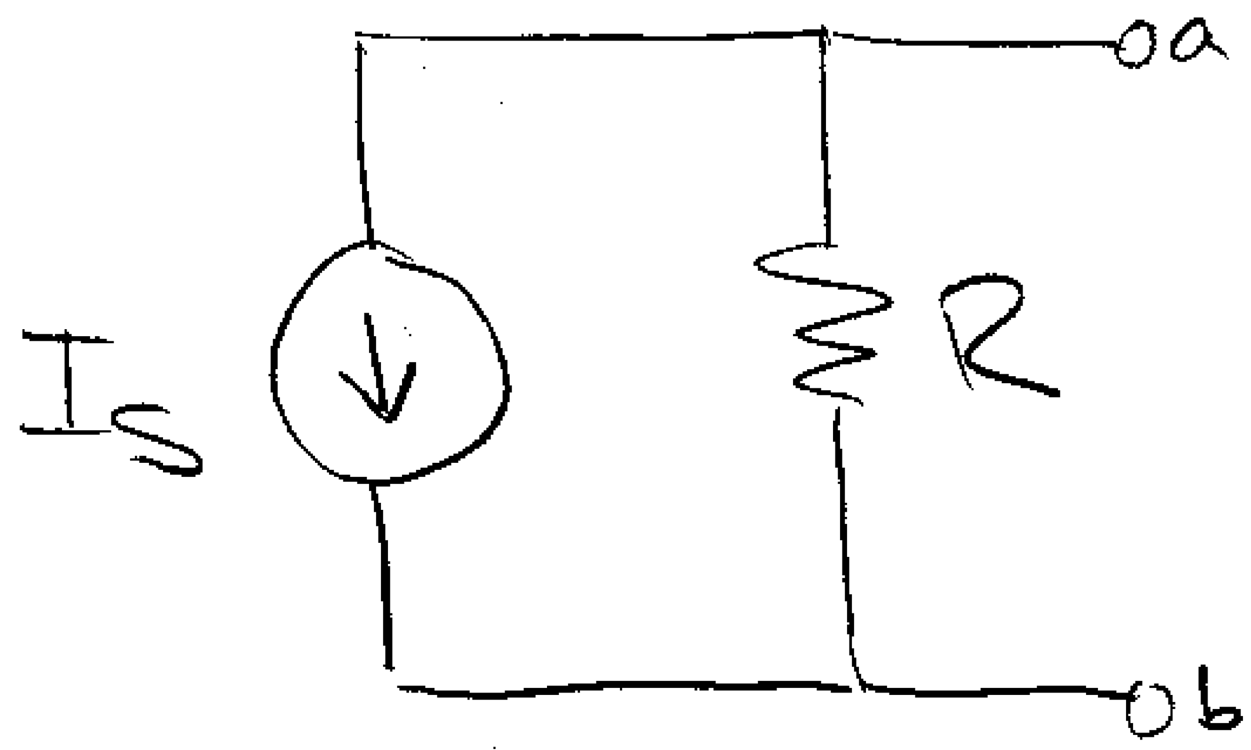
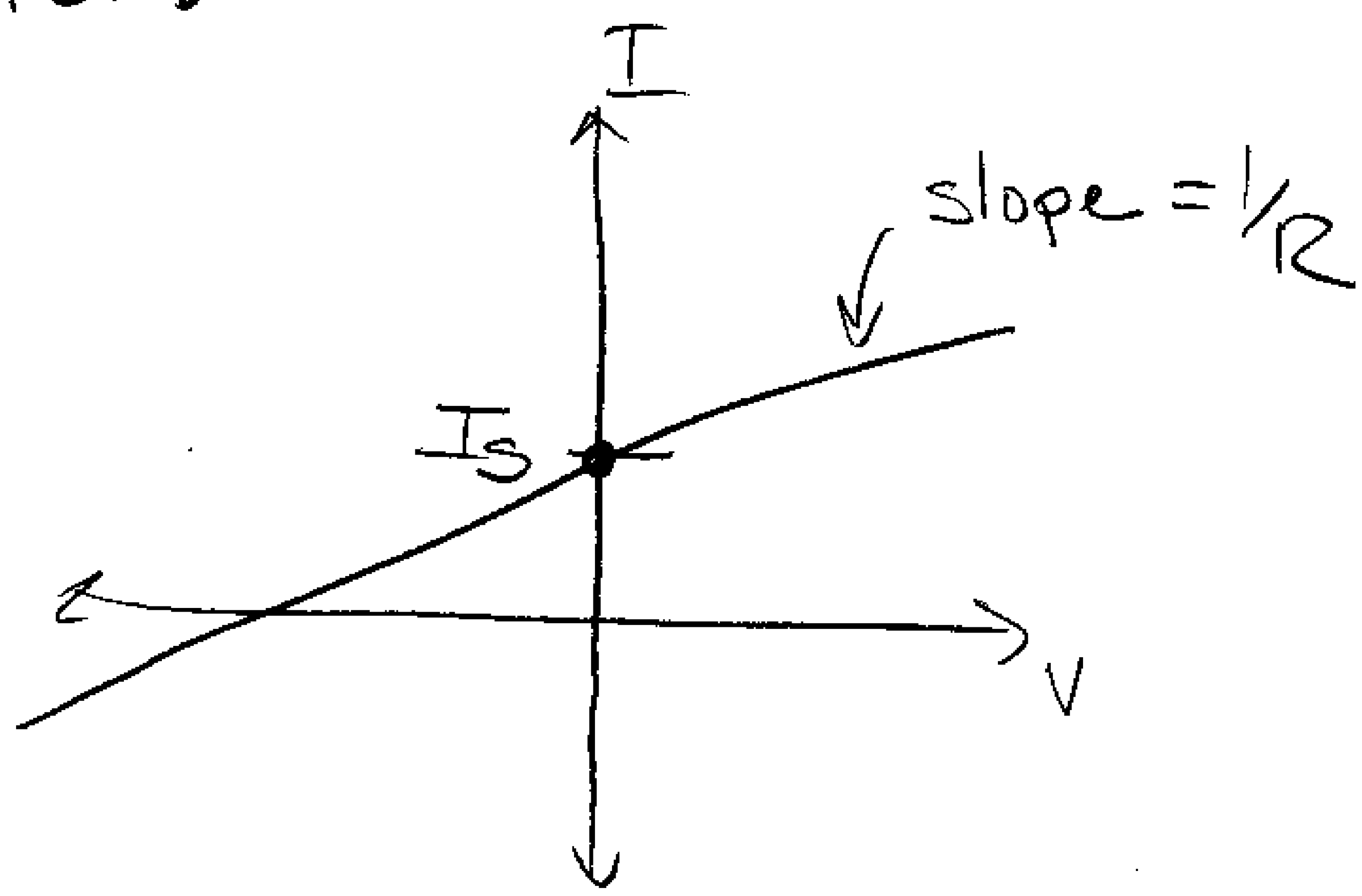
* What is the exception?

Let's also revisit the current source in parallel with the resistor.



