# Phasors: Eq. Circuits and Power

**Lecture 18, 10/10/05**

**OUTLINE**

- Frequency Response for Characterization
- Asymptotic Frequency Behavior
- Log magnitude vs log frequency plot
- Phase vs log frequency plot
- dB scale
- Transfer function example

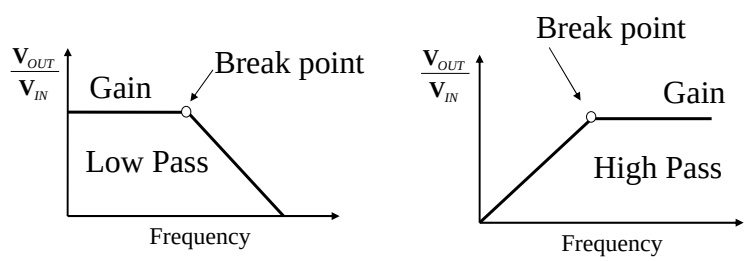
**Reading**

Hambley 6.1-6.4



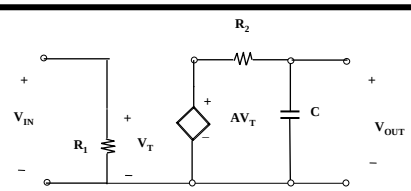
# Frequency Response

- The shape of the frequency response of the complex ratio of phasors $\mathbf{V}_{OUT}/\mathbf{V}_{IN}$ is a convenient means of classifying a circuit behavior and identifying key parameters.




FYI: These are log ratio vs log frequency plots

## Example Circuit




$$TransferFunction = \frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}}$$

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{AZ_c}{Z_R + Z_c}$$

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{A(1/jwC)}{R_2 + 1/j\omega C)} = \frac{A}{(1 + j\omega R_2 C)}$$

A = 100

$R_1$ = 100,000 Ohms

$R_2$ = 1000 Ohms

C = 1 uF



## Asymptotic Behavior of Transfer Functions

Ratio of polynomials form

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{A + j\omega B}{C + j\omega D}$$

Low frequency asymptotic limit

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{A + j\omega B}{C + j\omega D}_{\omega \to 0} = \frac{A}{C}$$

High frequency asymptotic limit

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{A + jwB}{C + j\omega D}_{\omega \to \infty} = \frac{B}{D}$$

Special cases

When A = 0 => $\omega^1$

When C = 0 => $\omega^{-1}$

When B = 0 => $\omega^{-1}$

When D = 0 => $\omega^{+1}$

## Break Points of Transfer Functions

Ratio of polynomials form

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{A + j\omega B}{C + j\omega D}$$

Numerator Break Point (slope change upward)

Occurs when A = |jωB| => $\omega_z$ = A/B

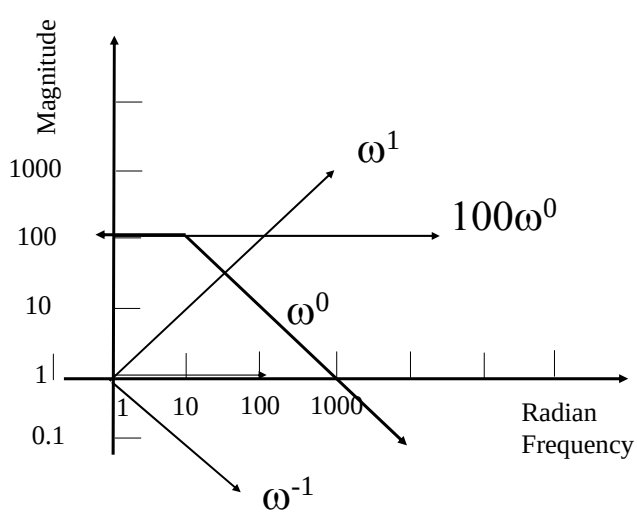
Denominator Break Point (slope change downward)

