Total (Steady State plus Transient) Solution

Lecture 35, 11/28/05

OUTLINE

- Overview of Solutions and Initial Conditions
- Example #1: RC Phasor Solution Lec #17
- Example #2: RC (sin and V_C= 1V) Fig 4.14
- Example #3: RC (cos and V_C = 0) Fig 4.18

Reading

Lecture #17 Slides 9-11

Hambley pp. 160-166

Lecture Plan for Week #14

This week we cover extensions of the mathematical circuit analysis that

- Add transient solutions to steady state solutions to meet initial conditions (such as the initial capacitor charge and inductor current)
- Generalize the transient solution to that for 2^{nd} Order Differential Equations such as occur when both an inductor and capacitor are present and produce under, critical and over damping of signals
- Revisit resonate LC circuits

Lecture Plan for Week #14 (Cont.)

Presentation Materials Issues

- The good news is that the lecture material is straight out of Hambley
- The bad news is that the material has not been re-typed into lecture notes that can be distributed

You may wish to lug your 5lb text to class or at least look at it before the lecture.

Total Solution Overview

- When we switch a source such as 60Hz line voltage into a circuit
  - We do not expect to be able to catch the voltage at just the correct phase
  - And in fact we often see a transient surge
- What we see is a transient transition into the steady-state sinusoidal response represented by
  - The steady-state response
  - And an added Transient response that assists in meeting the initial conditions

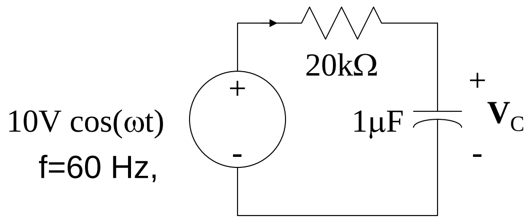
Total Solution Overview (Cont. #1)

- The time relationships between voltages and current are integrated and differentiated by capacitors and inductors.
- This results in Kirchhoff's laws producing integral-differential equations for the circuit response.
- By taking derivatives with respect to time these equations can be converted to differential equations with
  - Only the unknown node voltages or loop currents and their derivatives on the left side
  - And various derivatives of the independent voltage or current source on the right side as forcing functions

Total Solution Overview (Cont. #2)

- Since the circuit elements have linear time-invariant values the contributions of each of the forcing functions to the forced (particular) solution can be found individually and added (Superposition).
- It is likely that a natural (force free or complementary) solution to the differential equation will also need to be added to to the combined forced solution to correctly satisfy the initial conditions on the capacitor voltages and inductor currents,

Example: Lecture #17 with Initial Condition



Added initial condition

$V_C(0) = 0$

How do the circuit laws change?

Sinusoids

$$-10V\cos(wt) + i(t)R + V_c(t) = 0$$

$$RC\frac{dV_c(t)}{dt} + V_c(t) = -10V\cos(wt)$$

Forcing Function

Differential equation for Voltage

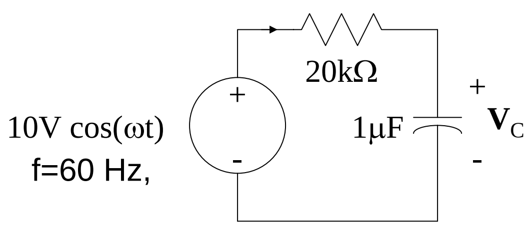
Phasors

$$\mathbf{V}_s = 10\angle 0$$

$$-\mathbf{V}_s + j\omega RC\mathbf{V}_c + \mathbf{V}_c = 0$$

$$\mathbf{V}_c = \frac{\mathbf{V}_s}{(1 + jwRC)}$$

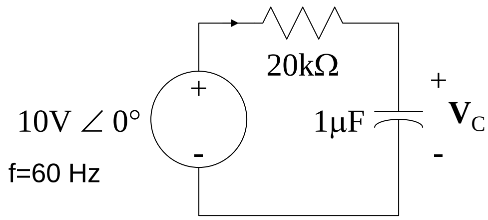
Circuit Analysis with Phasors



How and how hard is it to find $\mathbf{V}_C$?

The New Phasor Way:
Assume $V_C = Ve^{j\theta}$
Compute Impedances of R and C
Use impedances in a Voltage Divider
Evaluate the complex ratio to get $Ve^{j\theta}$

Phasor Anaylsis Result for Forced Solution



$\mathbf{Z}_R$ = R= 20kΩ = 20kΩ ∠ 0°

$\mathbf{Z}_C$ = 1/*j* (2πf x 1μF) = 2.65kΩ ∠ -90°

Divide in polar

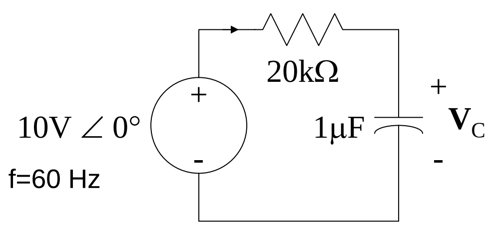
$$\mathbf{V}_C = 10\text{V} \angle 0° \left(\frac{2.65\text{k}\Omega \angle -90°}{2.65\text{k}\Omega \angle -90° + 20\text{k}\Omega \angle 0°} \right)$$