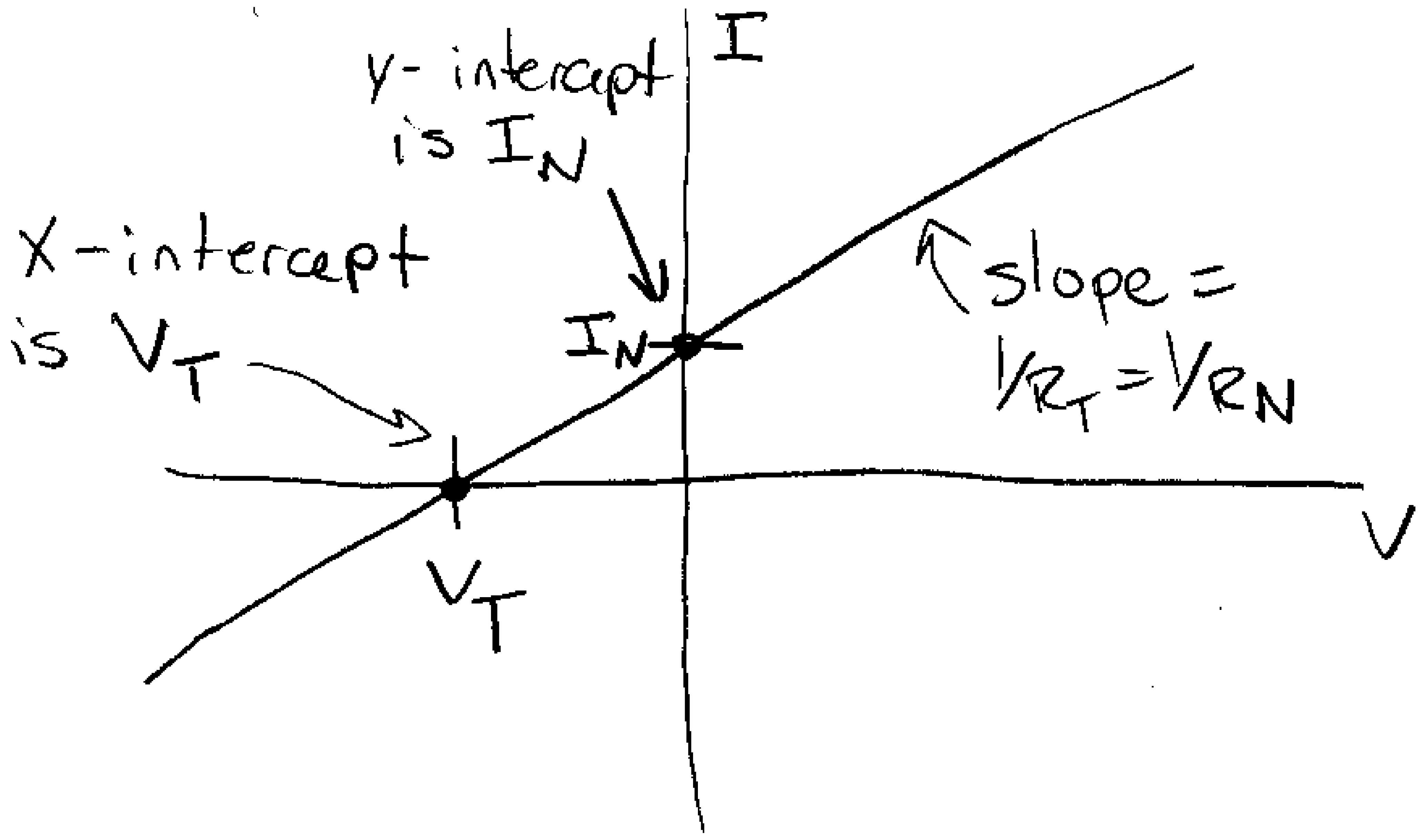


We can also replicate <u>any</u>* linear circuit through choice of I_S and R.

This is the <u>Norton</u> <u>equivalent</u> circuit.

The Thevenin equivalent voltage and resistance are called V_T and R_T, respectively.
The Norton equivalent current and resistance are called I_N and R_N, respectively.

Find them on the graph or analytically or measuring:

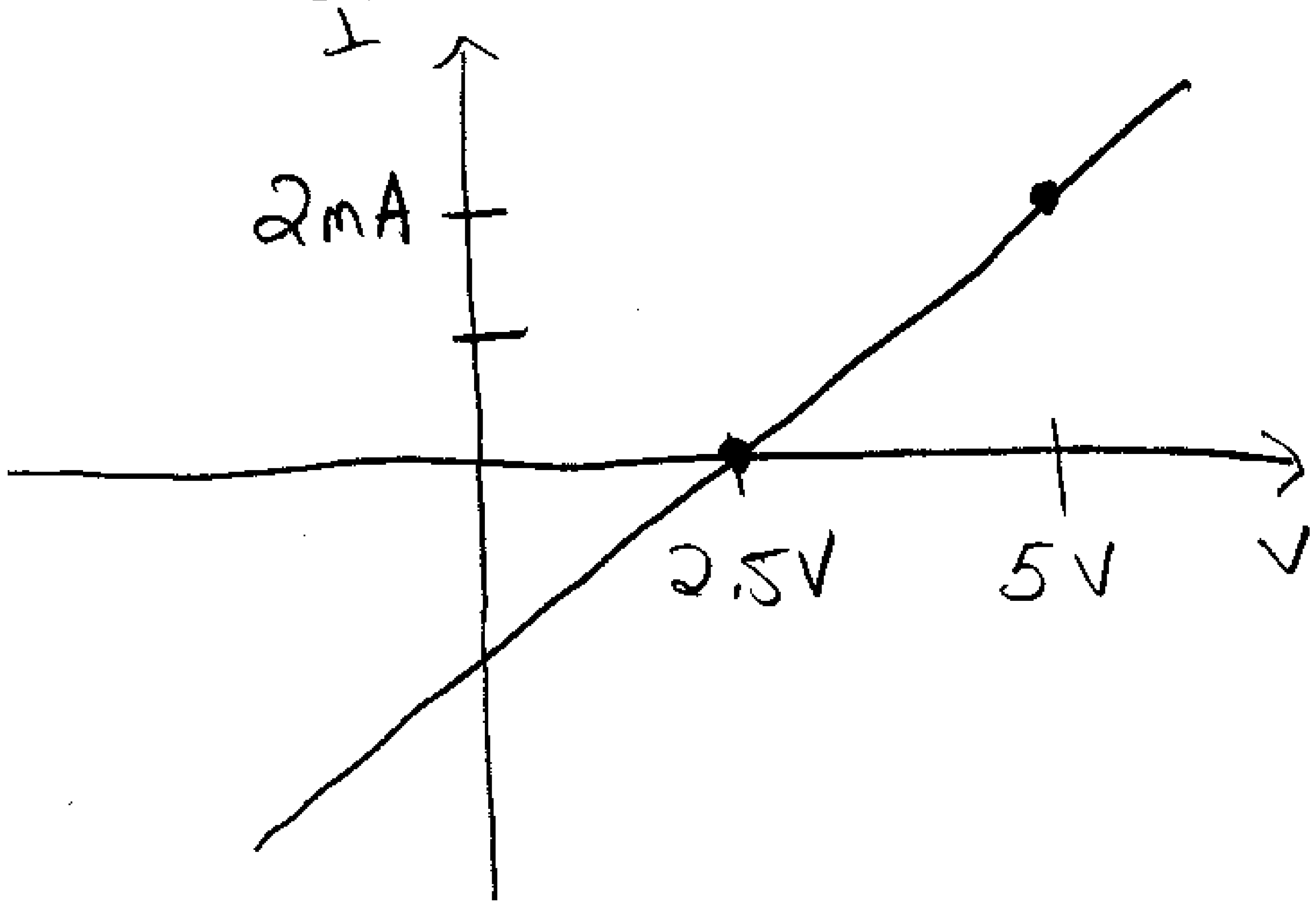


$V_T = V_{ab}$ when a-b <u>open</u>
(then $I = 0$)

$I_N = I_{b \to a}$ when a-b <u>shorted</u>
(then $V = 0$)

$$R_T = R_N = -\frac{V_T}{I_N}$$

Example: Find the Thevenin and Norton equivalents for our "mystery circuit" from before.



Equation for line:

