EE40 Fa05 HW #2 Solutions

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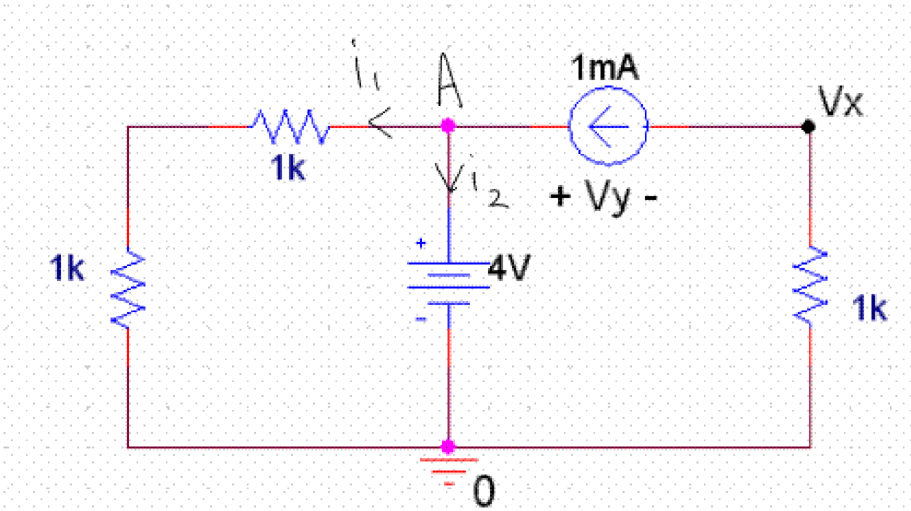
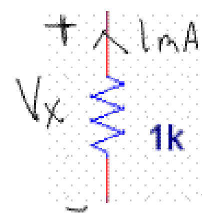


1 (a) KCL:

Node A: $1mA = i_1 + i_2$

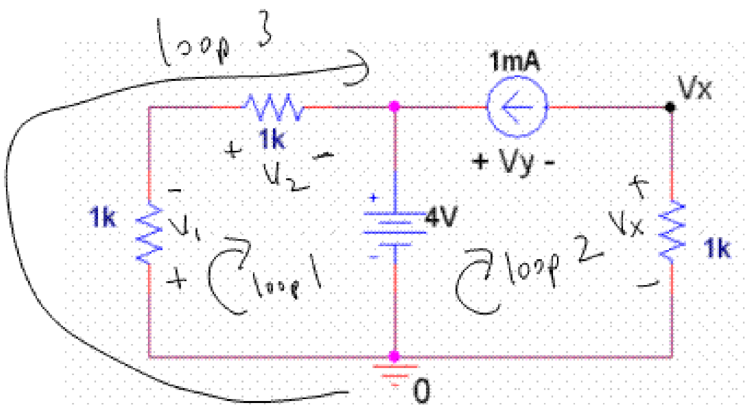
(b) $V_x = ?$

We use Ohm's law:



$\Leftrightarrow V_x = -(1mA)(1k)$

$\Rightarrow \boxed{V_x = -1\ V}$



(c) KVL:

loop 1: $V_1 + V_2 + 4 = 0$ — ①

loop 2: $4 = V_y + V_x$ — ② ①+②

loop 3: $V_1 + V_2 + V_y + V_x = 0$ — ③

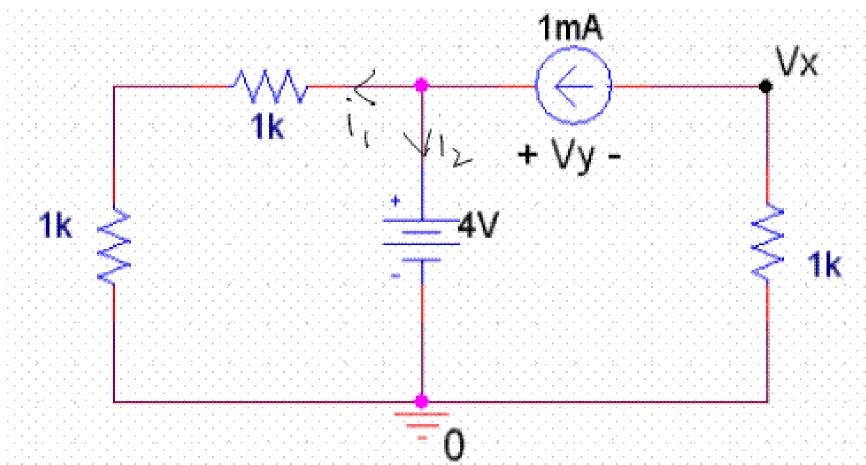
(d) V_y can be found from ②:

$$V_y = 4 - V_x = 4 - (-1) = 5\,V$$

$$\Rightarrow \boxed{V_y = 5\,V}$$

Notice how voltage across a current source need not be zero!

(e) We can use the fact that the 4 V is across a series combination of the two 1ks to find i_1



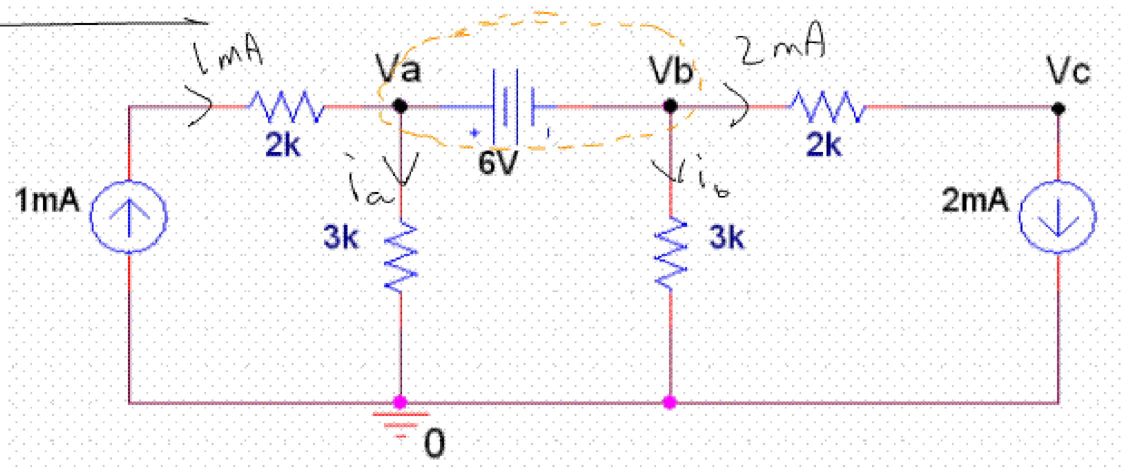
$$i_1 = \frac{4\,V}{2\,k} = 2\,mA$$

$$\therefore \quad i_2 = 1\,mA - i_1 = \underline{\underline{-1\,mA}}$$

Hence:

	$P_{delivered}$	$P_{absorbed}$
4V:	$vi = 4\,mW$	$i_1^2 R = 4\,mW$: 1k
1mA:	$vi = 5\,mW$	$i_1^2 R = 4\,mW$: 1k
		$(1mA)^2 R = 1\,mW$: 1k
checks! ← Total:	9 mW	= 9 mW

(2) Nodal analysis



Notice that we have a 6 V floating voltage source

$\Rightarrow$ Supernode!

KCL at supernode:

$$\sum(\text{currents in}) = \sum(\text{currents out})$$

$$1\,mA = i_a + i_b + 2\,mA$$

Applying Ohm's law for i_a and i_b:

$$\boxed{1\,mA = \frac{V_a}{3k} + \frac{V_b}{3k} + 2\,mA} \quad \text{—} (1)$$

Now, we need the constraint equation, which we get by "looking inside" the supernode:

$$\boxed{V_a - V_b = 6\,V} \quad \text{—} (2)$$

Finally, we can use Ohm's law to find V_c: $\boxed{2\,mA = \frac{V_b - V_c}{2k}}$ — (3)

