

## EE 40

## Homework #2

## Solutions

Problem 1:

$$a) E = \int_0^{5RC} P(t) dt = \int_0^{5RC} R i(t) i(t) dt$$

$$i(t) = C \frac{dV_{out}}{dt} = C \frac{d}{dt} (5(1 - e^{-t/RC}))$$

$$= \frac{5C}{RC} e^{-t/RC} = \frac{5}{R} e^{-t/RC}$$

$$E = \int_0^{5RC} R \left( \frac{5}{R} e^{-t/RC} \right)^2 dt = \int_0^{5RC} \frac{25}{R} e^{-\frac{2t}{RC}} dt$$

$$= \frac{25}{R} \left( \frac{RC}{2} \right) \left( e^{-10} - e^0 \right) \stackrel{\text{approx}}{\approx} 0$$

$$= 12.5 \cdot 5 \cdot 10^{-12} = 62.5 \text{ pJ}$$

b) Following the same procedure, we find that

$i(t) = -\frac{5}{R} e^{-t/RC}$ , the negative of the prev current.

Since we have  $i(t)^2$  as the integrand for  $E$ ,

we obtain the same solution,  $E = 62.5 \text{ pJ}$ .

c)  $5T = 5RC = 5 \cdot 2 \cdot 10^3 \cdot 5 \cdot 10^{-12} = 50\text{ns}$

$$\frac{1\text{s}}{50\text{ns/computation}} = 2 \times 10^7 \text{ or } 20 \text{ million computations}$$

d) In a) and b), we showed that  $62.5\text{ pJ}$  is absorbed by the resistor during either transition.

$$2 \times 10^7 \text{ transitions} \cdot 62.5 \cdot 10^{-12} \frac{\text{J}}{\text{transition}} = 1.25 \text{ mJ}$$

### Problem 2:

Computation Speed:

Ted:  $5T = 5RC = 5 \cdot 400 \cdot 5 \cdot 10^{-12} = 10\text{ns} \Rightarrow 100 \text{ million computations/s}$

Dilbert:  $5RC = 5 \cdot 2 \cdot 10^3 \cdot 2 \cdot 10^{-12} = 20\text{ns} \Rightarrow 50 \text{ million computations/s}$

Power dissipated by resistor:

Ted:  $P(t) = R i(t)^2 = \frac{25}{R} e^{-\frac{2t}{RC}} \text{ (see Problem 1)}$   
 $= 62.5 \times 10^{-3} e^{-\frac{t}{5\text{ns}}}$

Dilbert:  $\frac{25}{R} e^{-\frac{2t}{RC}} = 12.5 \times 10^{-3} e^{-\frac{t}{10\text{ns}}}$

Peak power is 5x higher for Ted

Average power  $= \frac{E}{5RC} = \frac{0.5}{R} = \begin{cases} 6.25 \text{ mW} & \text{Ted} \\ 1.25 \text{ mW} & \text{Dilbert} \end{cases}$   
 (over 1 transition)