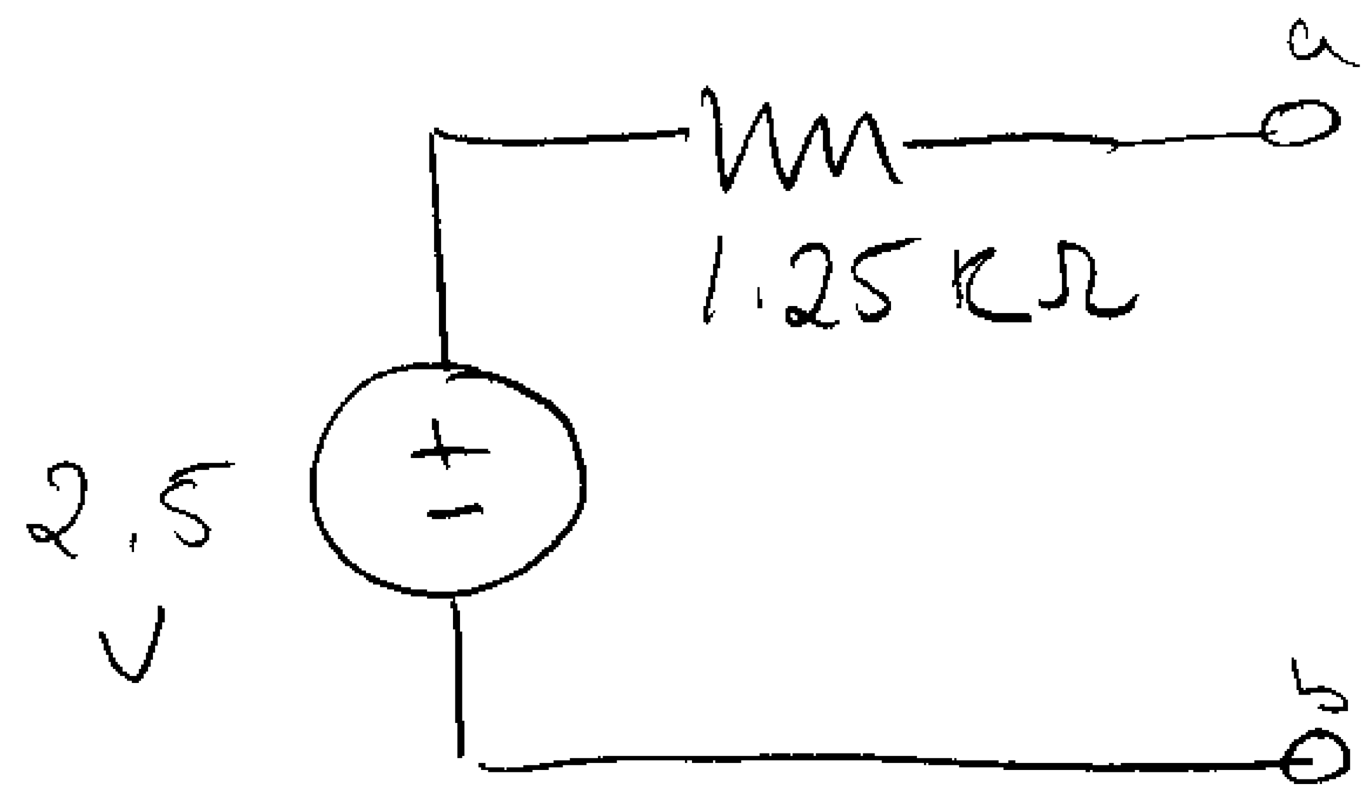
$$I = \frac{2mA}{5-2.5\,V}(V-2.5V)$$

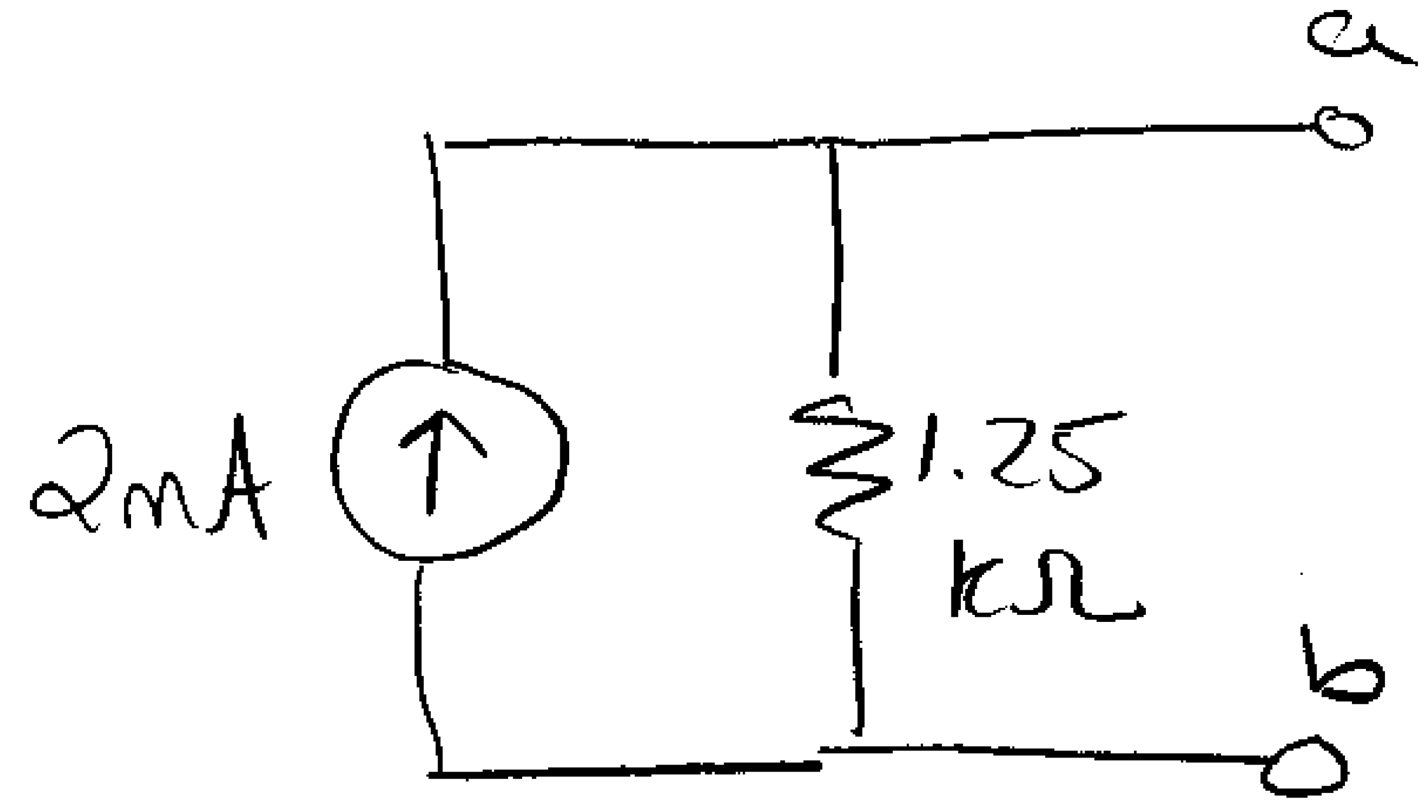
$$= 0.8mS\,V - 2mA$$

X-intercept $= V_T = 2.5\ V$

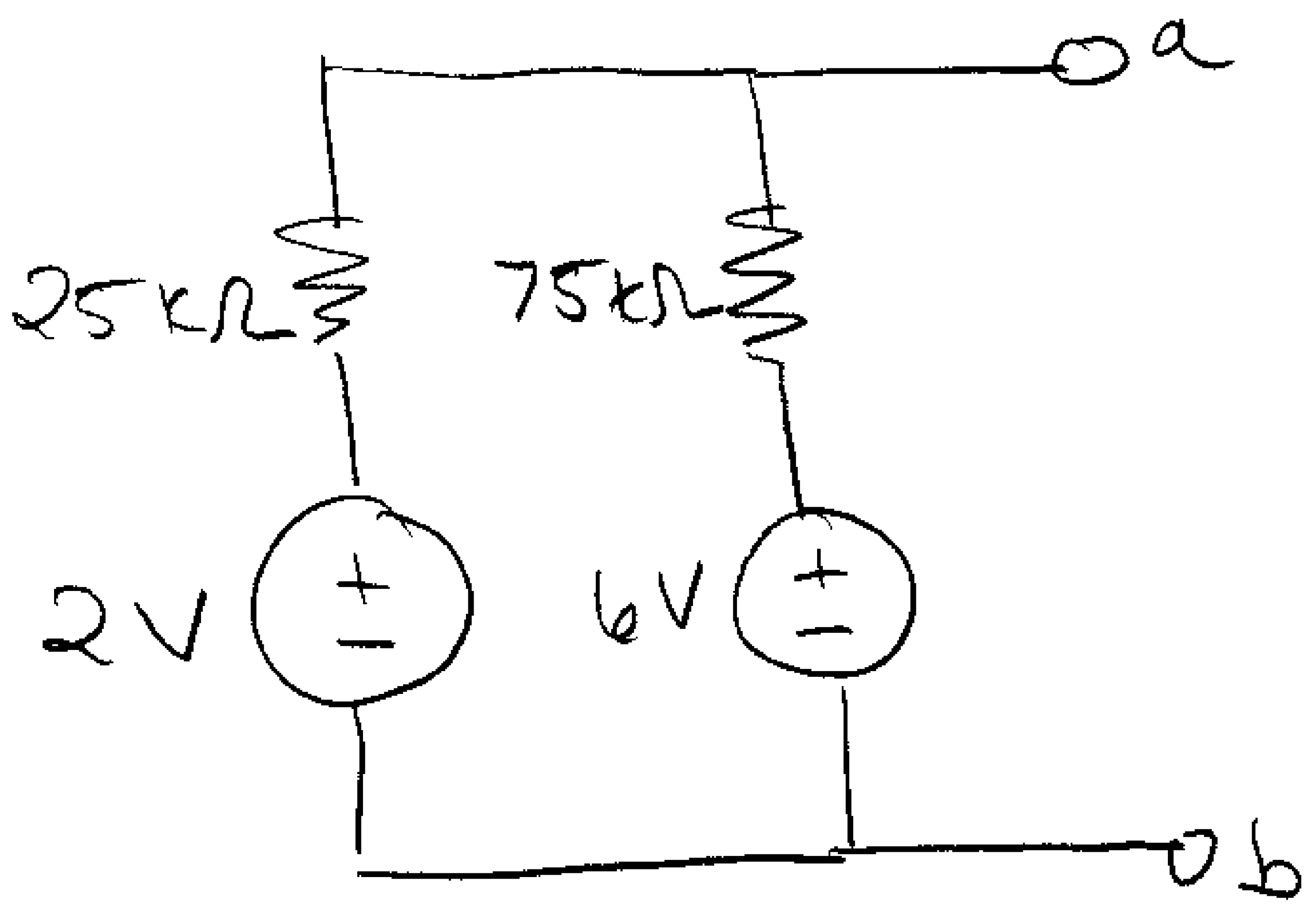
y-intercept $= I_N = -2\ mA$

slope $= 1/R_N = 1/R_T = 0.8\ mS \Rightarrow R_N = R_T = 1.25\ k\Omega$





Example: Find the Thevenin and Norton equivalents for the following:



$V_T = V_{ab}$ when a-b open

Nodal analysis: b is ground

$\frac{V_a - 2}{25k} + \frac{V_a - 6}{75k} = 0 \qquad V_a = V_{ab} = 3\ V$

$I_N = I_{b \to a}$ when a-b shorted

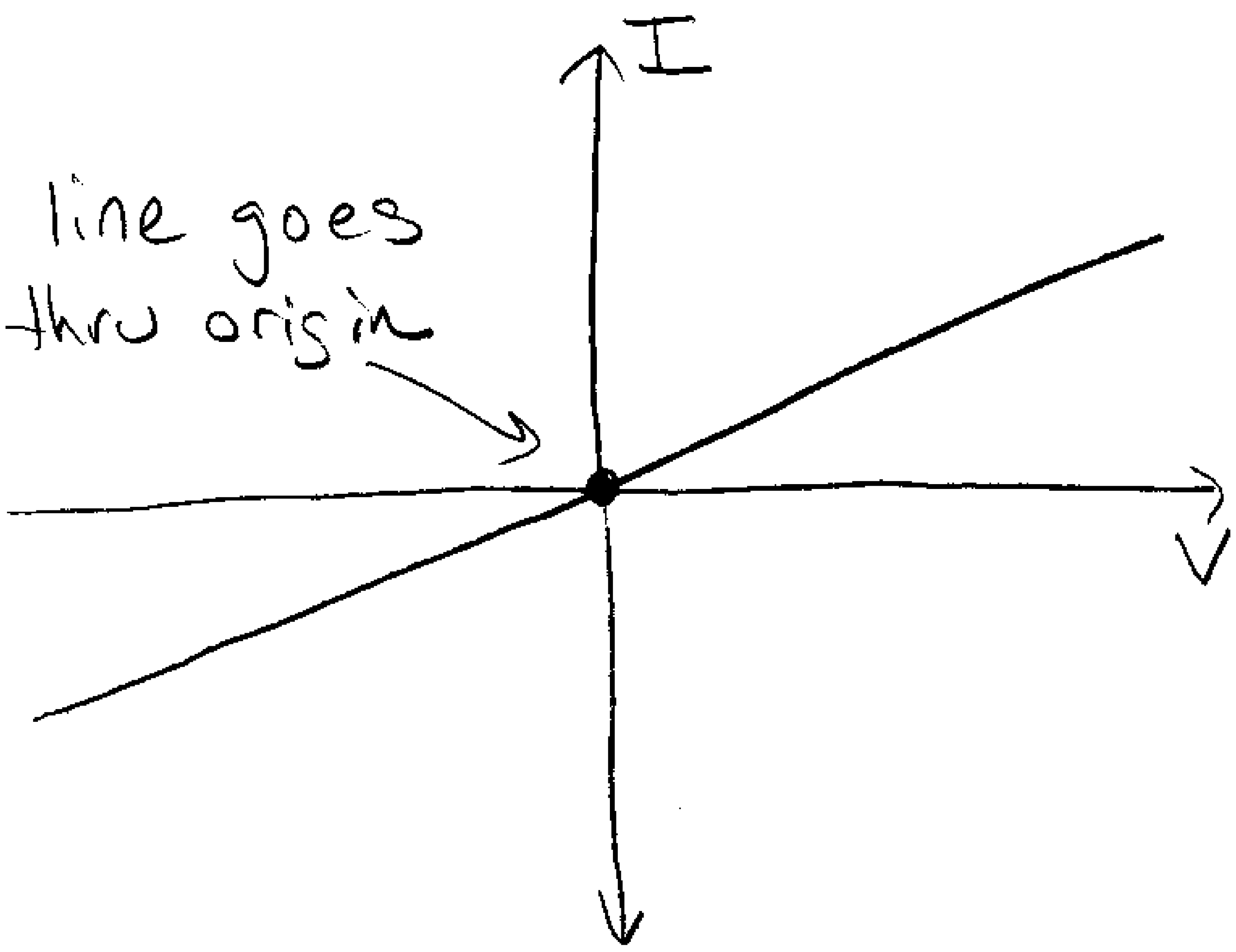
KVL: $25k I_1 + 2 = 0 \qquad 75k\, I_2 + 6 = 0$

$I_1 = -80\,\mu A \qquad I_2 = -80\,\mu A$

$I_N = I_1 + I_2 = -160\,\mu A$

$R_T = R_N = -\frac{V_T}{I_N} = -\frac{3\ V}{-160\,\mu A} = 18.75\ k\Omega$

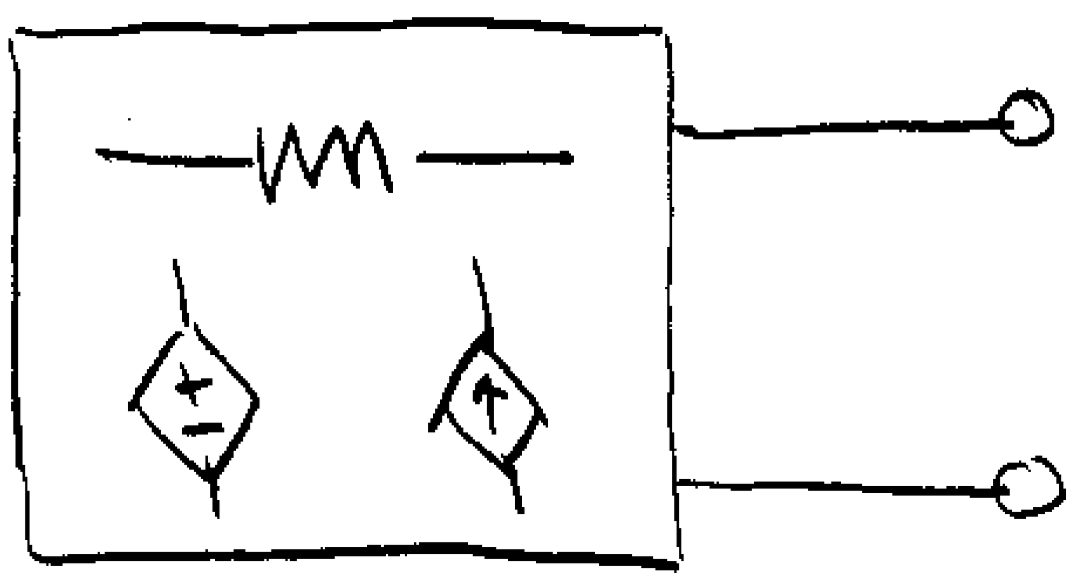
What happens when there are <u>no</u> <u>independent</u> <u>sources</u>?



$I_N = V_T = 0$

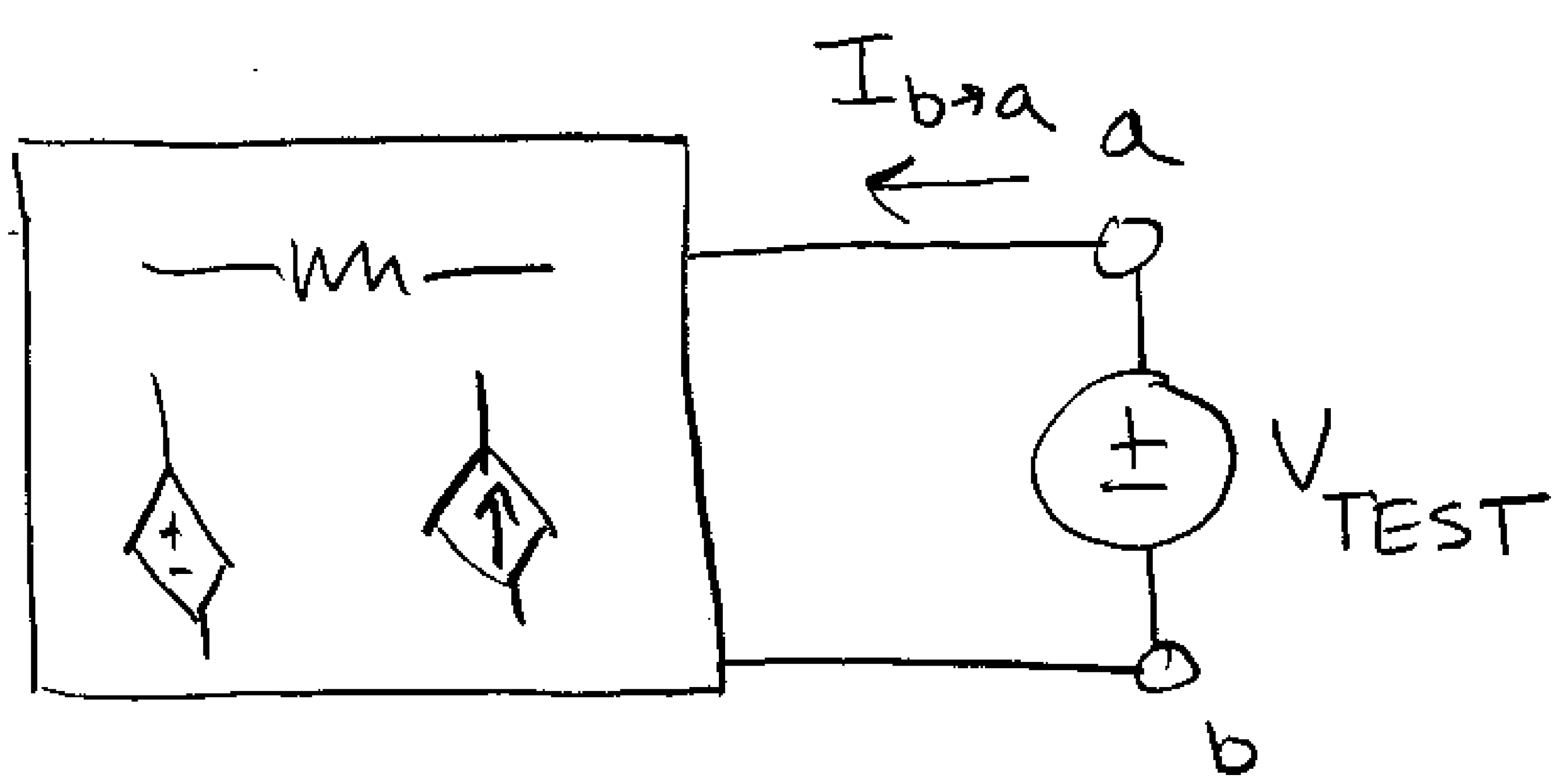
so $R_T = R_N = -\dfrac{V_T}{I_N}$

is undefined.