Occurs when C = |jωD| => $\omega_p$ = C/D

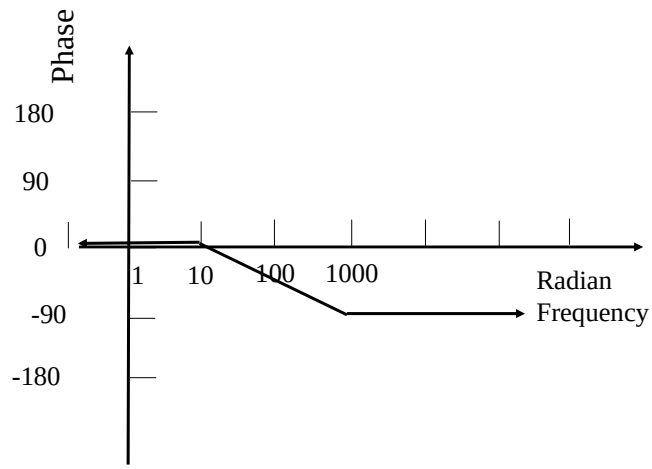
FYI: z is for zero of the numerator and p is for zero of the denominator giving one over zero or a pole. Filter design consists of specifying frequency locations of zeros and poles.



## Log magnitude versus log frequency plot



## Phase versus log frequency






## Example: Circuit in Slide #3 Magnitude

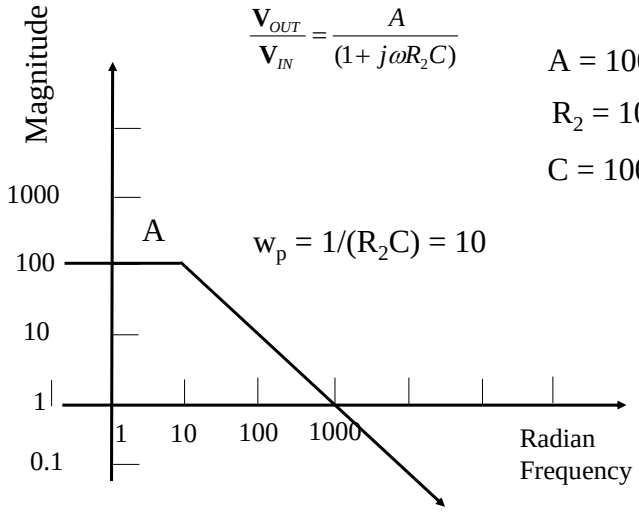
$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{A}{(1 + j\omega R_2 C)}$$

