Phasors: Eq. Circuits and Power

OUTLINE

• Phasor Re-Re-Cap
• Re-Cap Circuit Analysis with Phasors
• Phasor Equivalent Circuits
• Instantaneous and Time-Average Power
• Power and Reactive Power
• Maximum Power Transfer

Reading
Hambley 5.5-5.6

Midterm #1 Results

• Summary of grade will appear here once
Exams are graded. Hopefully the exams
will be graded in time to return results on
Oct. 10th.

Phasors

• Assuming a source voltage is a sinusoid time-varying function

\[ v(t) = V \cos(\omega t + \theta) \]

Note: Amplitude is V, frequency f is \( \omega/2\pi \), period T is 1/f, head start is \( \theta \).

• We can write:

\[ v(t) = V e^{j \omega t} = V e^{j(\omega t + \theta)} \]

Define Phasor as \( V e^{j\theta} \)

Computing the Current

\[ i(t) = C \frac{dv(t)}{dt} = C \frac{d}{dt} \text{Re}\{Ve^{j(\omega t + \theta)}\} \]

\[ i(t) = \text{Re}\{j\omega CV e^{j(\omega t + \theta)}\} = \text{Re}\{YVe^{j(\omega t + \theta)}\} = \text{Re}\{YVe^{j\omega t}\} \]

The Complex Ratio is Admittance. The inverse is Impedance.

\[ Y = j\omega C \quad Z = \frac{1}{Y} = \frac{1}{j\omega C} \]

Note: The differentiation and integration operations become algebraic operations

\[ \int \frac{dt}{j\omega} = \frac{1}{j\omega} \quad \frac{d}{dt} \Rightarrow j\omega \]

Phasor Diagrams

• A phasor diagram helps to visualize the relationships between currents and voltages.
  - Capacitor: I leads V by 90°
  - Inductor: V leads I by 90°

\[ i(t) = V \cos(\omega t + \theta) \]

\[ i(t) = j\omega V \]

\[ \theta \]

Voltage

Phase

Voltage

\[ 7 \cos(\omega t + \theta) \]

\[ 7 \sin(\omega t + \theta) \]

\[ 7 \sin(\omega t + \theta) \]

\[ 7 \cos(\omega t + \theta) \]

\[ \pi \]

\[ \pi \]

\[ \pi \]

\[ \pi \]

\[ \pi \]

\[ \pi \]

\[ \pi \]

\[ \pi \]

\[ \pi \]

\[ \pi \]

\[ \pi \]
Circuit Analysis with Sinusoids

\[ 10V \cos(\omega t) \]

\[ f=60 \text{ Hz,} \]

How and how hard is it to find \( V_C \)?

The Old Way
Assume \( V_C = A \cos(\omega t) + B \sin(\omega t) \)
Substitute and take derivatives
Match time variations to get equations for \( A \) and \( B \)
Solve for \( A \) and \( B \)

\( f=60 \text{ Hz,} \]

How and how hard is it to find \( V_C \)?

The New Phasor Way:
Assume \( V_C = V e^{j\theta} \)
Compute Impedances of \( R \) and \( C \)
Use impedances in a Voltage Divider
Evaluate the complex ratio to get \( V e^{j\theta} \)

Example: Single Loop Circuit

\[ 10V \angle 0^\circ \]

\[ f=60 \text{ Hz} \]

\( Z_{TH} \)

Steady-State AC Node-Voltage Analysis

- Nodal analysis or mesh?
- What are the nodes (or meshes)?
- What happens if the sources are at different frequencies?
**Root Mean Square (rms) Values**

- **rms valued defined as**
  \[ \text{rms}_{\text{v}} = \frac{1}{T} \int_{0}^{T} |v(t)|^2 dt \quad T = \text{period} \]
- **Assuming a sinusoid gives**
  \[ \text{rms}_{\text{v}} = \frac{1}{T} \int_{0}^{T} v(t)^2 dt \]
- **Using an identity gives**
  \[ \text{rms}_{\text{v}} = \frac{1}{2T} \left[ \cos(2\omega T + \theta) \right] \]
- **Evaluating at limits gives**
  \[ \text{rms}_{\text{v}} = \frac{1}{2T} \left[ \cos(2\omega T + \theta) \right] \]

\[ \text{v}_{\text{rms}} = \frac{\sqrt{2}}{2} \left( \cos(2\omega T + \theta) \right) \]
\[ \text{v}_{\text{rms}} = \frac{V}{\sqrt{2}} \]

**Power: Instantaneous and Time-Average**

**For a Resistor**
- The instantaneous power is
  \[ p(t) = v(t)i(t) = \frac{v(t)^2}{R} \]
- The time-average power is
  \[ P_{\text{rms}} = \frac{1}{T} \int_{0}^{T} p(t) dt = \frac{1}{T} \int_{0}^{T} \left( \frac{v(t)^2}{R} \right) dt = \frac{V^2}{2R} \]

**For an Impedance**
- The instantaneous power is
  \[ p(t) = v(t)i(t) \]
- The time-average power is
  \[ P_{\text{rms}} = \frac{1}{T} \int_{0}^{T} p(t) dt = \frac{1}{T} \int_{0}^{T} \left( v(t)i(t) \right) dt \]
- The reactive power at \( 2\omega \) is
  \[ Q = V_{\text{rms}} I_{\text{rms}} \]
  \[ P_{\text{rms}} + Q = (V_{\text{rms}} I_{\text{rms}}) \]

**Maximum Average Power Transfer**

- Maximum time average power occurs when
  \[ Z_{\text{LOAD}} = Z_{\text{TH}} \]
- This presents a resistive impedance to the source
  \[ Z_{\text{LOAD}} = Z_{\text{TH}} + Z_{\text{TH}} \]
- Power transferred is
  \[ P_{\text{TH}} = V^2 \frac{1}{2R} \]

\[ \frac{\sqrt{2}}{2} \]