$$(1) \Rightarrow \quad 3 = V_a + V_b + 6 \Rightarrow V_a + V_b = -3$$

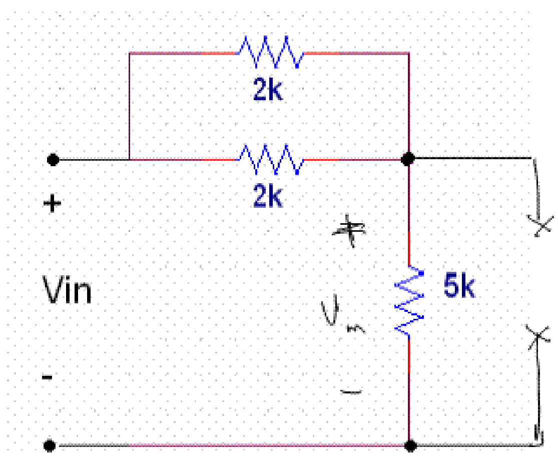
$$\Rightarrow (6 + V_b) + V_b = -3 \quad [\text{from } (2)]$$

$$\Rightarrow V_b = \frac{-9}{2} \Rightarrow \boxed{V_b = -4.5\,V}$$

∴ (2) $\Rightarrow$ $V_a = 6 + V_b \Rightarrow \boxed{V_a = 1.5\ V}$

(3) $\Rightarrow$ $V_c = V_b - 4 \Rightarrow \boxed{V_c = -8.5\ V}$

(3)

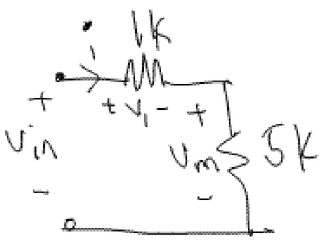


(a) This part is easy!

Voltmeter is ideal, therefore voltmeter internal resistance $\rightarrow \infty$

And, you can combine resistors in series & parallel:

$$2k \| 2k = 1k$$

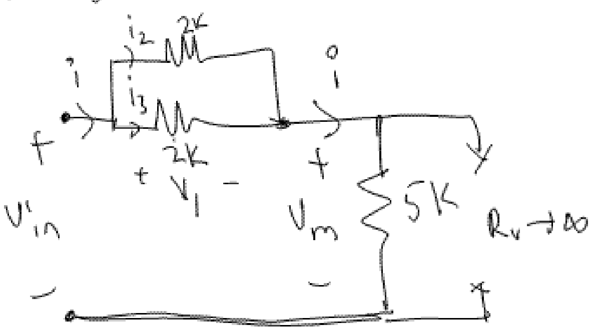


$$\therefore V_{in} = V_1 + V_m \Rightarrow V_{in} = i(1k) + i(5k)$$

$$\Rightarrow i = \frac{V_{in}}{6k}$$

$\therefore \quad V_m = i(5k) \Rightarrow \boxed{V_m = \frac{5 V_{in}}{6}}$

(b) Now, we have to use KCL, KVL & ohm's law:



$$i = i_2 + i_3 \quad (KCL)$$

$$i = \frac{V_1}{2k} + \frac{V_1}{2K}$$

Now, $V_m = i(5k)$ (ohm's law)

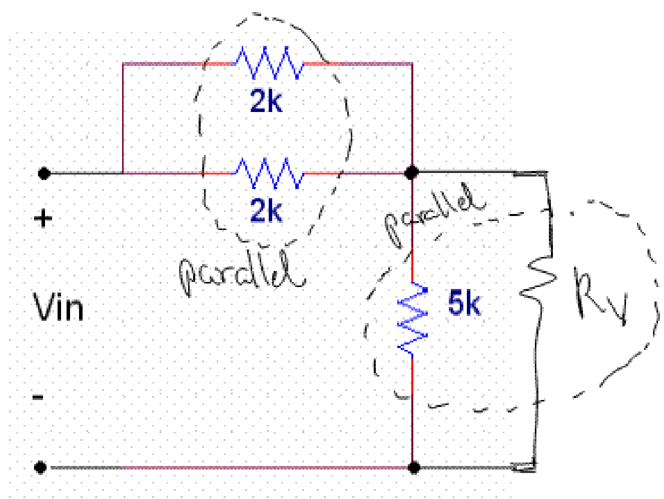
$$= \left(\frac{V_1}{2k} + \frac{V_1}{2k}\right) 5K \quad \text{— } 1$$