A = 100

$R_2$ = 1000 Ohms

C = 100 uF

$w_p = 1/(R_2C) = 10$



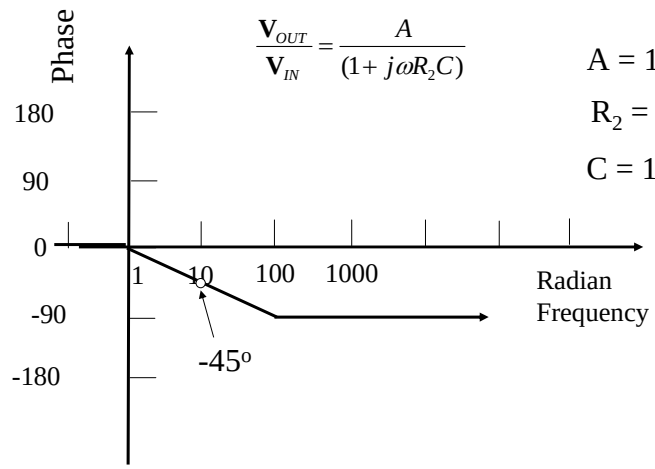
## Example: Circuit in Slide #3 Phase

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{A}{(1 + j\omega R_2 C)}$$

A = 100

$R_2$ = 1000 Ohms

C = 100 uF






## Bel and Decibel (dB)

- A **bel** (symbol **B**) is a unit of measure of ratios of power levels, i.e. relative power levels.
  - The name was coined in the early 20th century in honor of Alexander Graham Bell, a telecommunications pioneer.
  - The bel is a logarithmic measure. The number of bels for a given ratio of power levels is calculated by taking the logarithm, to the base 10, of the ratio.
  - one bel corresponds to a ratio of 10:1.
  - B = $\log_{10}(P_1/P_2)$ where $P_1$ and $P_2$ are power levels.
- The bel is too large for everyday use, so the **decibel (dB)**, equal to 0.1B, is more commonly used.
  - 1dB = 10 $\log_{10}(P_1/P_2)$
- dB are used to measure
  - Electric power, Gain or loss of amplifiers, Insertion loss of filters.

## Logarithmic Measure for Power

- To express a power in terms of decibels, one starts by choosing a reference power, $P_{reference}$, and writing

  Power P in decibels = $10 \log_{10}(P/P_{reference})$
- Exercise:
  - Express a power of 50 mW in decibels relative to 1 watt.
  - P (dB) =$10 \log_{10} (50 \times 10^{-3})$ = - 13 dB
- Exercise:
  - Express a power of 50 mW in decibels relative to 1 mW.
  - P (dB) =$10 \log_{10} (50)$ = 17 dB.
- dBm to express absolute values of power relative to a milliwatt.
  - dBm = $10 \log_{10}$ (power in milliwatts / 1 milliwatt)
  - 100 mW = 20 dBm
  - 10 mW = 10 dBm



## Break Point Values

- **When dealing with resonant circuits it is convenient to refer to the frequency difference between points at which the power from the circuit is half that at the peak of resonance.**
- **Such frequencies are known as "half-power frequencies", and the power output there referred to the peak power (at the resonant frequency) is**
- **$10\log_{10}(P_{half\text{-}power}/P_{resonance}) = 10\log_{10}(1/2) = -3$ dB.**

## Logarithmic Measures for Voltage or Current

From the expression for power ratios in decibels, we can readily derive the corresponding expressions for voltage or current ratios.

Suppose that the voltage $V$ (or current $I$) appears across (or flows in) a resistor whose resistance is $R$. The corresponding power dissipated, $P$, is $V^2/R$ (or $I^2R$). We can similarly relate the reference voltage or current to the reference power, as

$$P_{\text{reference}} = (V_{\text{reference}})^2/R \text{ or } P_{\text{reference}} = (I_{\text{reference}})^2 R.$$

*Hence,*

Voltage, V in decibels = $20\log_{10}(V/V_{\text{reference}})$
Current, I, in decibels = $20\log_{10}(I/I_{\text{reference}})$



## Logarithmic Measures for Voltage or Current

**Note that the voltage and current expressions are just like the power expression except that they have 20 as the multiplier instead of 10 because power is proportional to the square of the voltage or current.**

**Exercise: How many decibels larger is the voltage of a 9-volt transistor battery than that of a 1.5-volt AA battery? Let $V_{\text{reference}}$ = 1.5. The ratio in decibels is**

$$20 \log_{10}(9/1.5) = 20 \log_{10}(6) = 16 \text{ dB}.$$

## Logarithmic Measures for Voltage or Current

The gain produced by an amplifier or the loss of a filter is often specified in decibels.

The input voltage (current, or power) is taken as the reference value of voltage (current, or power) in the decibel defining expression:

Voltage gain in dB = $20 \log_{10}(V_{output}/V_{input})$
Current gain in dB = $20\log_{10}(I_{output}/I_{input}$
Power gain in dB = $10\log_{10}(P_{output}/P_{input})$

Example: The voltage gain of an amplifier whose input is 0.2 mV and whose output is 0.5 V is

$$20\log_{10}(0.5/0.2\text{x}10^{-3}) = 68 \text{ dB}.$$



## Bode Plot: Label as dB

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{A}{(1 + j\omega R_2 C)}$$

A = 100

$R_2$ = 1000 Ohms

C = 100 uF