No current, no voltage when left alone.

Must "excite" circuit with external voltage to see how circuit works!



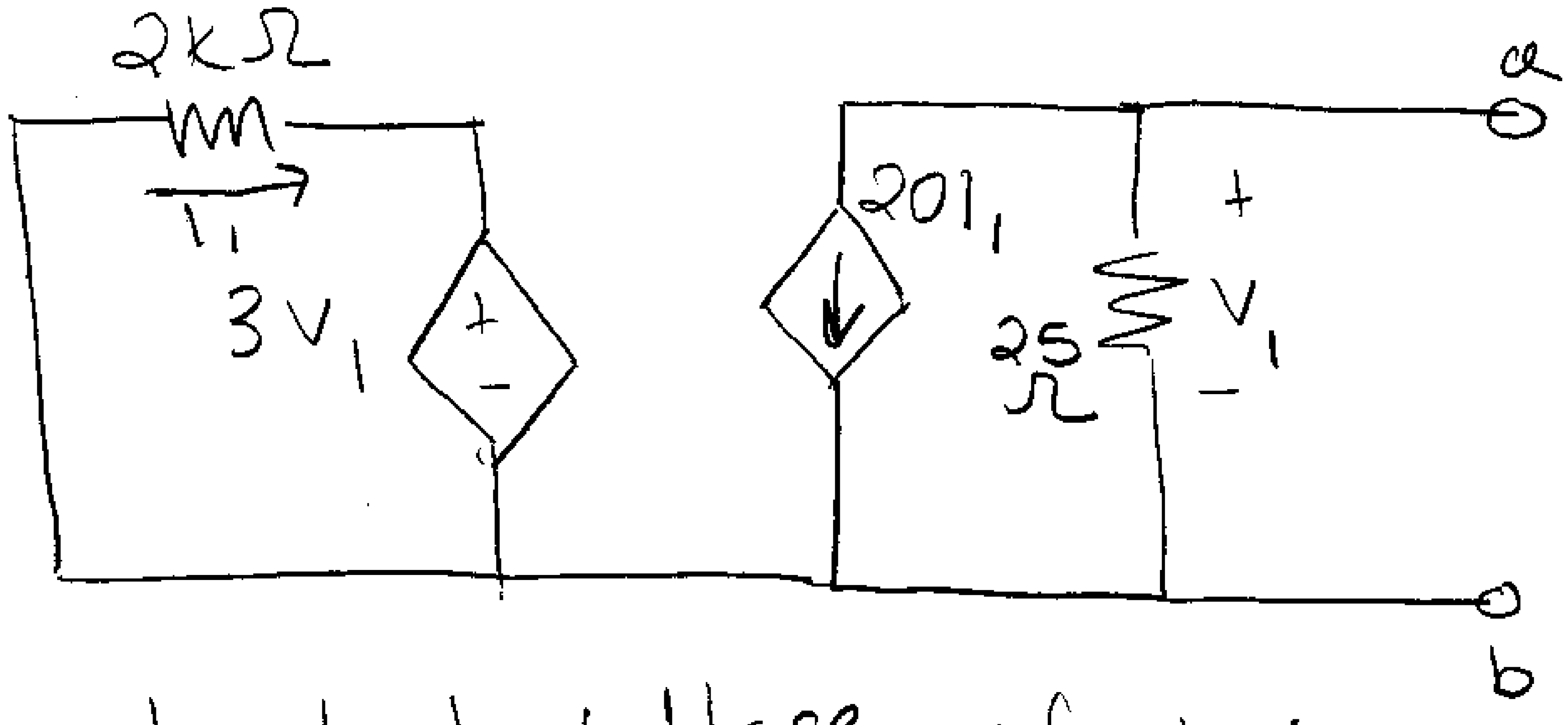
Apply a "test" voltage, measure $I_{b \to a}$.

This gives us a second point on the line.

$$R_T = R_N = \frac{V_{TEST}}{I_{b \to a}}$$

If just resistors, easier to find Req.

Example: Find the Thevenin and Norton equivs.



Apply test voltage of 1V.

We want to find I_{TEST}.

KCL @ node "a":

$$20I_1 + \frac{V_1}{25} = I_{TEST}$$

$$V_1 = V_{TEST} = 1V$$

How to find I_1? KVL at left loop:

$$3V_1 + 2k I_1 = 0 \qquad I_1 = -\frac{3}{2k}V_1 = -\frac{3}{2k}$$

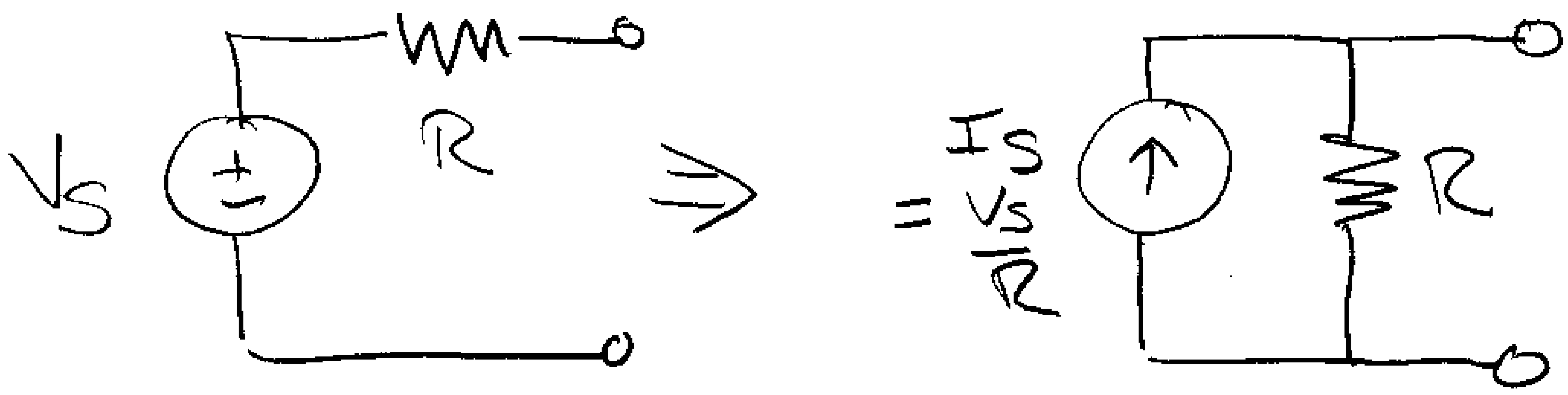
$$I_{TEST} = 20\left(\frac{-3}{2k}\right) + \frac{1}{25} = 10mA$$

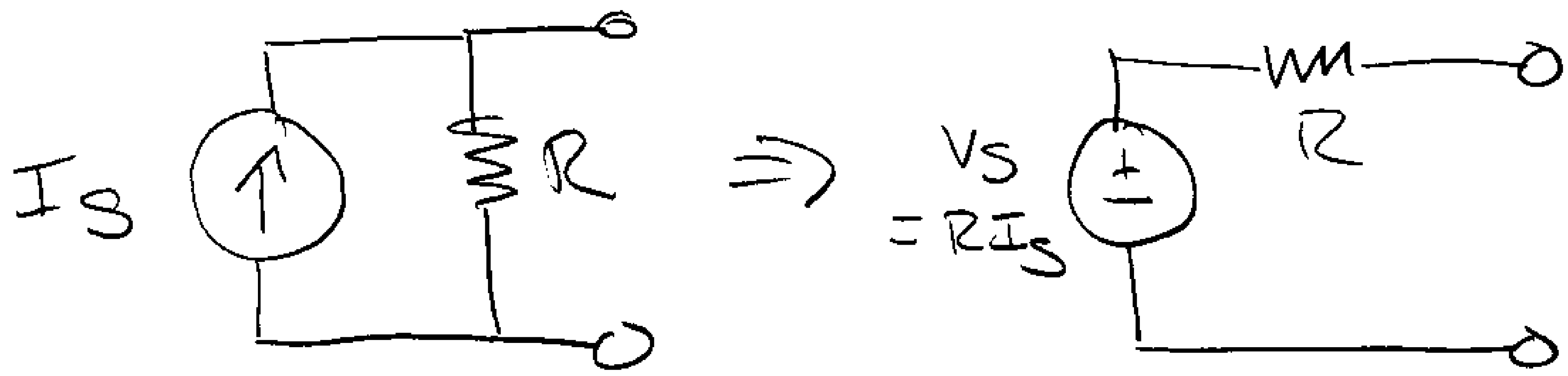
$$\frac{V_{TEST}}{I_{TEST}} = \frac{1V}{10mA} = \boxed{100\Omega = R_T = R_N \quad V_T = 0V \quad I_N = 0A}$$

