Filter Transfer Functions vs. Frequency

OUTLINE

• Transfer Function
• Common Filter Types
• First Order Lowpass and Highpass
• Bode Plots using notation of Hambley
• LC Filter Resonance

Reading
Hambley 6.4-6.7

Transfer Function

• Transfer function is a function of frequency
  – Complex quantity
  – Both magnitude and phase are function of frequency

\[ H(f) = \frac{V_{out}}{V_{in}} \angle \theta \]

\[ H(f) = \frac{V_{out}}{V_{in}} \angle (\theta_{out} - \theta_{in}) \]

Hambley Notation

Filters

• Circuit designed to retain a certain frequency range and discard others
  Low-pass: pass low frequencies and reject high frequencies
  High-pass: pass high frequencies and reject low frequencies
  Band-pass: pass some particular range of frequencies, reject other frequencies outside that band
  Notch: reject a range of frequencies and pass all other frequencies

Common Filter Transfer Function vs. Freq

First-Order Lowpass Filter

\[ H(f) = \frac{V_C}{V_i} = \frac{1/(j\omega C)}{1/(j\omega C) + R} = \frac{1}{1 + j\omega RC} = \frac{1}{\sqrt{1 + (\omega RC)^2}} \angle -\tan^{-1}(\omega RC) \]

Let \( \omega_a = \frac{1}{RC} \) and \( f_a = \frac{1}{2\pi RC} \)

\[ H(f) = H(f_a) \angle \theta \]

Hambley Notation

First-Order Highpass Filter

\[ H(f) = \frac{V_C}{V_i} = \frac{R}{1/(j\omega C) + R} = \frac{1/(j\omega RC)}{1 + j\omega RC} = \frac{1}{\sqrt{1 + (\omega RC)^2}} \angle \frac{\pi}{2} \tan^{-1}(\omega RC) \]

\[ H(f_a) = \frac{1}{\sqrt{2}} = 2^{-1/2} \]

\[ 20\log_{10}\left(\frac{H(f_a)}{H(0)}\right) = 20\left(-\frac{1}{2}\right)\log_{10}2 = -3 \text{ dB} \]
First-Order Lowpass Filter
\[
H(f) = \frac{1}{\sqrt{\frac{1}{f_c^2} + 1}} \frac{f}{f_c} - \tan^{-1} \left( \frac{f}{f_c} \right)
\]

Let \( f_c = \frac{R}{L} \) and \( f_0 = \frac{R}{2\pi L} \).

First-Order Highpass Filter
\[
H(f) = \frac{1}{\sqrt{\left( \frac{f}{f_0} \right)^2 + 1}} \frac{f}{f_0} - \tan^{-1} \left( \frac{f}{f_0} \right)
\]

Change of Voltage or Current with A Change of Frequency

One may wish to specify the change of a quantity such as the output voltage of a filter when the frequency changes by a factor of 2 (an octave) or 10 (a decade).

For example, a single-stage RC low-pass filter has at frequencies above \( \omega = 1/RC \) an output that changes at the rate -20dB per decade.

Bode Plot
- Plot of magnitude of transfer function vs. frequency
  - Both x and y scale are in log scale
  - Y scale in dB
- Log Frequency Scale
  - Decade \( \rightarrow \) Ratio of higher to lower frequency = 10
  - Octave \( \rightarrow \) Ratio of higher to lower frequency = 2

High-frequency asymptote of Lowpass filter
The high frequency asymptote of magnitude Bode plot assumes -20dB/decade slope

As \( f \rightarrow \infty \)
\[
H(f) \approx -20 \log_{10} \left( \frac{f}{f_0} \right)
\]

Hambley Notation
**Bode Plot: Label as dB**

![Bode Plot Diagram](image)

**Low-frequency asymptote of Highpass filter**

As $f \to 0$

$$H(f) = \left(\frac{f}{f_0}\right) \to \left(\frac{f}{f_0}\right)$$

$$20 \log_{10} \left(\frac{H(f)}{H(0.1f_0)}\right) = 20\text{dB}$$

The low frequency asymptote of magnitude

Bode plot assumes 20dB/decade slope

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**Series Resonance**

Voltage divider

Substitute branch elements

Arrange in resonance form

Maximum when $w^2 = 1/(LC)$

**Parallel Resonance**

Admittance

Substitute branch elements

Arrange in resonance form

Maximum = $I_R/R$ when $w^2 = 1/(LC)$

Bandwidth is $f_0/Q$