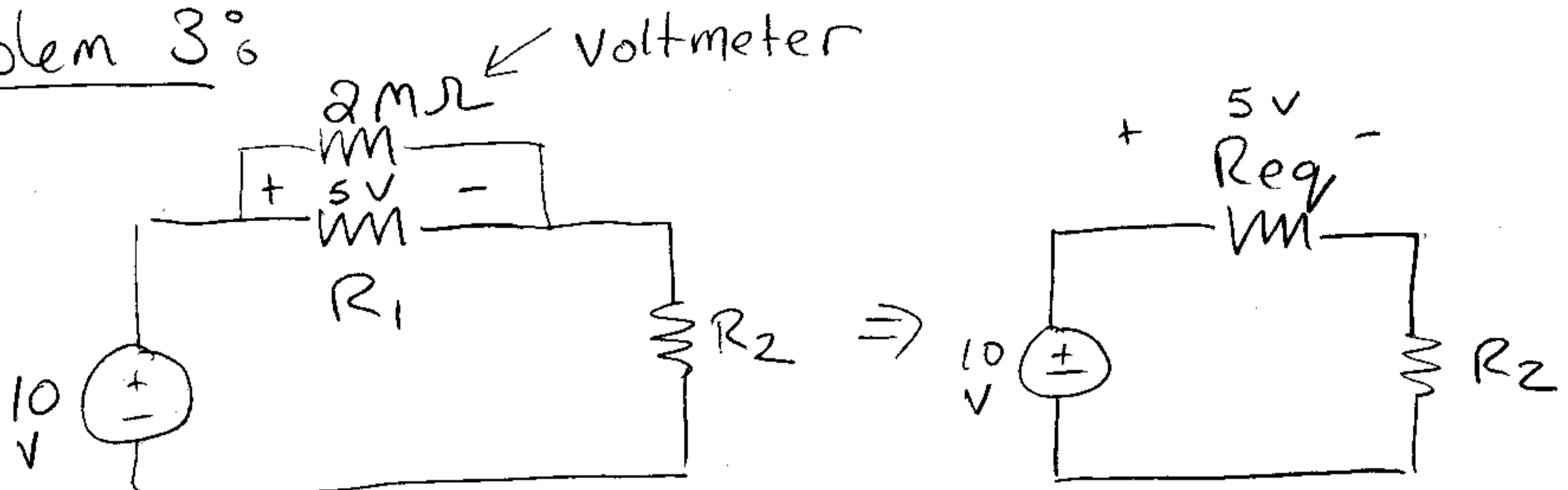
(3)

So Ted's solution is better if speed is more important than power consumption.

Dilbert's solution is better if power consumption is a big concern.

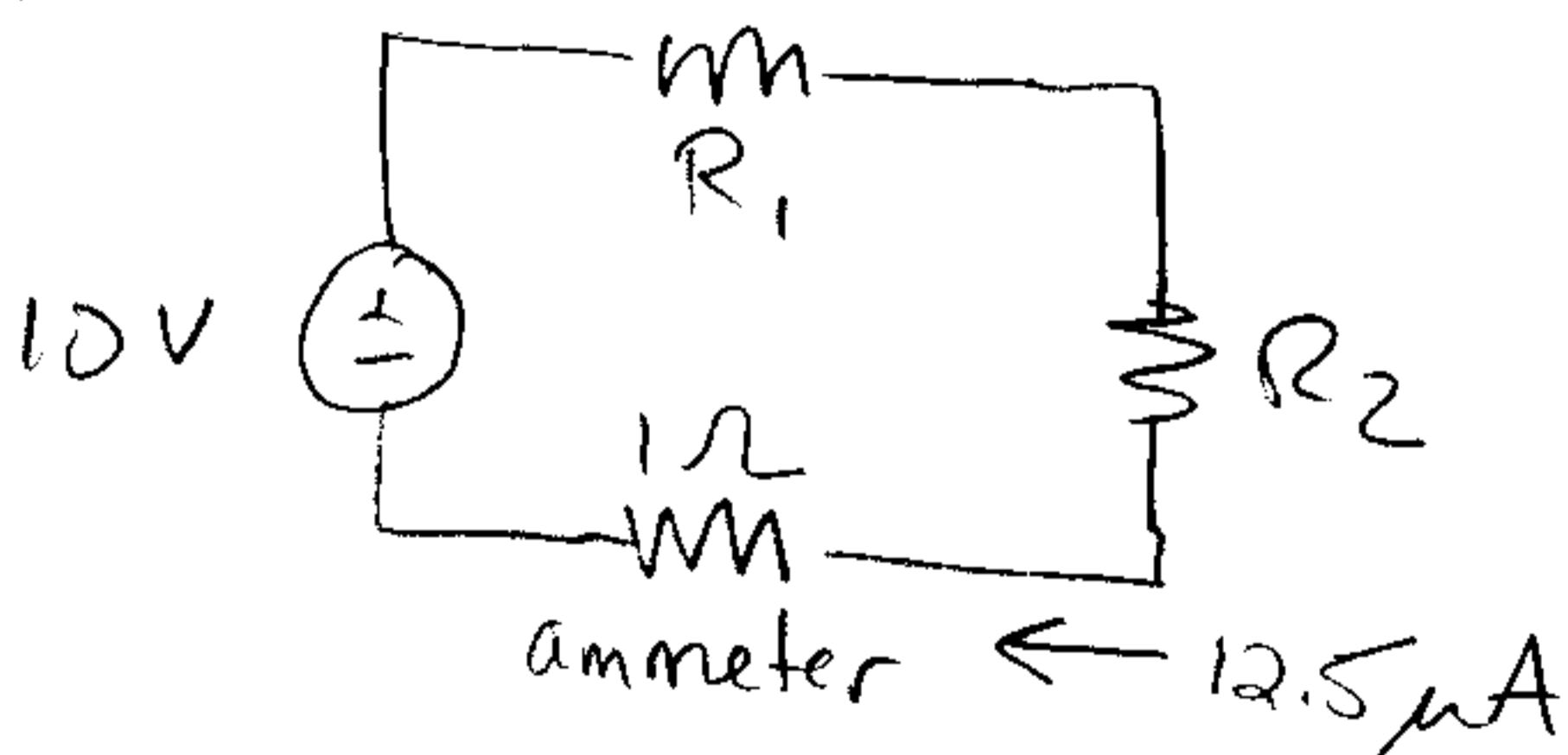
Problem 3:

a)



$$5V = 10V \frac{Reg}{Reg + R_2} = 10V \cdot \frac{\frac{R_1 \cdot 2M\Omega}{R_1 + 2M\Omega}}{\frac{R_1 \cdot 2M\Omega}{R_1 + 2M\Omega} + R_2}$$

$$= 10V \frac{R_1 \cdot 2M\Omega}{R_1 \cdot 2M\Omega + R_2(R_1 + 2M\Omega)}$$



$$\frac{10V}{12.5\mu A} = 800k\Omega = R_1 + R_2 + 1\Omega$$

$$R_1 + R_2 \approx 800k\Omega$$

$$R_2 = 800k\Omega - R_1$$

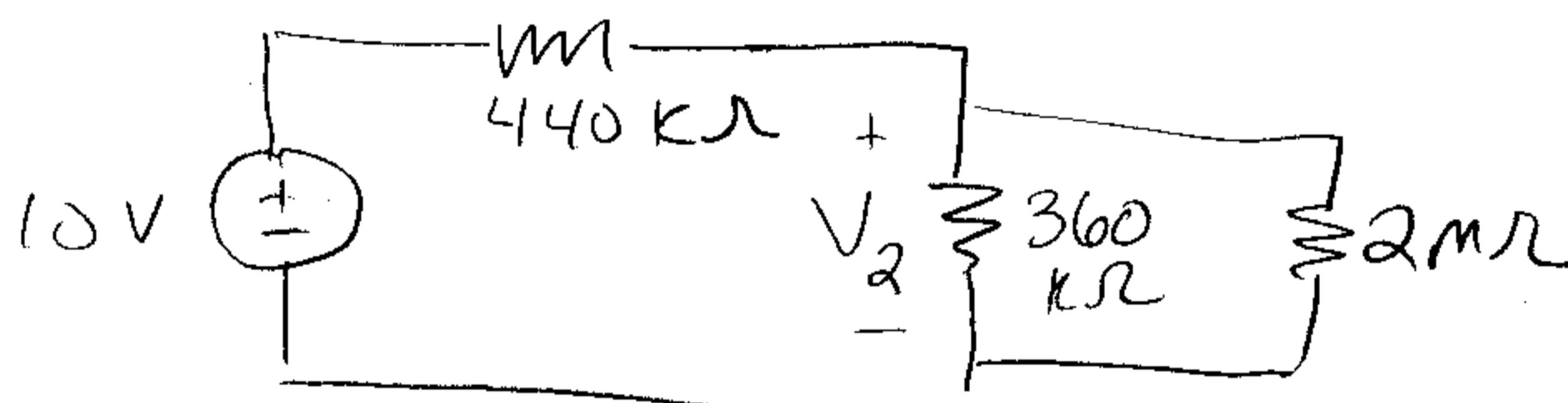
$$5V = 10V \left( \frac{2M\Omega R_1}{2M\Omega R_1 + (800k\Omega - R_1)(2M\Omega + R_1)} \right)$$

