Phasors: Eq. Circuits and Power

Lecture 17, 10/10/05

OUTLINE

- Phasor Re-Re-Cap
- Re-Cap Circuit Analysis with Phasors
- Phasor Equivalent Circuits
- Instantaneous and Time-Average Power
- Power and Reactive Power
- Maximum Power Transfer

Reading

Hambley 5.5-5.6

Midterm #1 Results

- Summary of grade will appear here once Exams are graded. Hopefully the exams will be graded in time to return results on Oct. 10th.

Phasors

- Assuming a source voltage is a sinusoid time-varying function

$$v(t) = V\cos(\omega t + \theta)$$

Note: Amplitude is V, frequency f is $\omega/2\pi$, period T is 1/f head start is θ.

- We can write:

$$v(t) = V\cos(\omega t+\theta) = V\,\mathrm{Re}\left[e^{j(\omega t+\theta)}\right] = \mathrm{Re}\left[Ve^{j(\omega t+\theta)}\right]$$

$$\textit{Define Phasor as } Ve^{j\theta} = V\angle\theta$$

Computing the Current

$$i(t) = C\frac{dv(t)}{dt} = C\frac{d\,\mathrm{Re}\{\mathbf{V}e^{j\omega t}\}}{dt}\,C\frac{d\,\mathrm{Re}\{V_M e^{j\omega t+j\theta}\}}{dt}$$

$$i(t) = \mathrm{Re}\{j\omega C V_M e^{j\omega t+j\theta}\} = \mathrm{Re}\{\mathbf{Y}V_M e^{j\omega t+j\theta}\} = \mathrm{Re}\{\mathbf{YV}e^{j\omega t}\}$$

The Complex Ratio is Admittance.

$$\mathbf{Y} = j\omega C$$

The inverse is Impedance.

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1}{j\omega C}$$

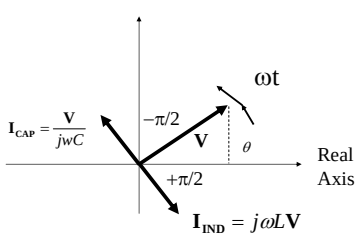
Note: The differentiation and integration operations become algebraic operations

$$\int dt \Rightarrow \frac{1}{j\omega} \qquad \frac{d}{dt} \Rightarrow j\omega$$

Phasor Diagrams

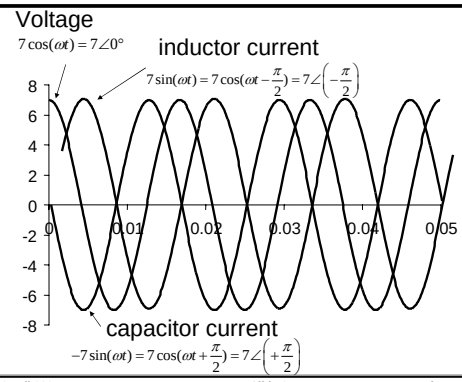
- A phasor diagram helps to visualize the relationships between currents and voltages.
  - Capacitor: I leads V by 90°
  - Inductor: V leads I by 90°

$$v(t) = V\cos(\omega t+\phi)$$

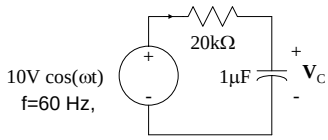


EL**I** the **I**C**E** man.

Phase



Circuit Analysis with Sinusoids



How and how hard is it to find $\mathbf{V}_C$?

The Old Way

Assume $\mathbf{V}_C$=Acos(ωt)+Bsin(ωt)

Substitute and take derivatives

Match time variations to get equations for A and B

Solve for A and B

Circuit Analysis Using Complex Impedances

- Suitable for AC steady state.
- KVL

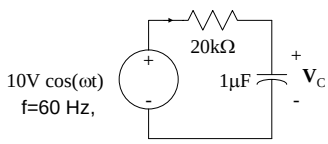
$$v_1(t)+v_2(t)+v_3(t)=0$$
$$V_1\cos(\omega t+\theta_1)+V_2\cos(\omega t+\theta_2)+V_3\cos(\omega t+\theta_3)=0$$
$$\mathrm{Re}\left[V_1e^{j(\omega t+\theta_1)}+V_2e^{j(\omega t+\theta_2)}+V_3e^{j(\omega t+\theta_3)}\right]=0$$

Phasor Form KVL

$$V_1e^{j(\theta_1)}+V_2e^{j(\theta_2)}+V_3e^{j(\theta_3)}=0$$
$$\mathbf{V_1}+\mathbf{V_2}+\mathbf{V_3}=0$$

- Phasor Form KCL $\mathbf{I_1}+\mathbf{I_2}+\mathbf{I_3}=0$
- Use complex impedances for inductors and capacitors and follow same analysis as in chap 2.

Circuit Analysis with Phasors



How and how hard is it to find $\mathbf{V}_C$?