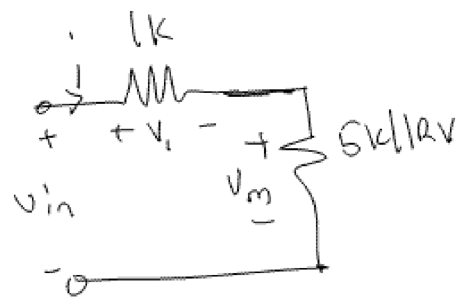
But, $V_1 = V_{in} - V_m$ (KVL) — 2

Substituting (2) in (1) and simplifying:

$$\boxed{V_m = \frac{5}{6} V_{in}}$$

(C) Now, we assume $R_V \not\to \infty$ (i.e. R_V is finite):





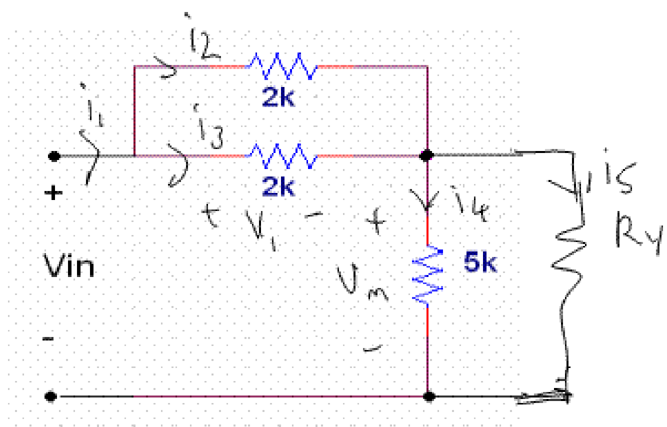
Now, $V_{in} = i\,(1k + 5k\|R_V)$

and $V_m = i\,(5k\|R_V)$

Therefore: $$V_m = \frac{V_{in}}{1k + 5k\|R_V}\, 5k\|R_V \Rightarrow \boxed{V_m = \frac{5k\,R_V}{6k\,R_V + 5k \cdot 1k} \cdot V_{in}}$$

Check: $R_V \to \infty$, $$\boxed{V_m = \frac{5}{6} V_{in}}$$

(d)



KCL: $i_1 = i_2 + i_3$ — ①, $i_1 = i_4 + i_5$ — ②

KVL: $V_{in} = V_1 + V_m$ — ③

Ohm's law: $V_m = i_4(5k)$

$= i_5(R_y)$

$V_1 = i_3(2k) = i_2(2k)$

Want: $V_m = f(V_{in}, R_y)$

① $\Rightarrow$ $i_1 = i_2 + i_3$

$\Rightarrow$ $i_1 = \dfrac{V_1}{2k} + \dfrac{V_1}{2k}$ — ④

② $\Rightarrow$ $i_1 = i_4 + i_5$

$\Rightarrow$ $i_1 = \dfrac{V_m}{5k} + \dfrac{V_m}{R_y}$ — ⑤

③ $\Rightarrow$ $V_{in} = V_1 + V_m$

$\Rightarrow$ $V_1 = V_{in} - V_m$ — ⑥

Therefore, ④ = ⑤

$$\Rightarrow \quad \frac{V_m}{5k} + \frac{V_m}{R_v} = \frac{V_1}{2k} + \frac{V_1}{2k}$$

Substituting for V_1 from (6) & Simplifying:

$$\boxed{V_m = \frac{5k\,R_v}{6k\,R_v + 5k \cdot 1k} \cdot V_{in}}$$

(e) If $R_v = 20\,M$,

$$V_m = \left[\frac{5k}{6k + \frac{5k \cdot 1k}{R_v}}\right] V_{in} \Rightarrow V_m = \left[\frac{5k}{6k + \frac{5k \cdot 1k}{20M}}\right] V_{in}$$

$$\boxed{V_m = \frac{5k}{6k + 0.25} V_{in} \approx 0.833299}$$

Actual: $V_m = \frac{5}{6} V_{in} \Rightarrow \boxed{V_m = 0.8\overline{3}\, V_{in}}$

$\therefore$ % error = $\frac{|\text{observed} - \text{actual}|}{\text{actual}} \times 100$

$\boxed{\% \text{ error} \approx 4.2 \times 10^{-3} \%}$