(4)

$$5V(-R_1^2 + 800k\Omega R_1 + 800k \cdot 2M\Omega) = 10V \cdot 2M\Omega \cdot R_1$$

Solve quadratic to find  $R_1 = 440k\Omega$   
 $R_2 = 360k\Omega$

b) The meter should read



$$V_2 = 10V \cdot \frac{360k\Omega || 2M\Omega}{360k\Omega || 2M\Omega + 440k\Omega}$$

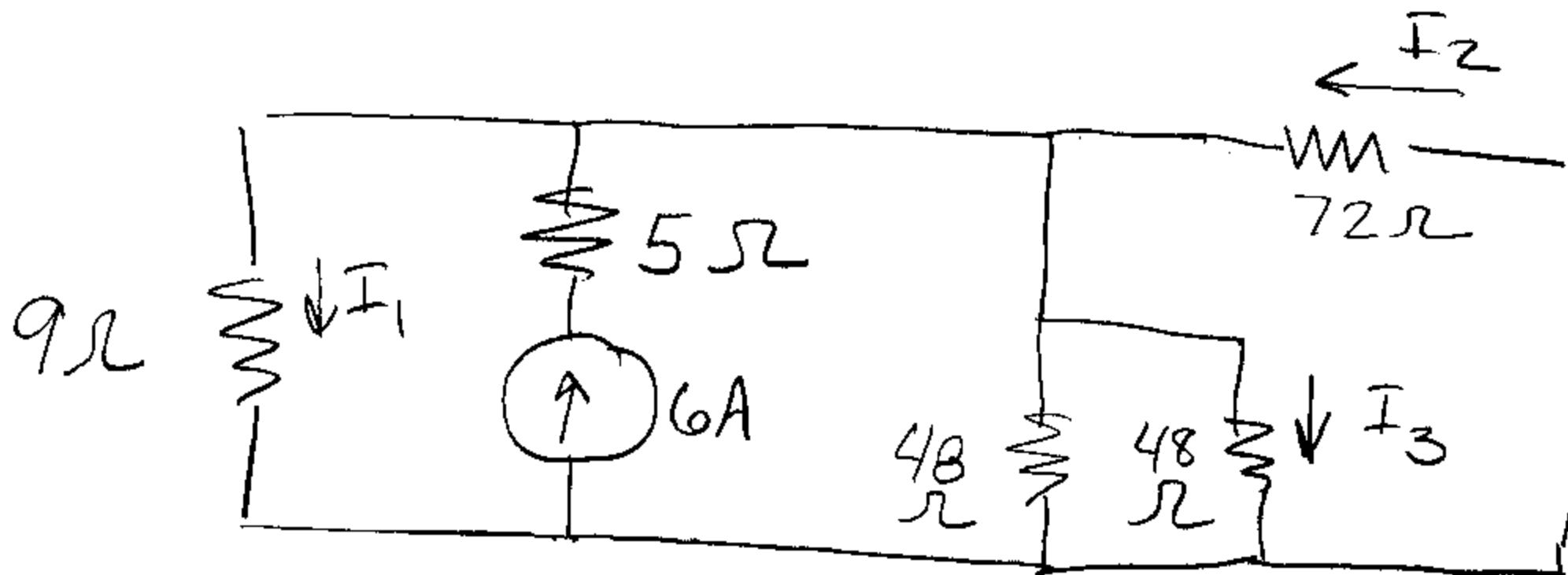
$$= 10V \cdot \frac{305k\Omega}{305k\Omega + 440k\Omega} = 4.1V$$

There is something wrong with the meter!

Moral of the story: Before you attribute a measurement error to "loading effects", etc, do a "reality check".