Some tricks to make things easier:

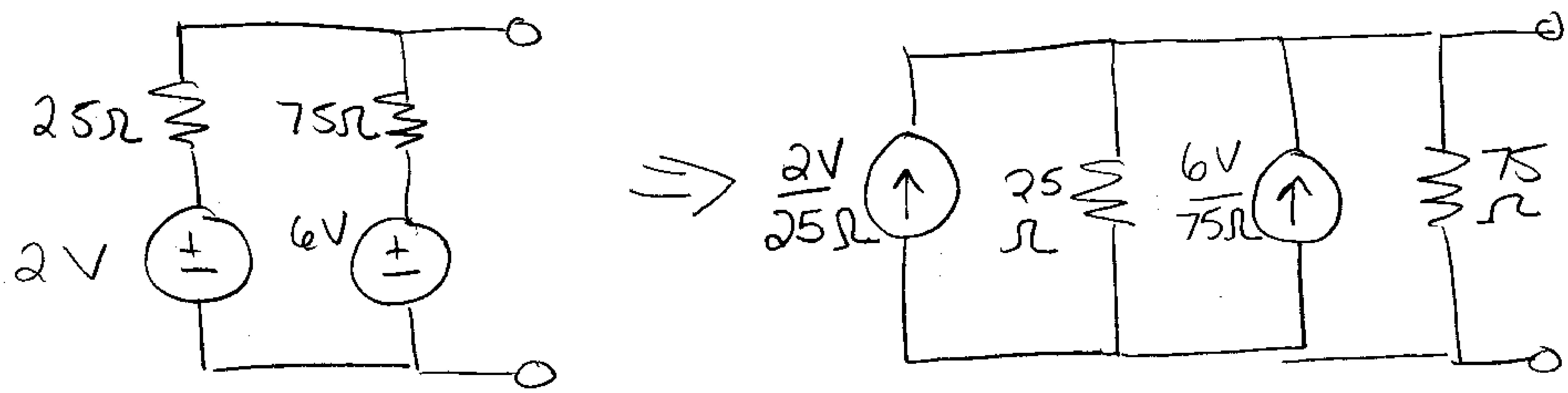
Trick #1: Exploit the "duality" of the voltage source-resistor series and the current source-resistor parallel combo to simplify circuits.

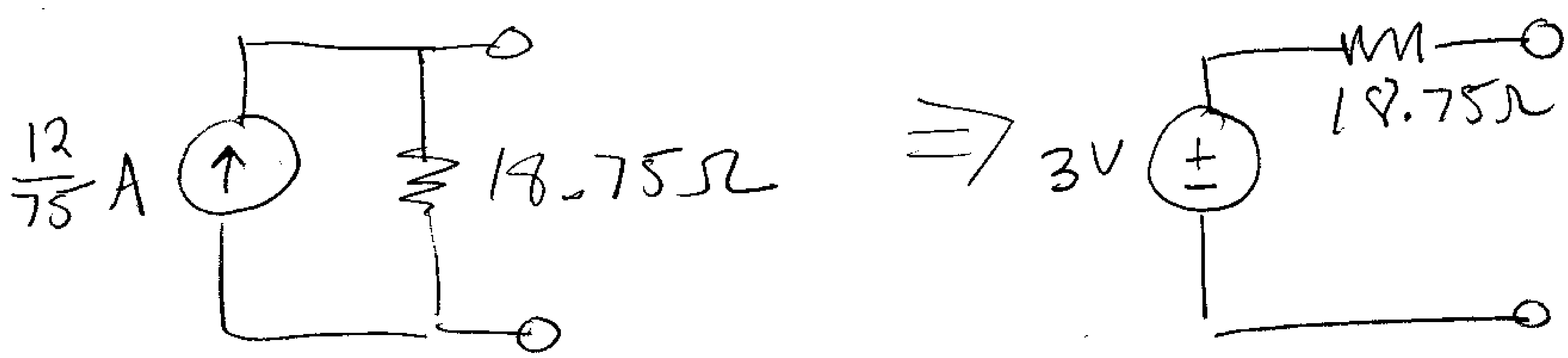
aka "Source Transformations"





Example: Revisit earlier Thevenin/Norton example.



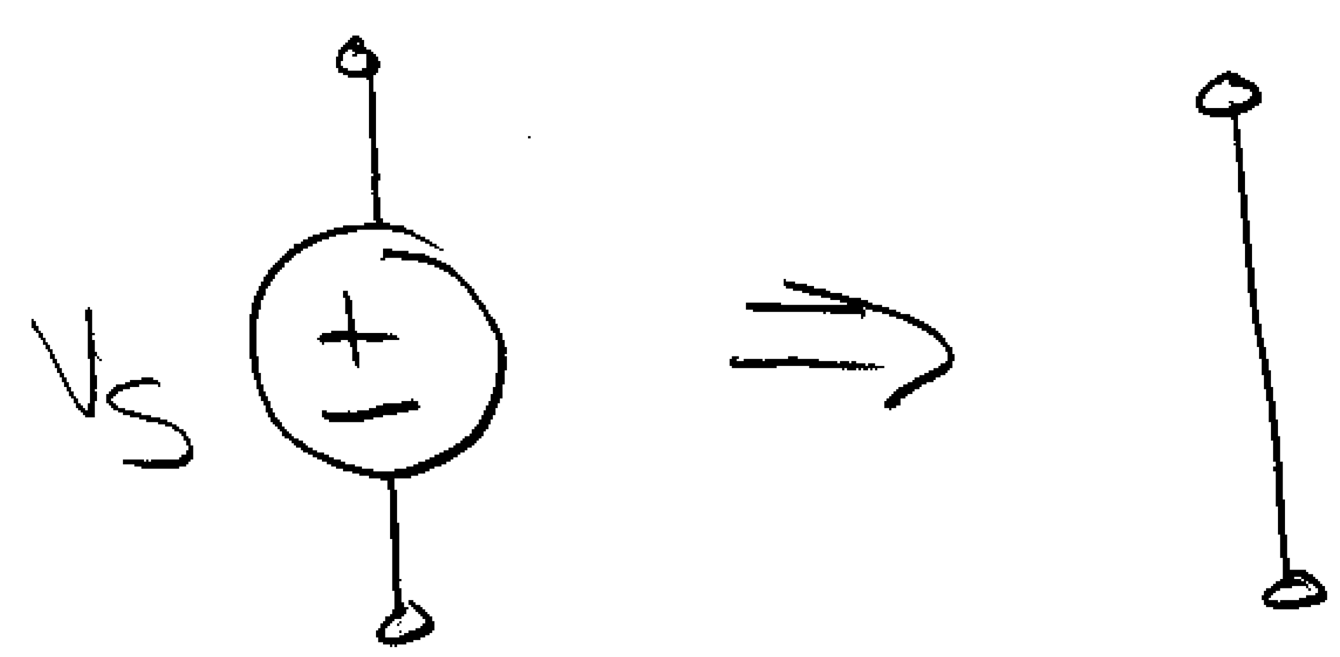


Trick #2: Instead of finding R_T or R_N via $-\frac{V_T}{I_N}$, turn off independent sources and find R_T or R_N for remainder of circuit.

This works because the slope of the I-V graph is not affected by the values of the independent sources! It depends on resistances/dep sources only.

Suggested use: When there are no dep sources.