$$\mathbf{V}_C = 1.31\text{V} \angle -82.4°$$

Add in rectangular

$V_{C\_FORCED}(t) = 1.31\text{V} \cos(\omega t - 82.8^{o})$

Forced Solution has $V_C(0)$ non Zero



Added initial condition

$V_C(0) = 0$

$V_{C\_FORCED}(t) = 1.31\text{V} \cos(\omega t - 82.8^{o})$

But at t =0

$V_{C\_FORCED}(0) = 1.31\text{V} \cos(-82.8^{o}) = 0.164\text{V}$

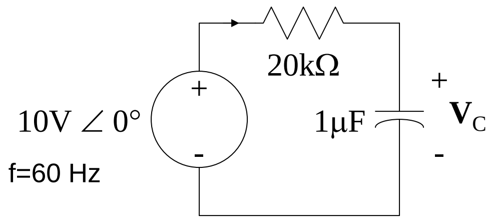
Add Complementary Solution

$V_{C\_COMP}(t) = A + Be^{-t/\tau}$

τ is given by Thevenin Resistance seen by the capacitor = $R_{TH}C$

A = 0 because when source is turned to zero capacitor discharges

Total Solution has $V_C(0)$ Zero



Added initial condition

$V_C(0) = 0$

$$V_{C\_TOTAL}(t) = 1.31V\cos(\omega t - 82.8^o) + Be^{-t/\tau}$$

Now at t =0

$$V_{C\_TOTAL}(0) = 0.164V + B = 0 \;\; => B = -0.164V$$

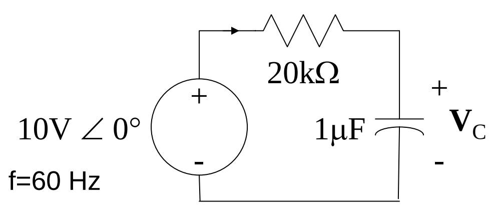
Thus

$$V_{C\_TOTAL}(t) = 1.31V\cos(\omega t - 82.8^o) - 0.164Ve^{-t/\tau}$$

Forced Response   Natural Resoponse

τ is given by Thevenin Resistance seen by the capacitor = $R_{TH}C$

Comments on the Total Solution



Added initial condition

$V_C(0) = 0$

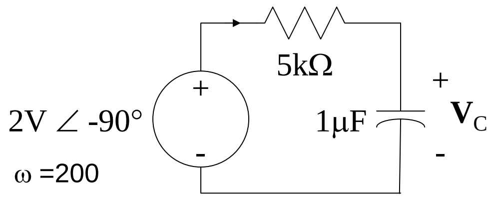
$$V_{C\_TOTAL}(t) = 1.31V\cos(\omega t - 82.8^o) - 0.164Ve^{-t/RC}$$

0.02s

Forced Response   Natural Resoponse

- Other circuit variables such as the resistor voltage or current can be found from $V_C(t)$
- It is also possible to formulate the solution directly for other circuit variables such as the resistor voltage or current but the initial conditions for these variables must be carefully derived from the **more fundamental initial conditions on C and L**.

Hambley Example #1 Fig 4.14



Added initial condition

$V_C(0) = 1$

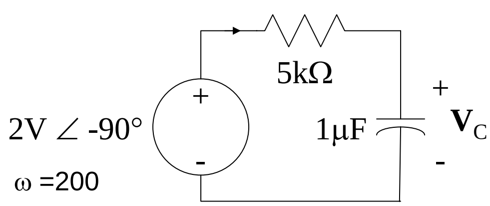
$\mathbf{Z}_R$ = R= 5kΩ = 5kΩ ∠ 0°
$\mathbf{Z}_C$ = 1/*j* (200 x 1μF) = 5kΩ ∠ -90°

$$I_C = \left(\frac{2\text{V} \angle -90°}{5\text{k}\Omega \angle \text{-}90° + 5\text{k}\Omega \angle 0°} \right) = 200\sqrt{2} \angle -45° \mu A$$

$I_{C\_TOTAL}(t) = 200\cos(200t) + 200\sin(200t) + Ke^{-t/RC}$

Forced Response Natural Response

Hambley Example #1 Fig 4.14



Added initial condition

$V_C(0) = 1$

At t = 0, V_C = 1V pushes current backward through R.

(0-1V)/5KΩ= -200 = 200cos(0) + 200sin(0) + K

K = -400 μA

0.005s

$I_{C\_TOTAL}(t) = 200\cos(200t) + 200\sin(200t) - 400e^{-t/RC}$

Forced Response Natural Response

Hambley Graphs and Exercise

- Figure 4.16 shows the individual components in the total solution as a distinct sinusoid (forced) soultion and a distinct transient (natural) solution rising from -400 μA with time constant 5 ms.
- Figure 4.17 shows that the sum forms a somewhat sinusoidal response that rises from negative 200 μA and converges to the steady state sinusoid.
- Exercise 4.7 Fig. 4.18 considers the same circuit with a cos forcing function and an initial condition of zero volts on the capacitor that is similar to our previous example from Lec. #17.

EE40 Value Added to Hambley

- For sinusoidal excitations **Phasors are more efficient** than the differentiation of sinusoids.
- Since the most fundamental initial conditions are capacitor voltages and inductor currents it is generally advantageous to **formulate in terms of capacitor voltages and inductor currents**
- The time constant for the natural can be found from the **Thevenin resistance seen looking out from the capacitor or inductior** into the rest of the circuit with all independent sources turned to zero.