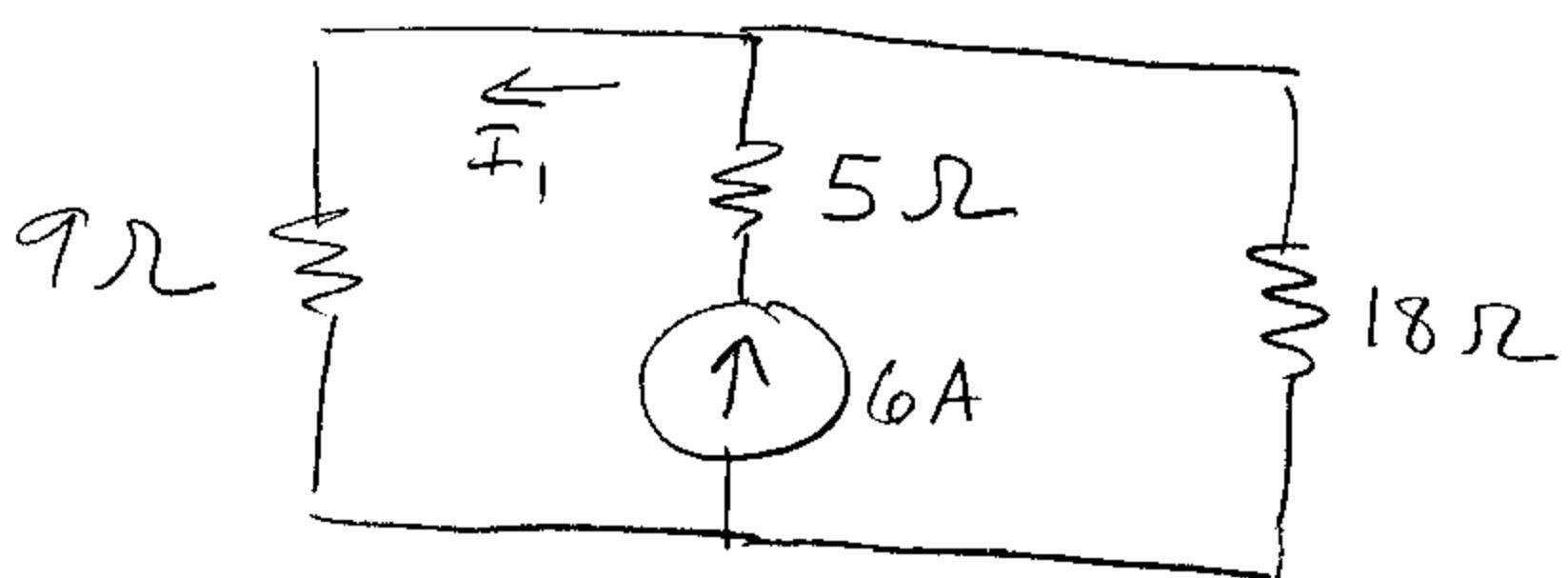
(5)