The New Phasor Way:

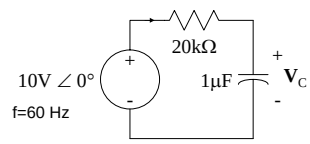
Assume $\mathbf{V}_C$= $\mathbf{V}e^{j\theta}$

Compute Impedances of R and C

Use impedances in a Voltage Divider

Evaluate the complex ratio to get $\mathbf{V}e^{j\theta}$

Example: Single Loop Circuit



$\mathbf{Z}_R$ = R= 20kΩ = 20kΩ ∠ 0°

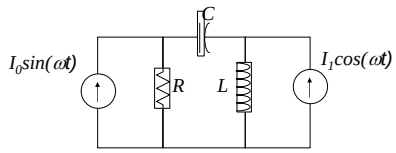
$\mathbf{Z}_C$ = 1/*j* (2πf x 1μF) = 2.65kΩ ∠ -90°

$$\mathbf{V}_C = 10\text{V}\angle 0°\left(\frac{2.65\text{k}\Omega\angle -90°}{2.65\text{k}\Omega\angle -90° + 20\text{k}\Omega\angle 0°}\right)$$

$$\mathbf{V}_C = 1.31\text{V}\angle -82.4°$$

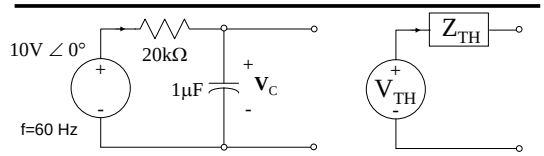
$V_C(t)$ = 1.31V cos(ωt-82.8°)

Steady-State AC Node-Voltage Analysis



- Nodal analysis or mesh?
- What are the nodes (or meshes)?
- What happens if the sources are at different frequencies?

Thevenin Equivalent



$\mathbf{Z}_R$ = R= 20kΩ = 20kΩ ∠ 0°

$\mathbf{Z}_C$ = 1/*j* (2πf x 1μF) = 2.65kΩ ∠ -90°

$$\mathbf{V}_{TH} = \mathbf{V}_{OC} = 10\text{V}\angle 0°\left(\frac{2.65\text{k}\Omega\angle -90°}{2.65\text{k}\Omega\angle -90° + 20\text{k}\Omega\angle 0°}\right) = 1.31\angle -82.4$$

$$\mathbf{Z}_{TH} = \mathbf{Z}_R \,||\, \mathbf{Z}_C = °\left(\frac{20\text{k}\Omega\angle 0°\cdot 2.65\text{k}\Omega\angle -90°}{2.65\text{k}\Omega\angle -90° + 20\text{k}\Omega\angle 0°}\right) = 2.62\angle -82.4$$

Root Mean Square (rms) Values

- rms valued defined as

$$v_{RMS} = \sqrt{\frac{1}{T}\int_o^T v^2(t)dt} \qquad \text{T = period}$$

- Assuming a sinusoid gives

$$v_{RMS} = \sqrt{\frac{1}{T}\int_o^T v_m{}^2 \cos^2(\omega t + \theta)dt}$$

- Using an identity gives

$$v_{RMS} = \sqrt{\frac{v_m{}^2}{2T}\int_o^T [1 + \cos(2\omega t + 2\theta)]dt}$$

- Evaluating at limits gives

$$v_{RMS} = \sqrt{\frac{v_m{}^2}{2T}[T + \frac{1}{2\omega}\sin(2\omega T + 2\theta) - \frac{1}{2w}\sin(2\theta)]} \qquad v_{RMS} = \frac{v_m}{\sqrt{2}}$$

Power: Instantaneous and Time-Average

For a Resistor

- The instantaneous power is $p(t) = v(t)i(t) = \frac{v(t)^2}{R}$
- The time-average power is

$$P_{AVE} = \frac{1}{T}\int_0^T \underbrace{p(t)}_{v(t)i(t} dt = \frac{1}{T}\int_0^T \frac{v(t)^2}{R}dt = \frac{1}{R}[\frac{1}{T}\int_0^T v(t)^2 dt] = \frac{v_{rms}^2}{R}$$

For an Impedance

- The instantaneous power is

$$p(t) = v(t)i(t)$$

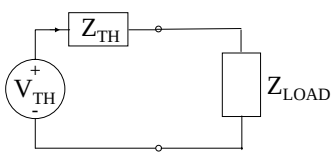
- The time-average power is

$$P_{AVE} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T v(t)i(t)dt = \mathbf{V}_{rms} \cdot \mathbf{I}_{rms}{}^*$$

- The reactive power at 2ω is

$$Q = \mathbf{V}_{rms} \cdot \mathbf{I}_{rms} \qquad P_{AVE}^2 + Q^2 = (V_{rms} \cdot I_{rms})^2$$

Maximum Average Power Transfer



- Maximum time average power occurs when

$$\mathbf{Z}_{LOAD} = \mathbf{Z}_{TH}^*$$

- This presents a resistive impedance to the source

$$\mathbf{Z}_{total} = \mathbf{Z}_{TH} + \mathbf{Z}_{TH}^*$$

- Power transferred is

$$P_{AVE} = \mathbf{V}\mathbf{I}^* = \mathbf{V}\frac{\mathbf{V}^*}{2R} = \tfrac{1}{2}\frac{V_{rms}^2}{R}$$