Node and Mesh Analysis

OUTLINE

- Node Analysis
  - Examples
  - Supernode
- Mesh Analysis
  - Method
  - Examples
  - Supermesh

Reading
Chapter 2.4-2.5

Node-Voltage Circuit Analysis Method

1. Choose a reference node ("ground")
   Look for the one with the most connections!
2. Define unknown node voltages
   those which are not fixed by voltage sources
3. Write KCL at each unknown node,
   expressing current in terms of the node voltages
   (using the I-V relationships of branch elements)
   Special cases: floating voltage sources
4. Solve the set of independent equations
   $N$ equations for $N$ unknown node voltages

Nodal Analysis: Example #1

1. Choose a reference node.
2. Define the node voltages (except reference node and
   the one set by the voltage source).
3. Apply KCL at the nodes with unknown voltage.
4. Solve for unknown node voltages.

Nodal Analysis: Example #2

Challenges:
Determine number of nodes needed
Deal with different types of sources

Nodal Analysis w/ "Floating Voltage Source"
A "floating" voltage source is one for which neither side is
connected to the reference node, e.g. $V_{LL}$ in the circuit below:

Problem: We cannot write KCL at nodes a or b because
there is no way to express the current through the voltage
source in terms of $V_a-V_b$.

Solution: Define a "supernode" – that chunk of the circuit
containing nodes a and b. Express KCL for this supernode.
Incorporate voltage source constraint into KCL equation.

Nodal Analysis: Example #3

Eq’n 1: KCL at supernode
Substitute property of voltage source:
Formal Circuit Analysis Methods

**NODAL ANALYSIS**
("Node-Voltage Method")

0) Choose a reference node
1) Define unknown node voltages
2) Apply KCL to each unknown node, expressing current in terms of the node voltages
   \( \Rightarrow \) \( N \) equations for \( N \) unknown node voltages
3) Solve for node voltages
   \( \Rightarrow \) determine branch currents

**MESH ANALYSIS**
("Mesh-Current Method")

1) Select \( M \) independent mesh currents such that at least one mesh current passes through each branch*
   \( M = \# \text{branches} - \# \text{nodes} + 1 \)
2) Apply KVL to each mesh, expressing voltages in terms of mesh currents
   \( \Rightarrow \) \( M \) equations for \( M \) unknown mesh currents
3) Solve for mesh currents
   \( \Rightarrow \) determine node voltages

*Simple method for planar circuits
A mesh current is not necessarily identified with a branch current.

Mesh Analysis: Example #1

1. Select \( M \) mesh currents.
2. Apply KVL to each mesh.
3. Solve for mesh currents.

Mesh Analysis with a Current Source

Problem: We cannot write KVL for meshes a and b because there is no way to express the voltage drop across the current source in terms of the mesh currents.

Solution: Define a "supermesh" – a mesh which avoids the branch containing the current source. Apply KVL for this supermesh.

Mesh Analysis: Example #2

Eq’n 1: KVL for supermesh

Eq’n 2: Constraint due to current source:

Source Combinations

- Voltage sources in series can be replaced by an equivalent voltage source:
  \[ v_1 + v_2 \]

- Current sources in parallel can be replaced by an equivalent current source:
  \[ i_1 + i_2 \]

A Note of Caution

- These two resistive circuits are equivalent for voltages and currents external to the Y and \( \Delta \) circuits. Internally, the voltages and currents are different.

\[ R_i = \frac{R_i R_j}{R_i + R_j} \]
\[ R_2 = \frac{R_2 R_3}{R_2 + R_3} \]
\[ R_3 = \frac{R_3 R_4}{R_3 + R_4} \]

Brain Teaser Category: Important for motors and electrical utilities.
Circuit Simplification Example

Find the equivalent resistance $R_{ab}$:

\[ R_{ab} = 2 \Omega \]

Circuit Analysis Approaches

- The Node-Voltage method can always be used to solve a circuit, but techniques for simplifying circuits (using “equivalent circuits”) are useful:
  - series and parallel combination reductions
  - Δ-Y and Y-Δ conversions
  - source transformations
  - Superposition (to be covered in Lecture 6)
  - Thevenin and Norton equivalent circuits (to be covered in Lecture 6)