### Problem 4:



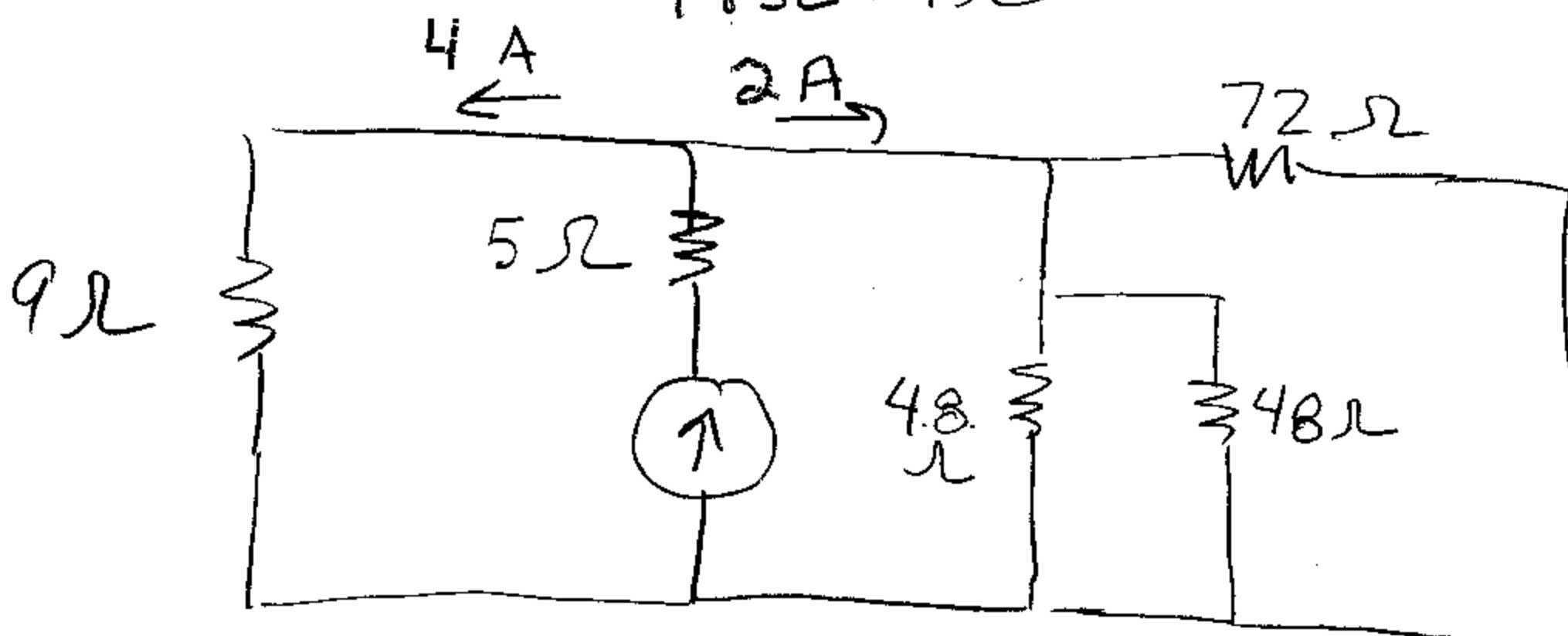
The  $9\Omega$ ,  $48\Omega$ , and  $72\Omega$  resistors are all in parallel.

$$48\Omega \parallel 48\Omega \parallel 72\Omega = 18\Omega$$



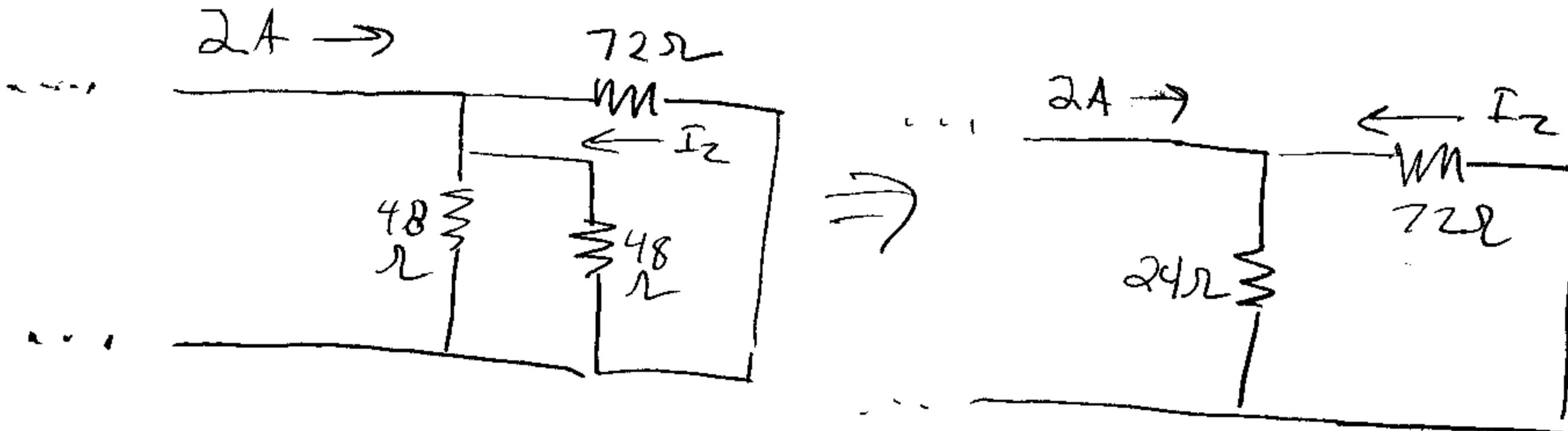
The  $6A$  is split between the  $9\Omega$  and  $18\Omega$ :  
(all  $6A$  goes right through the  $5\Omega$  resistor)