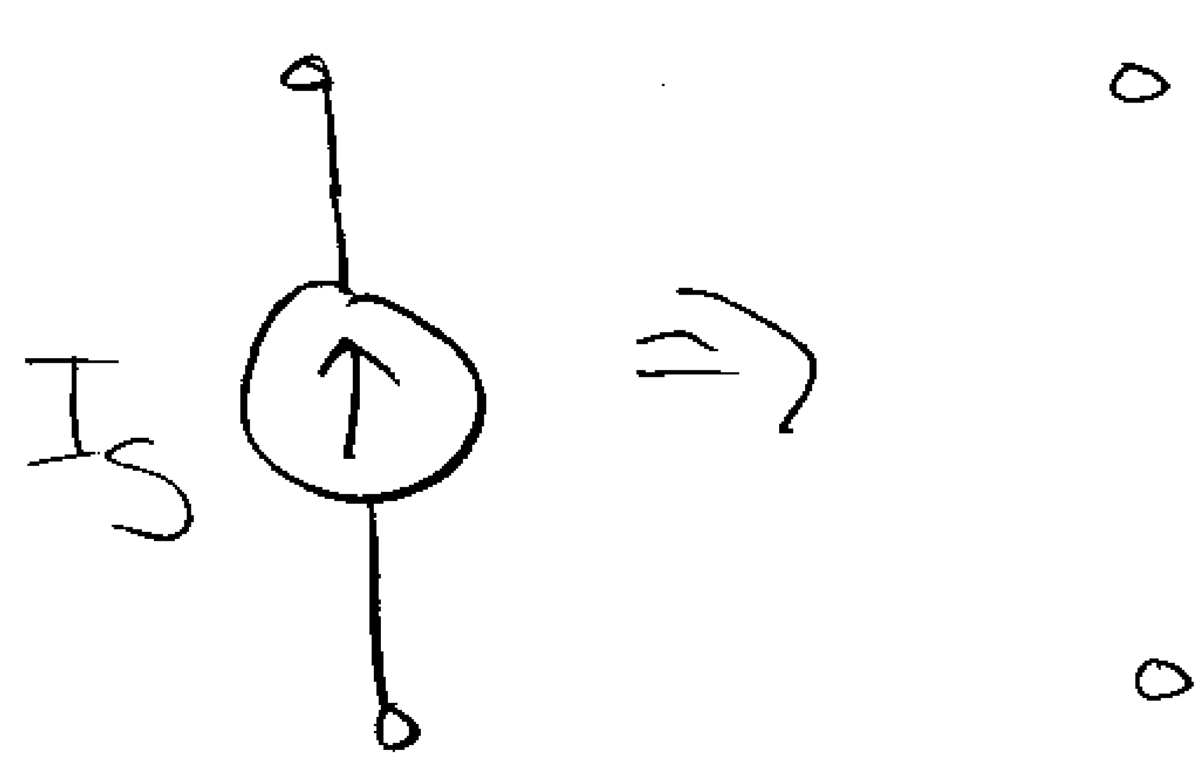
Turning off a voltage source:



Voltage sources ⇒ Short circuit

A wire has zero voltage!

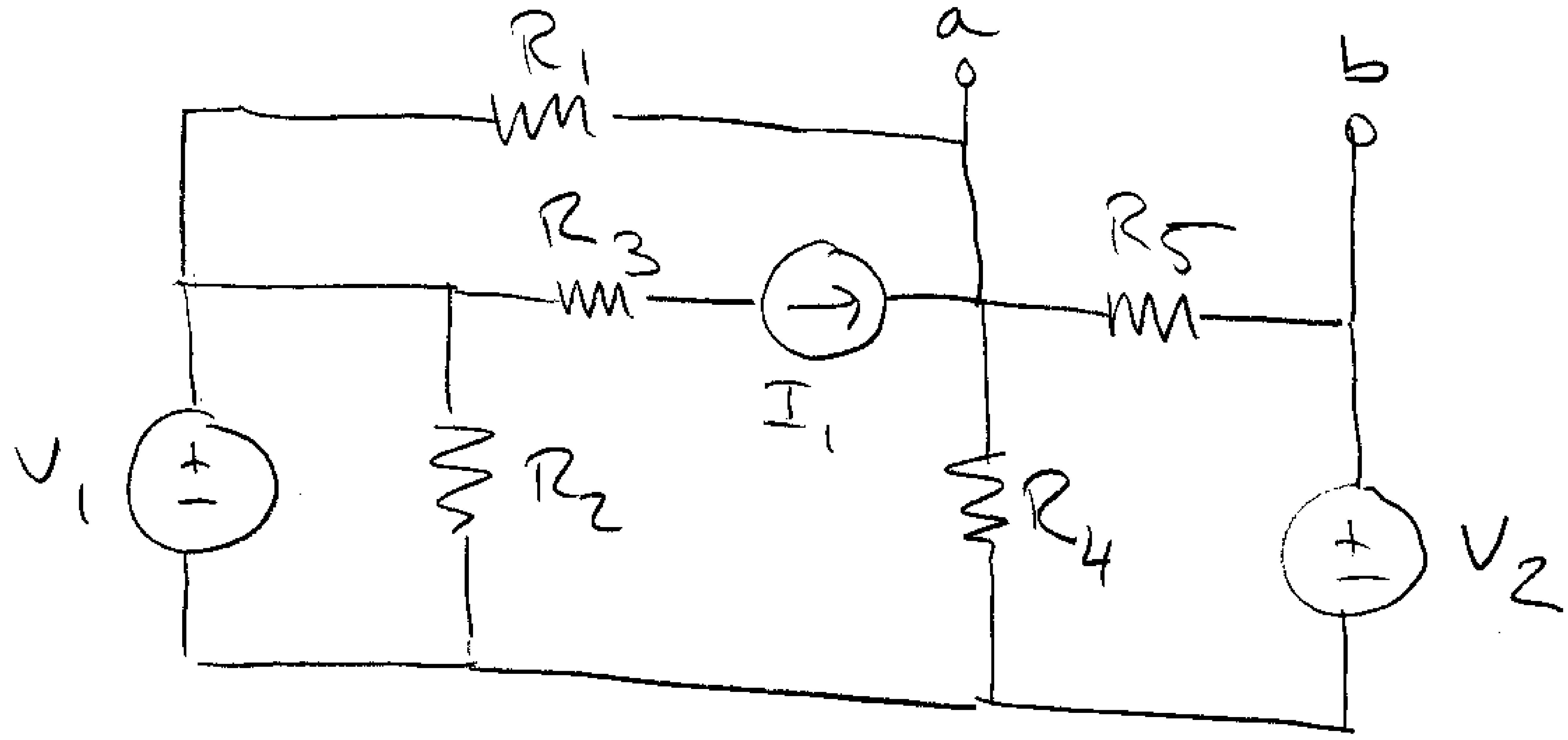
Turning off a current source:

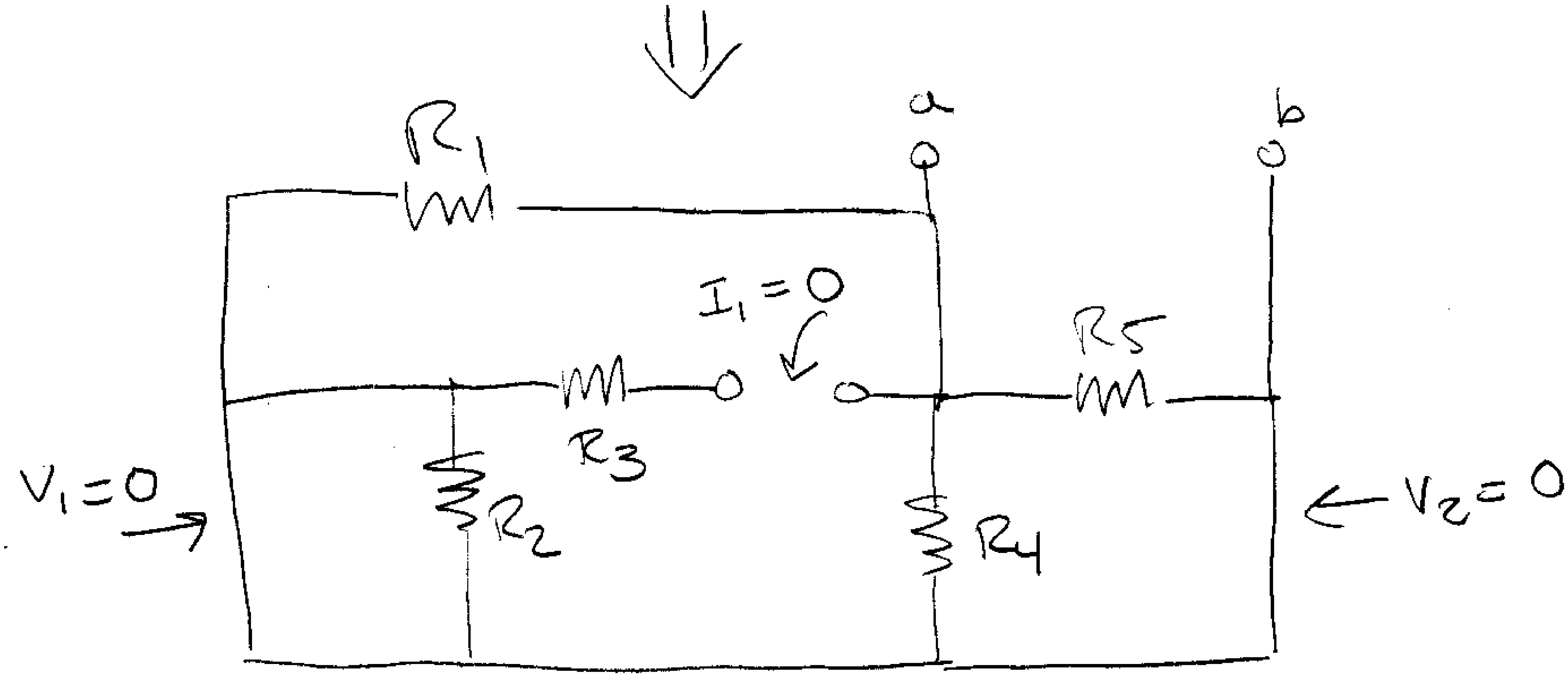


Current sources ⇒ open circuit

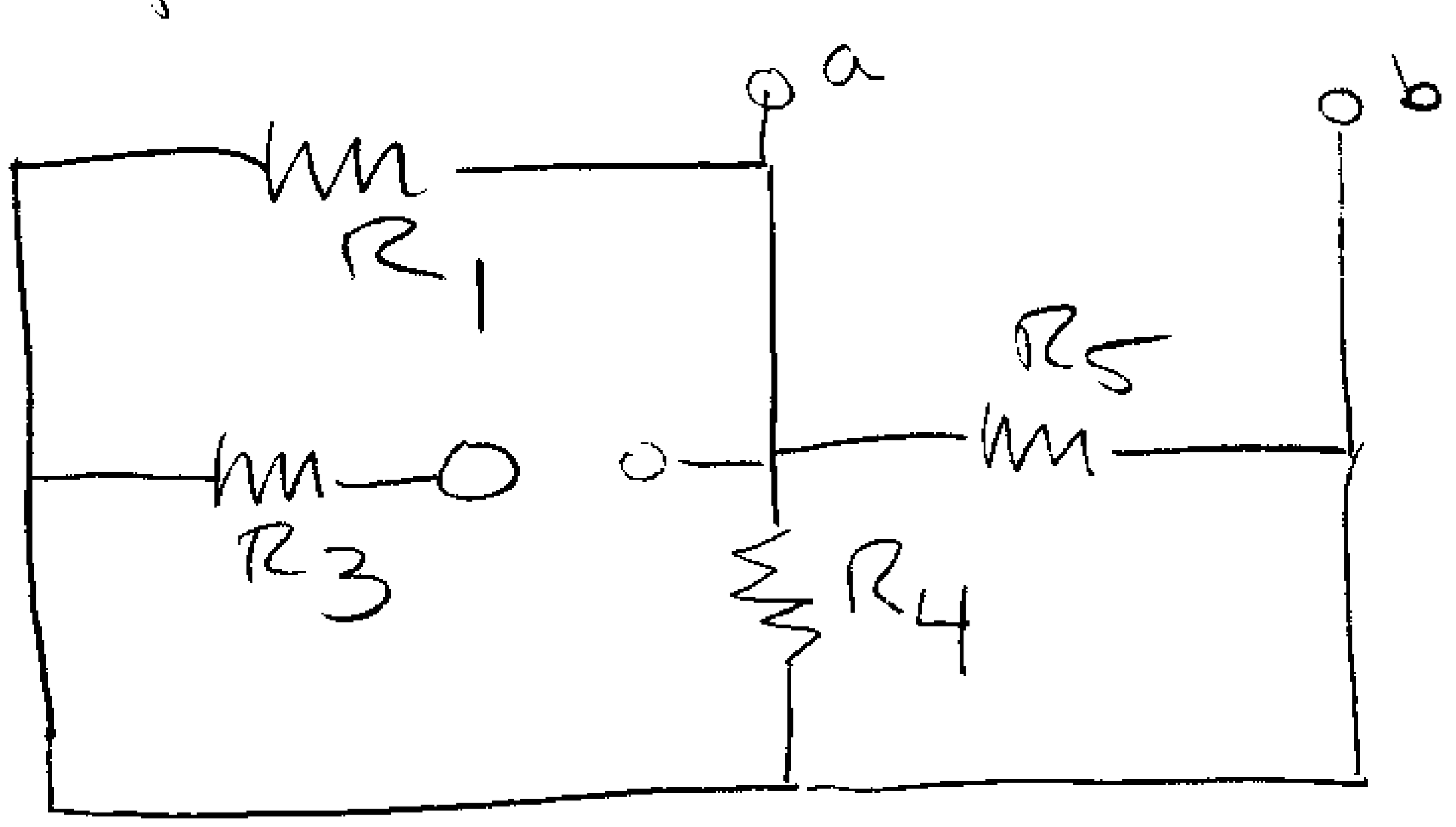
Air has zero current!

Example: Find R_T.



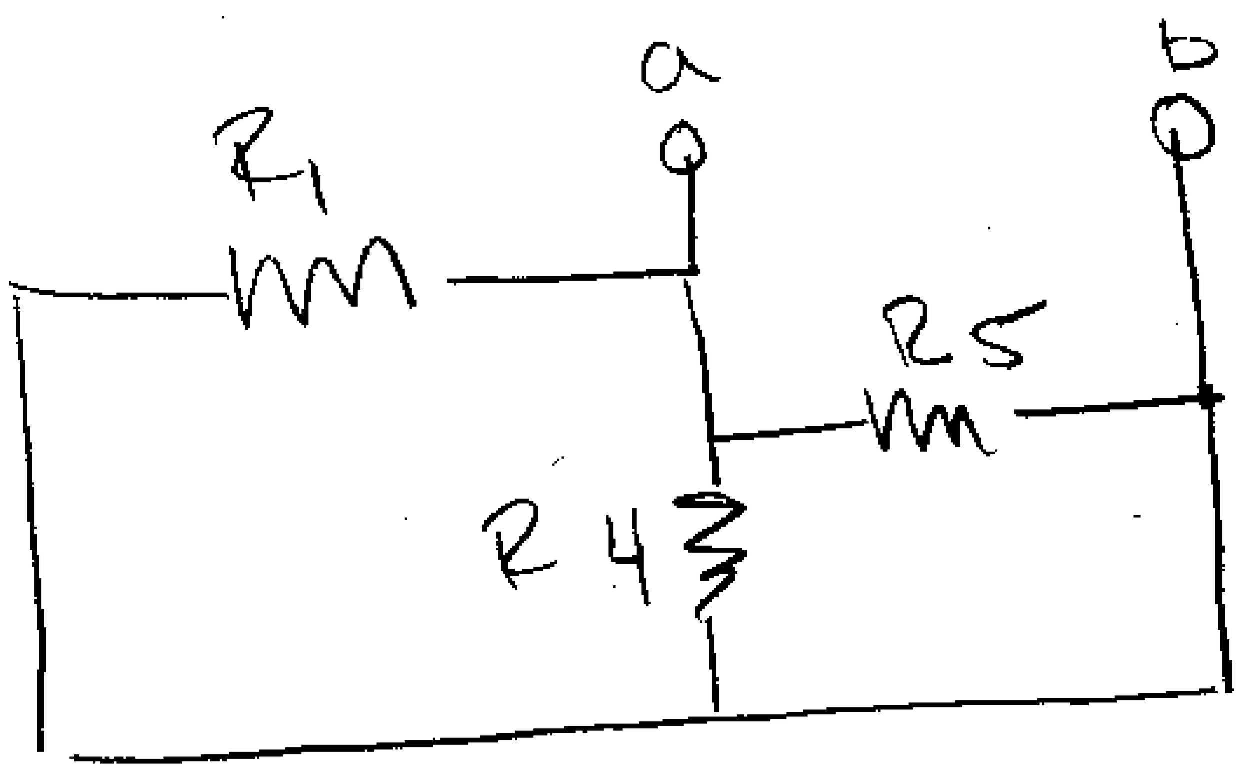


R_2 is in parallel with a wire (0Ω).
The parallel combination is 0Ω.



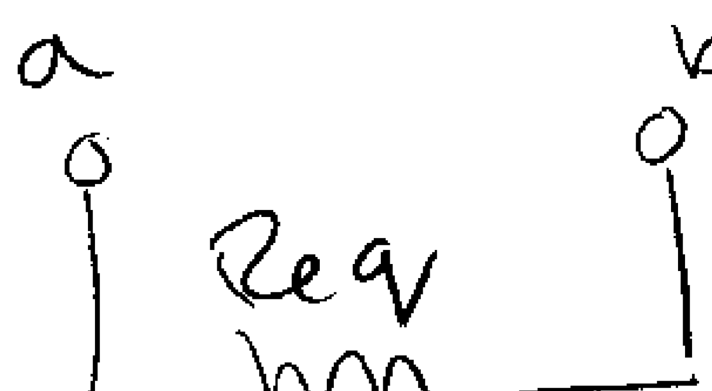
We want to find out how this network of resistors affects a device connected to a + b, i.e., the I-V characteristic.

Since R_3 is disconnected on one end, it does not carry current or contribute to the I-V relationship. We can leave it out.



Each resistor is connected at point a, and point b.

The resistors are in parallel.



$$R_{eq} = R_T = \left(R_1^{-1} + R_4^{-1} + R_5^{-1}\right)^{-1}$$