$$I_1 = 6A \cdot \frac{18\Omega}{18\Omega + 9\Omega} = 4A$$



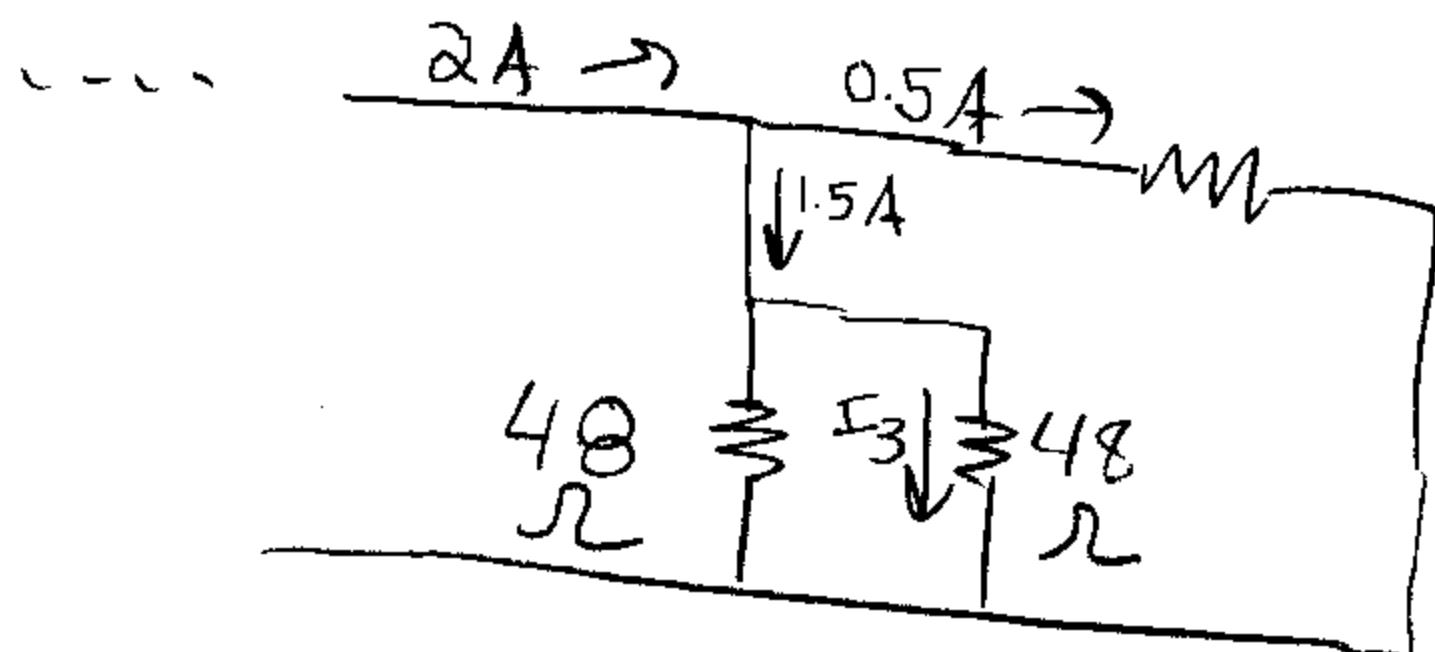
(6)

$2A$  divides between the  $72\Omega$  resistor  
and the  $48\Omega \parallel 48\Omega$  combination!



$$I_2 = -2A \cdot \frac{24\Omega}{24\Omega + 72\Omega} = -0.5A$$

Negative since portion of  $2A$  flows L to R,  
 $I_2$  flows R to L



$1.5A$  splits between  $48\Omega$  resistors.

$I_3$  must split equally:

$$I_3 = 1.5A \cdot \frac{48\Omega}{48\Omega + 48\Omega} = 0.75A$$